Distributed Opportunistic Scheduling for Ad-Hoc Communications Under Delay Constraints

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Abstract—With the convergence of multimedia applications and wireless communications, there is an urgent need for developing new scheduling algorithms to support real-time traffic with stringent delay requirements. However, distributed scheduling under delay constraints is not well understood and remains an under-explored area. A main goal of this study is to take some steps in this direction and explore the distributed opportunistic scheduling (DOS) with delay constraints. Consider a network with M links which contend for the channel using random access. Distributed scheduling in such a network requires joint channel probing and distributed scheduling. Using optimal stopping theory, we explore DOS for throughput maximization, under two different types of average delay constraints: 1) a network-wide constraint where the average delay should be no greater than α; or 2) individual user constraints where the average delay per user should be no greater than α_m, $m = 1, \ldots, M$. Since the standard techniques for constrained optimal stopping problems are based on sample-path arguments and are not applicable here, we take a stochastic Lagrangian approach instead. We characterize the corresponding optimal scheduling policies accordingly, and show that they have a pure threshold structure, i.e. data transmission is scheduled if and only if the rate is above a threshold. Specifically, in the case with a network-wide delay constraint, somewhat surprisingly, there exists a sharp transition associated with a critical time constant, denoted by α^*. If α is less than α^*, the optimal rate threshold depends on α; otherwise it does not depends on α at all, and the optimal policy is the same as that in the unconstrained case. In the case with individual user delay constraints, we cast the threshold selection problem across links as a non-cooperative game, and establish the existence of Nash equilibria. Again we observe a sharp transition associated with critical time constants $\{\alpha_m^*\}$, in the sense that when $\alpha_m \geq \alpha_m^*$ for all users, the Nash equilibrium becomes the same one as if there were no delay constraints.

I. Introduction

Channel-aware scheduling for achieving the rich diversity inherent in wireless communications has recently emerged as a promising technique for improving spectrum efficiency in wireless networks. Most existing studies along this line require centralized scheduling [1], [2], [3], [4], and little work has

been done on developing distributed algorithms to provide diversity gains for ad hoc communications, partially due to the challenge of distributed learning of time-varying channel information.

Recent work [5] has taken some initial steps to develop distributed opportunistic scheduling (DOS) to reap multiuser diversity and time diversity in wireless ad hoc networks. The basic idea of DOS is that once a successful channel probing has been made (through a successful channel contention), the successful link may decide to continue data transmission if the observed channel condition is "good"; otherwise, it may skip the transmission, and let all the links re-contend for the channel, in the hope that some links with better channel conditions can transmit after the re-contention. Intuitively speaking, for time varying channel conditions, different links at different time slots experience different channel conditions. It is likely that after further probing, the channel can be taken by a link with a better channel condition, resulting in higher network throughput. Hence, the multiuser diversity across links and the time diversity across slots can be exploited in a joint manner. On the other hand, each channel probing comes with a cost in terms of time. Clearly, there is a *tradeoff* between the throughput gain from better channel conditions and the cost for further channel probing. The desired tradeoff boils down to judiciously choosing the optimal stopping time for channel probing and the transmission rate, in the sense of maximizing the overall network throughput. Using the optimal stopping theory [6], it has been shown in [5] that the optimal scheme turns out to be a pure threshold policy, and the threshold is the optimal network throughput.

While significant progress has been made on opportunistic scheduling algorithms, scheduling with delay guarantees is still not well understood. Many wireless applications, such as multimedia traffic, have stringent delay requirements. For example, a VOIP application typically requires an average delay less than 200ms to maintain a normal conversation. For other applications, such as live video streaming or monitoring traffic, the tolerance to delay is even smaller, and there may be a limited lifetime during which a packet's information remains valid. Then, the delivery of such packets has to be before the deadline, because otherwise the information would become

outdated and useless.

Unfortunately, little work has been done on developing distributed opportunistic scheduling while taking delay into consideration. Without delay constraints, it is possible that the system may spend an arbitrarily long period of time on channel probing, looking for better channel conditions. This may significantly degrade the QoS performance of delay-sensitive applications, for which the delay performance is of critical importance. Therefore, distributed opportunistic scheduling must strive not only to maximize the throughput, but also to meet the delay constraints. Needless to say, delay-driven scheduling is challenging, considering the distributed nature of ad hoc communications. A main objective of this study is to obtain a rigorous understanding of DOS under delay constraints, which is known to be important but hard.

In this paper, we study DOS under the average delay constraint from two different perspectives, namely, network-wide average delay constraint and user-specific average delay constraint. Specifically, average delay constraint here refers to an ensemble constraint after taking average over many packet transmissions.

First, we consider the average delay constraint from a network-centric point of view, where links cooperate to maximize the overall network throughput, subject to the constraint that the *network-wide* average probing and transmission time is no greater than a given time constant α. We note that the *network-wide average delay constraint* is applicable to applications such as event monitoring by sensor networks where a group of sensor nodes observe the same phenomenon and try to deliver the same messages to the sink node. Thus, a network-wide constraint is to ensure that every message reaches the sink node by a given deadline. Optimal scheduling under this network-wide average delay constraint is equivalent to a constrained optimal stopping problem. However, the standard techniques for constrained optimal stopping problems [7] cannot be used to solve our problem here. This is because those standard techniques are based on sample-path arguments, but the problem with average constraints involves averaging over many sample paths. Instead, we take a stochastic Lagrangian approach to transform the constrained problem into an unconstrained one. For the case where the rate follows a continuous distribution, we are able to show that the duality gap does not exist. Intuitively speaking, the continuity of channel fading distribution ensures that the channel exhibits sufficient randomization to close the gap between the constrained primal problem and the unconstrained dual problem [8]. We then characterize the corresponding threshold-based optimal scheduling algorithm and its throughput. Somewhat surprisingly, we find there exists a sharp transition for the optimal threshold tied to a critical time constant, α^*, in the sense that if α is less than α^*, the optimal threshold is upper-bounded by a function of α; otherwise, the imposed delay constraint has no impact on the optimal scheduling, and the optimal threshold is the same as that in the unconstrained case.

