Rate-Achievability Strategies for Two-Hop Interference Flows

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***Abstract*— We consider a basic model for two-hop transmissions of two information flows which interfere with each other. In this model, two sources simultaneously transmit to two relays (in the first hop), which then simultaneously transmit to two destinations (in the second hop). While the transmission during the first hop is essentially the transmission over a classical interference channel, the transmission in the second hop enjoys an interesting advantage. Specifically, as a by-product of the Han-Kobayashi transmission scheme applied to the first hop, each of the relays (in the second hop) has access to some of the data that is intended to the other destination, in addition to its own data. As recently observed by Simeone *et al.*, this opens the door to cooperation between the relays. In this paper, we observe that the cooperation can take the form of distributed MIMO broadcast, thus greatly enhancing its effectiveness at high SNR. However, since each relay is only aware of *part* of the data beyond its own, full cooperation is not possible. We propose several approaches that combine MIMO broadcast strategies (including "dirty paper") with standard non-cooperative strategies for the interference channel. Numerical results are provided, which indicate that our approaches provide substantial benefits at high SNR.**

I. Introduction

Relay channels and interference channels model two fundamental forms of wireless networks. In a relay channel, as introduced by van der Meulen [15], a relay node assists the source in communicating data to a destination. In an interference channel, introduced by Ahlswede [1], two pairs of nodes, each consisting of a source and a destination, wish to communicate simultaneously. A defining property of this channel is that each of the two destinations experiences interference, resulting from the signal transmitted to the other destination.

In this paper, we consider a model building on these two models (see Fig. 1). In our model, two sources wish to communicate simultaneously with two destinations, and are aided by two relay nodes. We confine our attention to a two-hop scenario. That is, communication is performed in two consecutive "hops" (time intervals). During the first hop, the two sources communicate data to their respective relays, and during the second hop, the relays forward the data to the destinations. In our model, the relays are not able to transmit data during the first hop (we assume that they are half duplex), and the destinations are not able to hear the signal transmitted by the source. This models a situation where the destinations are too far away, or shaded by a barrier.

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This research is supported in part by Office of Naval Research through the grant N00014-05-1-0636 and National Science Foundation through the grants ANI-0238550 and CNS-0721820.

In designing communication strategies for this scenario, it is useful to begin by considering strategies for the interference channel. During the first hop in the model under consideration, the two sources simultaneously communicate their messages, each to its corresponding relay. That is, the relays function as virtual destinations, and each relay experiences interference resulting from the signal communicated to the other relay. The communication problem at hand is thus equivalent to transmission over an interference channel. The best known transmission strategy for this channel is due to Han and Kobayashi [7]. During the second hop, each relay functions as a virtual source, and communicates its corresponding message to the final destination. While Han-Kobayashi can be applied to this hop as well, Simeone *et al.* [13] applied the following technique to achieve better performance (on a similar model).

The Han-Kobayashi scheme achieves its remarkable performance by requiring each destination (of the interference channel) to decode, beyond its own message, also a part of the message intended to the other destination. That is, each source splits its message into two sub-messages: a *common* sub-message and a *private* sub-message. Each destination decodes both of the sub-messages transmitted by its corresponding source, and also the common sub-message communicated to the other destination. In the context of the interference channel scenario considered by Han and Kobayashi, the destination has no use for the additional common sub-message it decodes, and is expected to discard it once decoding is complete. However, in the context of the two-hop scenario considered in this paper, this additional knowledge, obtained by the relays in their roles as destinations in the first hop, can be exploited to enable cooperation between them, in their roles as sources during the second hop. Thus, the requirement that each node decode part of the message intended to the other, provides a benefit in both hops.

How can cooperation between the relays be achieved? In the approach of Simeone *et al.* [13], each relay transmits superimposed signals, corresponding to its own two sub-messages (the private one and the common one), as well as the alternate relay's common sub-message[1]. In this paper, we focus on a different set of approaches instead, which are based on the following observation.

Consider, for simplicity, the case where during the first hop, each of the common sub-messages constitutes the entire message transmitted by the corresponding source, and the

[1]More precisely, their model considers many sets of source-relay-destinations, and each relay is aware of *two* common sub-messages, beside its own.

private sub-messages degenerate to null. This means that at the end of this hop, the two relays have full knowledge of *all* of the data intended to *both* destinations. They may therefore function as a virtual antenna array, and the communication problem (during the second hop) coincides with multi-antenna (MIMO) broadcast[2]. Cooperation of this form between nodes is typically known as distributed MIMO (see e.g. [10], [11]). In particular, transmission is possible using dirty-paper coding (DPC) [4], which has recently been found to exhaust the capacity region of the multi-antenna broadcast channel [16]. Additional multi-antenna broadcast strategies are possible as well.

The above case, where the common sub-messages constitute all of the data transmitted during the first hop, is one extreme. In many cases of interest, requiring this incurs a sacrifice in rate during this hop (except when the interference is *strong* [5]). At the other extreme, it is the private messages that constitute all of the data during the first hop, and the common messages are reduced to null. In this case, cooperation between the relays during the second hop is not possible, and the best known communication strategy is again Han-Kobayashi.