Next, we explore distributed scheduling under the average delay constraint from a user centric perspective, where each link seeks to maximize its own throughput subject to the constraint that the expected user delay should be no greater than its own delay constraint $\alpha_m, m = 1, \ldots, M$. We treat the threshold selection problem under the individual delay constraint as a non-cooperative game. We show that the Nash equilibrium exists for this constrained non-cooperative game. Our results reveal that there exists a vector of critical time constants $\{\alpha_m^*\}$, as the counterpart to that in the network-centric case, such that only when all the delay parameters $\alpha_m \geq \alpha_m^*$ can the Nash equilibrium become the one for the unconstrained case. We further provide an iterative algorithm to find the Nash equilibrium and show the convergence.

To the best of our knowledge, this is the first channel-aware distributed opportunistic scheduling under the delay constraints. We believe that these initial steps are useful for developing distributed scheduling for delay sensitive applications.

The rest of the paper is organized as follows. We present the system model, and provide the background on the distributed opportunistic scheduling without delay constraints in Section II. In Section III, we study DOS under average delay constraints from the network-centric perspective and the user-centric perspective. The numerical results are presented in Section IV. Finally, we draw our conclusions and discuss the future work in Section V.

II. SYSTEM MODEL AND BACKGROUND

We consider a single-hop collocated random access network with M links [5], where link m contends for the channel with probability $p_m, m = 1, \ldots, M$. A collision model is assumed for the random access, where a successful channel contention of a link means that no other links transmit at the same time. Accordingly, the overall successful contention probability, p_s, is then given by $\sum_{m=1}^{M} (p_m \prod_{i \neq m} (1 - p_i))$ [9]. It is clear that the number of slots (denoted as K) for a successful channel contention is a Geometric random variable, i.e., $K \sim Geometric(p_s)$. Let τ denote the duration of a mini-slot for channel contention. It follows that the random duration corresponding to one round of successful channel contention is $K\tau$, with expectation of τ/p_s. For convenience, we call the random duration of achieving a successful channel contention as one round of channel probing.
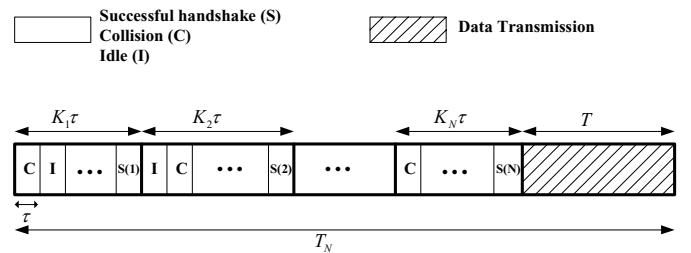

Fig. 1. Realization of channel probing and data transmission

To get a more concrete sense of the dynamics of joint channel probing and distributed scheduling, we depict in Fig.

1 a sample realization of N rounds of channel probing and one single data transmission. Let $s(n)$ denote the successful link at the n-th round of channel probing (successful channel contention) and $R_{n,s(n)}$ denote the corresponding transmission rate. Specifically, suppose after the first round of channel probing with a duration of K_1 slots, $R_{1,s(1)}$ is small due to poor channel condition, $s(1)$ will then give up its transmission opportunity and let all the links re-contend. This probing process continues for N rounds until link $s(N)$ transmit as link $s(N)$ has a high transmission rate with good channel condition.

In a wireless network, $R_{n,s(n)}$ is random since it depends on the time varying channel conditions. We assume that $R_{n,s(n)}, n = 1, 2, ...$ are statistically independent, which in general holds in many practical scenarios of interest. For convenience, let R_n denote the transmission rate corresponding to the n-th round successful channel probing, i.e., $R_n = R_{n,s(n)}$. Accordingly, R_n has the following compound distribution [5]:

$$P(R_n \leq r) = \sum_{m=1}^{M} \frac{p_{s,m}}{p_s} F_m(r), \tag{1}$$

where $p_{s,m} \triangleq p_m \prod_{i \neq m}(1 - p_i)$ is the successful probing probability of user m, and $F_m(\cdot)$ denotes the distribution for each link $m \in \{1, 2, \ldots, M\}$. In this study, we assume that $F_m(r)$ is continuous in r. Later, we will see that this continuous assumption on the channel statistics will ensure that there is no duality gap when the constrained problem is relaxed to an unconstrained dual problem, and hence is of critical importance to the existence of optimal solutions for the constrained problem.

A. Background: DOS Without Delay Constraint

In [5], we have studied distributed opportunistic scheduling (DOS) without delay constraint. Specifically, we have shown that the throughput maximization problem can be cast as a maximal rate of return problem in the optimal stopping theory [6], [10], where the rate of return is the average network throughput, x, given by

$$x = \frac{E[R_N T]}{E[T_N]} \quad \text{where} \quad T_n \triangleq \sum_{j=1}^{n} K_j \tau + T. \tag{2}$$

Note that N is a stopping time if $\{N = n\}$ and is $\mathcal{F}_n$-measurable, where $\mathcal{F}_n$ is the σ-field generated by $\{(\rho|h_j|^2, K_j), j = 1, 2, \ldots, n\}$.

The distributed opportunistic scheduling algorithm that maximize the network throughput is given by the optimal stopping rule, N^*, that solves the maximal rate of return problem in (2), i.e.,

$$N^* \triangleq \arg \max_{N \in Q} \frac{E[R_N T]}{E[T_N]}, \quad x^* \triangleq \sup_{N \in Q} \frac{E[R_N T]}{E[T_N]}, \tag{3}$$

where

$$Q \triangleq \{N : N \geq 1, E[T_N] < \infty\}. \tag{4}$$

We have shown that the optimal stopping rule for the above problem can be found if the channel coefficients $\{h_n, n =$

$1, 2, \ldots, \}$ are independent, and ρ is finite. For completeness, we restate the following result from [5].

Lemma 2.1: The optimal stopping rule N^* exists, and is given by

$$N^* = \min\{n \geq 1 : R_n \geq x^*\}. \tag{5}$$

The threshold in (5) is the maximum throughput x^*, which is the unique solution to

$$E(R - x)^+ = \frac{x\tau}{p_s T}, \tag{6}$$

where R is a random variable and has the same distribution as R_n.

III. DOS UNDER AVERAGE DELAY CONSTRAINTS

In this section, we generalize the above study to the case with average delay constraints. Specifically, we study the average time constraint from two different perspectives: a network-wide average delay constraint, or individual average delay constraints.

A. DOS under the Network-Centric Average Delay Constraint

In this section, we treat distributed opportunistic scheduling under average delay constraint from a network-centric perspective. To this end, we formulate the DOS as a cooperative game in which all the links collaborate to maximize the overall network throughput under the network-wide average delay constraint, which equivalent to impose an additional constraint on $E[T_N]$.

We have the following problem:

$$\mathbf{P_c} : \underset{N \in Q_\alpha}{\text{maximize}} \frac{E[R_N T]}{E[T_N]}, \tag{7}$$

where

$$Q_\alpha \triangleq \{N : N \geq 1, E[T_N] \leq \alpha\}. \tag{8}$$

Comparing Q_α with Q in (4), it can be seen that the newly imposed delay constraint dictates that the average duration of channel probing and transmission must be no greater than α.

For convenience, define

$$N_\alpha^* = \arg \max_{N \in Q_\alpha} \frac{E[R_N T]}{E[T_N]}, \quad x_\alpha^* \triangleq \sup_{N \in Q_\alpha} \frac{E[R_N T]}{E[T_N]}. \tag{9}$$

Clearly, α has to be greater than $\frac{\tau}{p_s} + T$, because even if the data transmission follows immediately after a successful channel contention regardless of the channel condition, it takes a duration of $\frac{\tau}{p_s} + T$ on average for the network to transmit a data packet. In other words, $\frac{\tau}{p_s} + T$ is the minimum achievable average delay.

Let $F_R(r)$ denote the continuous distribution of $\{R_n\}$ and α^* denote the unique solution to the following equation (in y):

$$E\left[\left(R_N - F_R^{-1}\left(1 - \frac{\tau}{p_s(y - T)}\right)\right)^+\right] \tag{10}$$

$$= \frac{\tau}{p_s T} F_R^{-1}\left(1 - \frac{\tau}{p_s(y - T)}\right).$$

We have the following result regarding the optimal scheduling policy N_α^* and the optimal throughput x_α^* for the constrained optimization problem $\mathbf{P_c}$.

Proposition 3.1: I) When $\alpha < \alpha^*$, the optimal scheduling policy N_α^* is given by

$$N_\alpha^* = \min\left\{ n \geq 1 : R_n \geq F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right)\right\}, \quad (11)$$

and the optimal throughput, x_α^*, is given by

$$
\begin{aligned}
x_\alpha^* \;=\; & \frac{T}{\alpha} F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right) \quad (12)\\
& + \frac{p_s T}{\tau}\left(1 - \frac{T}{\alpha}\right) E\left[\left(R_N - F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right)\right)^+\right].
\end{aligned}
$$

II) When $\alpha \geq \alpha^*$, we have that $N_\alpha^* = N^*$ and $x_\alpha^* = x^*$, where N^* and x^* are shown in (5) and (6), respectively.
The proof can be found in Appendix A.

Remarks:

1) The above result reveals that the optimal stopping rule under the network-wide average time constraint scenario, N_α^*, is a pure threshold policy and the threshold hinges heavily on the time constraint α. Interestingly, there exists a sharp transition associated with the critical time constant, α^*, in the sense that if α is less than α^*, the optimal threshold depends on α; otherwise, the imposed constraint has no impact on the optimal scheduling, and the optimal policy remains the same as if the time constraint were removed.

2) We observe from the proof that under the continuous assumption on the channel statistics, the strong duality holds. In general, $\psi(\alpha)$ is the concave hull of $\phi(\alpha)$ that is not necessarily concave [8] (see Fig. 2 for a pictorial illustration). Accordingly, there may exist duality gaps. Somewhat surprisingly, it turns out when the rate distribution is continuous, $\psi(\alpha)$ coincides with $\phi(\alpha)$. Our intuition is as follows: when the channel exhibits sufficient randomization, then there is no duality gap, and solving the relaxed problem is equivalent to solving the primal problem. In contrast, when the channel distribution is discrete, the duality gap is zero only at countably many points [11]. We note that the underlying ~~rationale~~ key idea behind the above result is ~~kin~~ akin to the hidden convexity property established in [12], [13].

3) Observe that to compute the optimal threshold offline, network-wide channel statistical information is required. We note that an online algorithm can be developed, based on local information only, to ~~find~~ estimate the optimal threshold similar to that in [5].

B. An Iterative Algorithm for Computing α^*

Comparing (6) and (10), it is easy to see that

$$x^* = F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha^* - T)}\right), \quad (13)$$
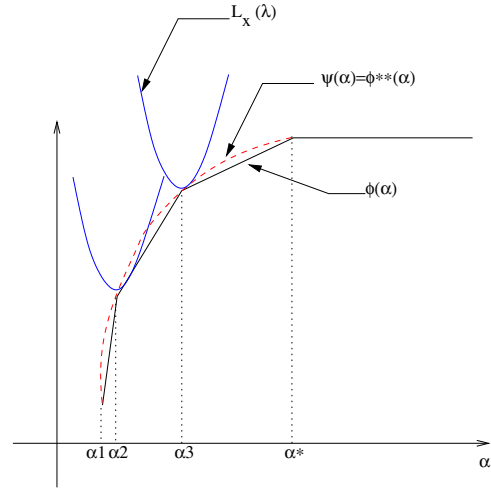

Fig. 2. Duality gap between the primal and the dual problems.

or equivalently,

$$\alpha^* = T + \frac{\tau}{p_s(1 - F_R(x^*))}. \quad (14)$$

Based on (14), we can use the iterative algorithm developed in [5] to find α^*. More specifically, we first use the following iterative algorithm to compute x^*:

$$x_{k+1} = \Phi(x_k) \quad \text{for } k = 0, 1, 2, ..., \quad (15)$$

where $\Phi(x) = \frac{\int_x^\infty r \, dF_R(r)}{\frac{\delta}{p_s} + 1 - F_R(x)}$ and x_0 is a positive initial value. Then, α^* can be obtained from (14).

Since $F_R^{-1}\left(1 - \frac{\tau}{p_s(x - T)}\right)$ is monotonically increasing in x, based on the above relationship of x^* and α^*, we can further simplify the optimal threshold policy presented in Prop. 3.1 as follows.

Corollary 3.1: The optimal scheduling policy N_α^* is given by

$$N_\alpha^* = \min\left\{ n \geq 1 : R_n \geq \min\left(x^*, F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right)\right)\right\}, \quad (16)$$

and the corresponding optimal throughput x_α^* is given by (17), $\mathbf{I}(\cdot)$ is an indicator function, and x^* and α^* are given in (6) and (10), respectively.

C. DOS under Individual Average Delay Constraints

In the above section, the average delay constraint is with respect to the whole network. In some cases, different users may have different individual delay requirements. Therefore, it is of great interest to study the impact of the individual average time constraint. More specifically, we focus on DOS from a user-centric perspective, where each link seeks to maximize its own throughput under its individual average delay constraint $E[T_m] \leq \alpha_m$, for $m = 1, ..., M$. To this end, we treat joint channel probing and distributed scheduling as a non-cooperative game, where each user chooses its threshold in a selfish manner to maximize its own throughput, subject to its own individual average time constraint.

$$x_\alpha^* = x^* \mathbf{I}(\alpha \geq \alpha^*) + \left\{ \frac{T}{\alpha} F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right) + \frac{p_s T}{\tau}\left(1 - \frac{T}{\alpha}\right) E\left[\left(R_N - F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right)\right)^+\right]\right\} \mathbf{I}(\alpha < \alpha^*). \quad (17)$$

For a given set of thresholds across links, $\{\phi_m, m = 1, 2, ..., M\}$, it is easy to see that the expected channel probing and transmission time for link m, $E[T_m]$, is given by [5]

$$E[T_m] = \frac{\tau + \sum_{i \neq m}^M p_{s,i}(1 - F_i(\phi_i))T}{p_{s,m}(1 - F_m(\phi_m))} + T. \quad (18)$$

Note that $E[T_m]$ is composed of two parts: 1) the effective channel probing time $\frac{\tau + \sum_{i \neq m}^M p_{s,i}(1 - F_i(\phi_i))T}{p_{s,m}(1 - F_m(\phi_m))}$, and 2) data transmission time, T. The average throughput of link m can be expressed as (see Lemma 4.1 in [5])

$$\vartheta_m(\boldsymbol{\varnothing}) = \frac{p_{s,m} \int_{\phi_m}^\infty r dF_m(r)}{\delta + \sum_{i=1}^M p_{s,i}(1 - F_i(\phi_i))}, \quad (19)$$

where $\boldsymbol{\varnothing} = \{\phi_1, \phi_2, ..., \phi_M\}$ is the threshold vector.

Following [5], we cast the threshold selection problem across different links as a non-cooperative game, in which each individual link chooses its threshold ϕ_m to maximize its own throughput, ϑ_m, given its own average probing and transmission time constraint, α_m. Let $G = [\{1, 2, ..., M\}, \times_{m \in \{1,2,...,M\}} A_m, \{\vartheta_m, m \in \{1, 2, ..., M\}\}]$ denote the non-cooperative threshold selection game. The links in $\{1, 2, ..., M\}$ are the players of the game, $A_m = \{\phi_m | 0 \leq \phi_m < \infty\}$ is the action set of the player m, and ϑ_m is the utility function for the player m. The non-cooperative game can be formulated as follows:

$$(G) \max_{\phi_m \in A_m} \vartheta_m(\boldsymbol{\varnothing}), \quad \text{subject to } E[T_m] \leq \alpha_m, \quad (20)$$
$$\forall m = 1, 2, ..., M.$$

D. Nash Equilibrium under Individual Average Delay Constraints

Next, we investigate the corresponding Nash equilibrium under individual average delay constraints.

Definition 3.1: A threshold vector $\boldsymbol{\varnothing}^* = \{\phi_1^*, \phi_2^*, ..., \phi_M^*\}$ is said to be a Nash equilibrium of game G if

$$\vartheta_m(\phi_m^*, \varnothing_{-m}^*) \geq \vartheta_m(\phi_m, \varnothing_{-m}^*), \forall \phi_m \in A_m, \quad (21)$$

where $\varnothing_{-m}^* \triangleq [\phi_1, ..., \phi_{m-1}, \phi_{m+1}, ..., \phi_m]^T$.
Definition 3.1 reveals that when the Nash equilibrium is achieved, no link can increase its throughput by changing its threshold from the equilibrium, given the thresholds of other links.

Along the same line as in network-centric case, we need the following conditions on each individual average delay constraint α_m:

$$\alpha_m \geq \frac{\tau + \sum_{i \neq m}^M p_{s,i} T}{p_{s,m}} + T. \quad (22)$$

To this end, we examine the existence of of Nash equilibrium. Using Prop. (3.1), we have the following result on the existence of the Nash equilibrium for the game G.

Proposition 3.2: Under the condition in (22), there exists a Nash equilibrium for the game G, and

$$\phi_m^* = \min\left(x_m^*, F_R^{-1}\left(1 - \frac{\tau + \sum_{i \neq m}^M p_{s,i}(1 - F_i(\phi_i^*))T}{p_s(\alpha_m - T)}\right)\right), \quad (23)$$

where x_m^* satisfies the following equation:

$$x_m^* = \frac{p_{s,m} \int_{x_m^*}^\infty r dF_m(r)}{\delta + \sum_{i=1}^M p_{s,i}(1 - F_i(\phi_i^*))}. \quad (24)$$

The proof follows directly from [5].

It is not surprising to see, Prop. 3.2 reveals that the optimal threshold in the Nash equilibrium is upper-bounded by a function of the time constraint α_m, and other users' thresholds. When $\alpha_m \to \infty, \forall m$, we have that

$$\phi_m^* \to x_m^* \to x_m'^*,$$

where $x_m'^*$ can be computed by

$$x_m'^* = \frac{p_{s,m} \int_{x_m'^*}^\infty r dF_m(r)}{\delta + \sum_{i=1}^M p_{s,i}(1 - F_i(x_i'^*))}, \forall m, \quad (25)$$

which boils down to the unconstrained case in [5].
Define for $m = 1, \ldots, M$,

$$\alpha_m^* \triangleq T + \frac{\tau + \sum_{i \neq m}^M p_{s,i}(1 - F_i(x_i'^*))T}{p_s(1 - F_m(x_m'^*))}. \quad (26)$$

As the counterpart to the critical time constant α^* in the network-centric case, the vector $\{\alpha_m^*\}$ defined above serves as the critical time constant vector for the individual average delay constraint case. Particularly, we have the following result:

Proposition 3.3:

$$\phi_m^* = x_m'^* \text{ if and only if } \alpha_m \geq \alpha_m^*, \forall m.$$

Proof: If $\alpha_m \geq \alpha_m^*, \forall m$, it is straightforward to examine that $\phi_m^* = x_m'^*$ satisfies (23) and (24). On the other hand, if there exists any m such that $\alpha_m < \alpha_m^*$, then it can be seen from (23) that $\phi_m^* < x_m'^*$, and the proof is concluded. ∎

Prop. 3.3 reveals that only when all the time constants are larger than the corresponding critical time constants can the Nash equilibrium points under the individual average delay constraint belong to that for the unconstrained case.

E. Iterative Algorithm for Finding Nash Equilibrium

Based on (23), we have the following best response strategy to compute the Nash equilibrium:

$$
\begin{aligned}
\phi_m(k+1) &= \min(x_m^*(k), F_R^{-1}(1 - \\
&\quad \frac{\tau + \sum_{i \neq m}^{M} p_{s,i}(1 - F_i(\phi_i(k)))T}{p_s(\alpha_m - T)})), (27)
\end{aligned}
$$

$$
\text{and } x_m^*(k) = \frac{p_{s,m} \int_{x_m^*(k)}^{\infty} r dF_m(r)}{\delta + \sum_{i=1}^{M} p_{s,i}(1 - F_i(\phi_i(k)))},
$$
$$
\text{for} \quad k = 0, 1, 2, \ldots \quad \forall m = 1, 2, \ldots, M. \quad (28)
$$

The following proposition establishes the convergence of the above best response strategy.

Proposition 3.4: Suppose that the Nash equilibrium is unique. Then, for any non-negative initial value $\boldsymbol{\varnothing}(0)$, the sequence $\{\boldsymbol{\varnothing}(k)\}$, generated by the iterative algorithm in (27), converge to the Nash equilibrium $\boldsymbol{\varnothing}^*$, as $k \to \infty$. The proof follows directly from [5].

IV. NUMERICAL RESULTS

In this section, we study, by numerical examples, the performance of the DOS under delay constraints. Unless otherwise specified, we assume that τ, T, p_m and M are chosen such that $\delta = \tau/T = 0.1, p_s = \exp(-1)$. We consider the continuous rate case only, assuming that the instantaneous rate is given by the Shannon channel capacity, i.e.,

$$
R_n = \log(1 + \rho|h_n|^2) \text{ nats/s/Hz}, \quad (29)
$$

where ρ is the normalized average SNR, and h_n is the random channel coefficient with a complex Gaussian distribution $\mathcal{CN}(0, 1)$. Accordingly, the distribution of the transmission rate is given by

$$
F_R(r) = 1 - \exp\left(-\frac{\exp(r) - 1}{\rho}\right). \quad (30)
$$

A. The Case with Network-Wide Average Delay Constraint

We first provide numerical examples for DOS under the network-wide average delay constraint. It follows from (17) and (30) that the maximal throughput, x_α^*, can be expressed as in (31), where $\gamma = F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right)$, $C = \frac{p_s}{\tau}(1 - \frac{T}{\alpha})$, x^* satisfies the following fixed point equation:

$$
x^* = \frac{p_s}{\delta} \exp\left(\frac{1}{\rho}\right) E_1\left(\frac{\exp(x^*)}{\rho}\right), \quad (32)
$$

and $E_1(x)$ is the *exponential integral function* defined as

$$
E_1(x) \triangleq \int_x^{\infty} \frac{\exp(-t)}{t} dt. \quad (33)
$$

In Fig. 3, we compare the optimal throughput x_α^* under delay constraints, against that corresponding to two other schemes: 1) the throughput, denoted as x_α, corresponding to the naive threshold policy $\phi = F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right)$ that always enforces that the average delay equal the delay constraint, and 2) the throughput, denoted as x^*, corresponding to
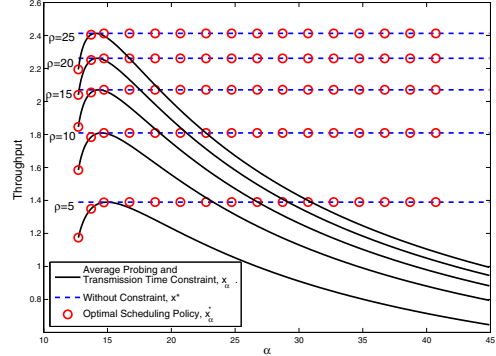

Fig. 3. Maximal throughput for different values of α at different ρ

the optimal threshold policy $\phi = x^*$ with no delay constraint. We plot them for different α at different average SNR $\rho = 5, 10, 15, 20, 25$. It is clear that x_α^* is an increasing function of the average SNR ρ for a given α as expected. Another important observation from Fig. 3 is that x_α intersects with x^* at its peak point, which is exactly the critical time constant, α^*. Note that $x_\alpha = x_\alpha^*$ when $\tau/p_s + T \leq \alpha \leq \alpha^*$, and $x_\alpha^* = x^*$ when $\alpha > \alpha^*$. That is to say, α^* is the transition point beyond which, the optimal scheduling policy would be the same as that without the delay constraint. Intuitively speaking, the optimal scheduling policy under delay constraints can be viewed as a marriage between the naive threshold scheme and the optimal threshold scheme without considering the constraint.
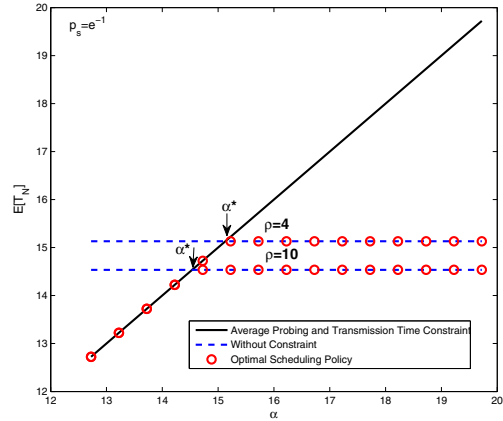

Fig. 4. $E[T_N]$ of network versus average probing and transmission time constraint of α

To gain a deeper understanding of the critical time point, α^*, we plot in Fig. 4 the average probing and transmission time,

$$
E[T_N] = \frac{\tau}{p_s(1 - F_R(\phi))} + T, \quad (34)
$$

under the above mentioned three schemes. It can be seen from Fig. 4 that the average probing and transmission time would always be the same value of α, under the threshold

$$x_\alpha^* = x^* \mathbf{I}(\alpha \geq \alpha^*) + \underbrace{\frac{\gamma T}{\alpha} + CT \exp\left(\frac{1}{\rho}\right) \left[\gamma \exp\left(-\frac{\exp(\gamma)}{\rho}\right) + E_1\left(\frac{\exp(\gamma)}{\rho}\right)\right] - CT\gamma \left(\exp\left(-\frac{\exp(\gamma)-1}{\rho}\right)\right)}_{\triangleq x_\alpha} \mathbf{I}(\alpha < \alpha^*). \quad (31)$$

policy with $\phi = F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha-T)}\right)$. For the threshold policy with $\phi = x^*$, the average probing and transmission time is a constant regardless of α. The critical constraint point, α^* is shown in this figure as the crossing point between these two threshold methods. This also explains the physical meaning of this critical point: when $\alpha \leq \alpha^*$, the threshold policy with $\phi = F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha-T)}\right)$ is optimal since it "pushes" the average time towards the allowed limit to achieve a higher throughput; when $\alpha > \alpha^*$, since the threshold policy with $\phi = x^*$ already satisfies the delay constraint while achieving the maximum throughput, it is also optimal in the case under the delay constraint. This phenomena can be also observed from Fig. 3: after the critical point, the throughput is decreasing if the scheduling policy still follows the original time constraint policy with $\phi = F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha-T)}\right)$. Therefore, the optimal scheduling policy (shown in circle) under the delay constraint should be the same as the case without the delay constraint, when $\alpha > \alpha^*$.
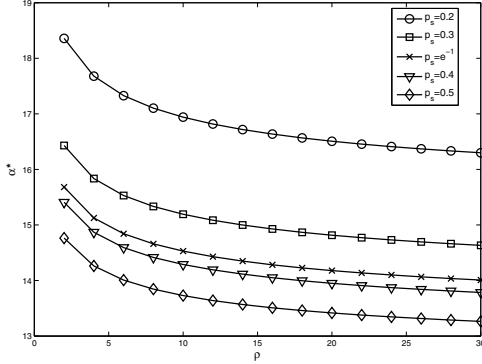

Fig. 5. α^* for different ρ at different p_s

It is clear from the above discussions that the network throughput suffers a loss due to the delay constraint when α falls into the range of $[\tau/p_s + T, \alpha^*)$. Thus, α^* could be used to measure the criticality of the delay constraint: the smaller α^*, the less the criticality of the delay constraint. To get a better understanding of the characteristics of the critical time, we plot in Fig. 5 α^* as a function of ρ and the contention successful probability p_s. It can be seen from the figure that α^* is a decreasing function of ρ for a given p_s. That means the effect of delay constraint is less critical at high SNR. We also observe that α^* is a decreasing function of p_s for a given ρ. This is because a larger p_s indicates that the users can access and probe the channel with smaller cost (in terms of contention time), and hence, can meet more stringent delay requirements. Overall, Fig. 5 suggests that the importance of our proposed

approach becomes increasingly prominent in low SNR and/or low successful contention probability regions.

B. The case with Individual Average Time Constraints

In this section, we examine the DOS performance with individual user's average delay constraint.

TABLE I
CRITICAL TIME FOR EACH INDIVIDUAL LINK

Link	α_m^* ($p_m = 0.125$)	α_m^* ($\rho_m = 6$)
Link 1	66.12 ($\rho_1 = 2$)	82.38 ($p_1 = 0.075$)
Link 2	64.81 ($\rho_2 = 4$)	69.51 ($p_2 = 0.100$)
Link 3	64.14 ($\rho_3 = 6$)	62.62 ($p_3 = 0.125$)
Link 4	63.70 ($\rho_4 = 8$)	58.84 ($p_4 = 0.150$)
Link 5	63.39 ($\rho_5 = 10$)	56.96 ($p_5 = 0.175$)

We first examine in Table I the behavior of α_m^* with fixed contention probability of $p_m = 0.125$ but with different average SNR (ρ_m) across links. The result shows that the link with higher average SNR has a lower α_m^* for the same reasoning explained in Fig. 5. We next examine the behavior of critical time with a fixed average SNR, but with the different contention probability (p_m) across links. It can be observed that the link with higher contention probability has a lower critical time constant. Similar to the reason for the network centric case, the link with higher contention probability has the lower unconstrained expected probing and transmission time, and therefore the time constraint is less stringent. The results imply that the effects of imposing a constraint on the individual link becomes less significant when the average SNR and/or the contention probability of the link increases.

TABLE II
CONVERGENCE BEHAVIOR OF THE BEST RESPONSE STRATEGY

Link	x_0	x_1	x_2	x_3	x_4	x^*
Link 1 ($\rho = 2$, $\alpha_m = 64.0$)	1.000	0.171	0.097	0.082	0.077	0.077
Link 2 ($\rho = 4$, $\alpha_m = 64.5$)	1.000	0.260	0.220	0.183	0.176	0.175
Link 3 ($\rho = 6$, $\alpha_m = 65.0$)	1.000	0.320	0.267	0.264	0.263	0.263
Link 4 ($\rho = 8$, $\alpha_m = 65.5$)	1.000	0.367	0.302	0.299	0.298	0.298
Link 5 ($\rho = 10$, $\alpha_m = 66.0$)	1.000	0.405	0.331	0.327	0.326	0.326

Needless to say, for a non-cooperative game, a key aspect of study is the characteristic of Nash equilibrium point. Table II illustrates the convergence behavior of the iterative best response strategy defined in section (III-E) with $M = 5$, $p_m = 0.125$ and different ρ and α_m for each individual link. It is clear from Table II that the thresholds for all the

links converge to the equilibrium point within a few (≈ 5) iterations. Note that the time constraint for each individual link, α_m, in this example is less than its corresponding critical time constant (refer to the corresponding critical point in table (I)). Thus, the table shows that the algorithm achieved convergence even when the imposed constraint is less than the corresponding critical time constant.

TABLE III
VERIFICATION OF PROP. 3.3

Link	α^* $(p_m = 0.125)$	$x_m'^*$	$x_m^*, \alpha_m > \alpha_m^*, \forall m$	$x_m^*, \alpha_m < \alpha_m^*, \forall m$
Link 1	66.12 $(\rho = 2)$	0.1504	0.1504 $(\alpha_m = 67.0)$	0.077 $(\alpha_m = 64.0)$
Link 2	64.81 $(\rho = 4)$	0.2188	0.2188 $(\alpha_m = 67.5)$	0.175 $(\alpha_m = 64.5)$
Link 3	64.14 $(\rho = 6)$	0.2652	0.2652 $(\alpha_m = 68.0)$	0.263 $(\alpha_m = 65.0)$
Link 4	63.70 $(\rho = 8)$	0.3006	0.3006 $(\alpha_m = 68.5)$	0.298 $(\alpha_m = 65.5)$
Link 5	63.39 $(\rho = 10)$	0.3293	0.3293 $(\alpha_m = 69.0)$	0.326 $(\alpha_m = 66.0)$

Prop. 3.2 shows that there exists a Nash equilibrium which satisfy the (23), which is verified by Table III. It is clear from the table that when the imposed constraint of all the links, α_m $m = 1, 2, ..., M$, are bigger than the corresponding critical time constraint of α_m^*, the threshold for each link with delay constraint (x_m^*, shown in forth column) will be the same as the scenario without the constraint ($x_m'^*$, shown in third column). When the imposed delay constraint is less than the α_m^*, the threshold (shown in fifth column) will be different than the corresponding $x_m'^*$.

V. CONCLUSIONS AND FUTURE WORK

In this paper, we considered an ad-hoc network where many links contend for the channel using random access, and studied distributed opportunistic scheduling (DOS) under average delay constraint from two different perspectives. First, we study DOS with delay constraints from a network-centric perspective, with the objective to maximize the overall throughput subject to the average constraint on the network-wide probing and transmission time. We showed that the optimal DOS strategy under such delay constraint is a pure threshold policy. Specifically, we found that there exist a critical time constant, α^*: if the imposed delay constraint is less than the α^*, the optimal threshold is a function of α; otherwise, the imposed delay constraint has no effect on the optimal scheduling and the optimal policy remains the same as if the delay constraint did not exist. We showed that the critical time α^* is a decreasing function of average SNR and contention probability. Next, from the user-centric constraint perspective, each individual link has its own average time constraint. In this case, the threshold selection for different links is treated as a non-cooperative game, in which every link strives to maximize its throughput subject to its own individual delay constraint. We explore the existence of the Nash equilibrium and showed that the Nash equilibrium can be

achieved by iterative algorithms. Similar to the network-centric case, we show that there exists a critical time vector $\{\alpha_m^*\}$ such that only when all delay constraints $\alpha_m \geq \alpha_m^*, \forall m$, then the delay constraints have no impacts on the optimal threshold in the Nash equilibrium.

In this paper, we considered the average delay constraint. It would also be of interest to consider individual packet lifetime constraint. Different from the average delay constraint, the packet life time constraint is imposed on scheduling of individual packets and is essentially a constraint on sample-path realizations. Our study along this line is underway.

In summary, we took some initial steps towards studying channel-aware distribution scheduling in ad hoc networks under a delay constraint for real-time traffic. In particular, we characterized the fundamental tradeoff between the throughput gain from better channel conditions and the cost for channel probing that may cause the delay. As expected, the scheduling policy tends to be more conservative by setting a smaller threshold when the time constraint becomes tighter.

APPENDICES

A. PROOF OF PROPOSITION 3.1

The proof of Proposition 3.1 hinges heavily on the tools in optimal stopping theory [6] and Lagrange duality theory [14]. We derive the optimal solution in the following four steps.

Step 1: Based on Theorem 1 in [6, Chapter 6], the main problem (7) is first transformed into the following problem:

$$\max E[R_N T] - x E[T_N], \text{ subject to } N \in Q_\alpha. \quad (35)$$

Let $\phi(\alpha)$ denote the optimal value for the primal problem (35).

Step 2: Next, consider the problem (35) with Lagrangian relaxation:

$$\max E[R_N T] - (x+\lambda)E[T_N] + \lambda\alpha, \text{ subject to } N \in Q. \quad (36)$$

Define

$$N^*(x, \lambda) \triangleq \arg\max_{N \in Q} E[R_N T] - (x + \lambda)E[T_N], \quad (37)$$

and

$$V^*(x, \lambda) \triangleq E[R_{(N^*(x,\lambda))} T] - (x + \lambda)E[T_{N^*(x,\lambda)}]. \quad (38)$$

Following the same procedure as in Lemma (2.1), it can be shown that

$$N^*(x, \lambda) = \min\{n \geq 1 : R_N \geq x + \lambda + \frac{V^*(x, \lambda)}{T}\}, \quad (39)$$

and

$$E\left[\left(R_N - (x+\lambda) - \frac{V^*(x, \lambda)}{T}\right)^+\right] = \frac{(x+\lambda)\tau}{T p_s}. \quad (40)$$

Consequently, from the corollary 3.1 of [5], we have that

$$E[T_{N^*(x,\lambda)}] = \frac{\tau}{p_s \left[1 - F_R \left(x + \lambda + \frac{V^*(x,\lambda)}{T}\right)\right]} + T. \quad (41)$$

Step 3: Solve the dual problem:

$$\psi(\alpha) \triangleq \min_\lambda L_x(\lambda), \text{ subject to } \lambda \geq 0, \quad (42)$$

where the dual objective is given by

$$L_x(\lambda) = V^*(x, \lambda) + \lambda\alpha. \tag{43}$$

Let $\lambda^*(x) \triangleq \underset{\lambda \geq 0}{\arg\min}\, L_x(\lambda)$. By the complementary slackness condition in Theorem 4 in [8] and (41), we have that

$$\lambda^*(x)\left[\frac{\tau}{p_s\left[1 - F_R\left(x + \lambda^*(x) + \frac{V^*(x,\lambda^*(x))}{T}\right)\right]}\right] \\ + \lambda^*(x)T - \lambda^*(x)\alpha = 0. \tag{44}$$

Step 4: By Theorem 1 in [6, Chapter 6], the optimal throughput x^* can be achieved if the the optimal value for the primal problem is 0. Accordingly, if there is no duality gap, the optimal value for the dual problem should be 0 too. Therefore, we characterize x_α^* by solving the following equation:

$$L_x(\lambda^*(x)) = 0. \tag{45}$$

To this end, we consider the following two cases.

Case 1: If $\lambda^*(x_\alpha^*) > 0$, then it follows from (44) that

$$x_\alpha^* + \lambda^*(x_\alpha^*) + \frac{V^*(x_\alpha^*, \lambda^*(x_\alpha^*))}{T} = F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right). \tag{46}$$

Combining (46) and (40) yields that

$$E\left[\left(R_N - F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right)\right)^+\right] = \frac{(x_\alpha^* + \lambda^*(x_\alpha^*))\tau}{p_s T}. \tag{47}$$

Using (47) in (46), we have that

$$V^*(x_\alpha^*, \lambda^*(x_\alpha^*)) = TF_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right) \\ - \frac{p_s T^2}{\tau}E\left[\left(R_N - F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right)\right)^+\right] \tag{48}$$

It then follows from (47) and (48),

$$L_x(\lambda^*(x_\alpha^*)) = V^*(x_\alpha^*, \lambda^*(x_\alpha^*)) + \lambda^*(x_\alpha^*)\alpha \tag{49}$$
$$= TF_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right) \\ - \frac{p_s T^2}{\tau}E\left[\left(R_N - F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right)\right)^+\right] \\ + \alpha\left[\frac{p_s T}{\tau}E\left[\left(R_N - F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right)\right)^+\right] - x_\alpha^*\right]$$

Since $L_x(\lambda^*(x)) = 0$, we have that

$$x_\alpha^* = \frac{T}{\alpha}F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right) \tag{50}$$
$$+ \frac{p_s T}{\tau}\left(1 - \frac{T}{\alpha}\right)E\left[\left(R_N - F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right)\right)^+\right].$$

Note that

$$\lambda^*(x_\alpha^*) = \frac{p_s T}{\tau} \tag{51}$$
$$\times E\left[\left(R_N - F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right)\right)^+\right] - x_\alpha^* > 0.$$

It follows that (51) is equivalent to

$$F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right) \tag{52}$$
$$< \frac{p_s T}{\tau}E\left[\left(R_N - F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right)\right)^+\right].$$

Case 2: Otherwise, if $\lambda^*(x_\alpha^*) = 0$, we have that

$$L_x(\lambda(x_\alpha^*)) = V^*(x_\alpha^*, 0) = 0. \tag{53}$$

It follows from (40) that

$$E\left[(R_N T - x_\alpha^* T)^+\right] = \frac{x_\alpha^* \tau}{p_s}, \tag{54}$$

which is exactly (6). Therefore, $x_\alpha^* = x^*$.

Observing that the left side of (53) is monotonically strictly increasing (by the continuous assumption of $F_R(r)$) in α from 0 to ∞ as α grows from $\frac{\tau}{p_s} + T$ to ∞, and the right side is monotonically strictly decreasing $\frac{p_s T}{\tau}E\left[R_{(n)}T\right]$ to 0, therefore, there exists a unique α^* that is the solution of (10). And when $\alpha < \alpha^*$, we have Case 1, and when $\alpha \geq \alpha^*$, we have Case 2. Using the above results, it can also by verified that the conditions in Theorem 4 in [8] are satisfied due to the continuity of $F_R(r)$, and therefore, there is no duality gap. The proof is concluded.

REFERENCES

[1] Z. Ji, Y. Yang, J. Zhou, M. Takai, and R. Bagrodia, "Exploiting medium access diversity in rate adaptive wireless LANs," in *Proceedings of MOBICOM'04*, 2004.

[2] X. Liu, E. K. Chong, and N. B. Shroff, "A framework for opportunistic scheduling in wireless networks," *Computer Networks*, vol. 41, no. 4, pp. 451–474, Mar. 2003.

[3] X. Qin and R. Berry, "Exploiting multiuser dieversity for medium access control in wireless networks," in *Proceedings of IEEE INFOCOM'03*, San Francisco, CA, 2003.

[4] P. Viswanath, D. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Transactions on Information Theory*, pp. 1277–1294, 2002.

[5] D. Zheng, W. Ge, and J. Zhang, "Distributed opportunistic scheduling for ad hoc networks with random access: An optimal stopping approach," *IEEE Transaction on Information Theory*, vol. 55, no. 1, pp. 205–222, Jan. 2009.

[6] T. Ferguson, *Optimal Stopping and Applications*. available at http://www.math.ucla.edu/~tom/Stopping/Contents.html, 2006.

[7] D. C. Nachman, "Optimal stopping with a horizon constraint," *Math Operat. Res.*, vol. 5, pp. 126–134, 1980.

[8] D. Kennedy, "On a constrained optimal stopping problem," *Journal of Applied Probability*, vol. 19, pp. 631–641, 1982.

[9] D. P. Bertsekas and R. Gallager, *Data Networks*. New Jersey: Prentice Hall, 1992.

[10] Y. S. Chow, H. Robbins, and D. Siegmund, *Great Expectations: Theory of Optimal Stopping*. Houghton Mifflin, 1971.

[11] D. Zheng, S.-S. Tan, J. Zhang, and J. Zeidler, "Distributed opportunistic scheduling under delay constraints," *Technical Report, School of ECEE, ASU*, Dec. 2009.

[12] Z.-Q. Luo and S. Zhang, "Dynamic spectrum management: Complexity and duality," *IEEE JASAC, Special Issue on Signal Processing and Networking for Dynamic Spectrum Access*, vol. 2, no. 1, pp. 57–73, Feb. 2008.

[13] D. Blackwell, "On a theorem of lyapunov," *Ann. Math. Statist.*, vol. 22, pp. 112–114, 1951.

[14] D. P. Bertsekas, *Nonlinear Programming*. Belmont, MA: Athena Scientific, 1995.