Our focus in this paper is on the continuum between these two extremes. That is, we propose partial-cooperation strategies, which are combinations of MIMO broadcast strategies and non-cooperative strategies for interference channels. In related work, [13] considers cooperative strategies to maximize the end-to-end rate, with the optimal power allocation being performed jointly by the sources and the relays. They restrict the cooperation of relays to linear strategies. We point out that, our focus is mainly on the second hop, where we explore cooperative strategies at the relays to enhance the achievable rate. Maric *et al.* [8] considered the case of an interference channel where the transmitters are interested in sending common information to both the destinations along with their own independent messages and presented the capacity region under the assumption of strong interference. In contrast, in our model, the two receivers are both interested in distinct portions of the common information. Furthermore, our focus is on the case with moderate interference (as defined in [6]) in the second hop. The authors [8] also examined the case where the available knowledge is asymmetric, in the sense that one transmitter has complete knowledge of the other's message, while the other transmitter knows only its own message. Oyman *et al.*[10] apply distributed MIMO techniques for dense MIMO interference networks with multiple MIMO relay nodes. They quantify the tradeoff between the power and the bandwidth efficiency. However, their study restricted to linear processing at the relays. Additional references are available in [13].

Notation: We use $C(x)$ to denote $\frac{1}{2}\log(1+x)$, $\|x\|$ to denote the Eucledian norm of x. Sequence $\{X[n]\}_{n=1}^{N}$ is denoted by X^N. $[A]_{i,j}$ refers to element in the (i,j) th position of matrix A; $\dagger$,$\mathsf{tr}(\cdot)$ and $\mathsf{det}(\cdot)$ respectively denote transpose, trace and determinant operations.

[2]Strictly speaking, each relay is subject to an individual power-constraint, rather than a total power constraint as typically assumed in multi-antenna communication problems. However, in this paper we focus on symmetric interference channels, and use a simple time-sharing argument (see Sec. IV) to circumvent this problem.

The rest of the paper is organized as follows. In Section II, we describe the system model. Section III deals with the first-hop communications. In Section IV, we investigate different relaying schemes for the second-hop communications. Numerical results are provided in Section V, and we conclude our work in Section VI.

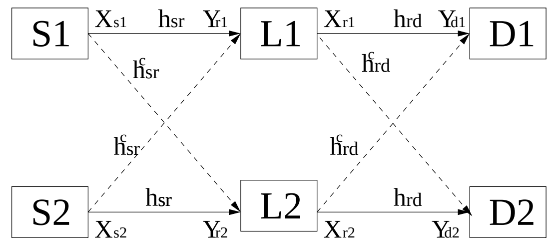

Fig. 1. A sketch of two-hop interference flows

II. System Model and Problem Statement

In this section, we describe the model to be used henceforth. The abstraction of the system under interest is illustrated in Fig. 1. All links are subjected to channel fading, and the channel gains remain constant for the duration of transmission. The transmitters as well as receivers in both the hops are assumed to have the complete state information about their direct links and interfering links. For the ease of exposition, we assume that channels in both hops to be symmetric. That is, in the first hop, the direct links (interfering links) $S_1 \to L_1$ $(S_1 \to L_2)$ and $S_2 \to L_2$ $(S_1 \to L_2)$ have the same gain h_{sr} (h_{sr}^c). In the second hop, the direct links (interfering links) $L_1 \to D_1$ $(L_2 \to D_1)$ and $L_2 \to D_2$ $(L_1 \to D_2)$ have the same gain h_{rd} (h_{rd}^c). We carry out our analysis under the assumption of moderate interference in both the hops, i.e., $h_{sr} > h_{sr}^c$ and $h_{rd} > h_{rd}^c$. We assume that the communications in both hops are slotted and synchronized. The mode of communication is half-duplex. That is, in the first slot, sources (S_1, S_2) transmit and relays (L_1, L_2) listen; and in the second slot, relays transmit and the mobile stations (D_1, D_2) listen. The source to relay $(S \to L)$ (relay to destination $(L \to D)$) transmission of one information flow interferes with the $S \to L$ $(L \to D)$ transmission of the other information flow.

To make our treatment more general, we assume that the first slot spans for a period of N_1 channel uses and the second slot spans for a period of N_2 channel uses, where N_1 and N_2 are chosen large enough to achieve reliable transmission. During the first slot, sources transmit and relays listen. Thus, the received signal at the relay L_1 for $n = 1, \ldots, N_1$ is given as

$$Y_{r,1}[n] = h_{sr} X_{s,1}[n] + h_{sr}^c X_{s,2}[n] + Z_{r,1}[n]. \quad (1)$$

where $Z_{r,1}[n] \sim \mathcal{N}(0, \sigma_r^2)$ is the AWGN process at the relay. In the second slot, the received symbol at the destination for $n = N_1+1, \ldots, N_1+N_2$ is given by

$$Y_{d,1}[n] = h_{rd} X_{r,1}[n] + h_{rd}^c X_{r,2}[n] + Z_{d,1}[n]. \quad (2)$$

where $Z_{d,1}[n] \sim \mathcal{N}(0, \sigma_d^2)$ is the AWGN process at the receiver D_1. A similar setup is employed for the second flow.

A principal goal is to characterize the *symmetric achievable rate*, defined as $R = \max\min\{R_1, R_2\}$, where the rate pair (R_1, R_2) belongs to the best known achievable rate region for the interference channel of interest, and the optimization is done over all possible choices of power allocations.

In the following sections, we brief about the transmission schemes and the power allocation strategies adopted by $S \rightarrow L \rightarrow D$ links in the network for the cooperative relaying and for the interference minimization.

III. Transmission from Sources: The First-hop Communications

In the first hop, the two parallel $S \rightarrow L$ channels represent the classical interference channel introduced by [1]. The capacity achieving strategy for a general interference channel is still unknown, and the best known strategy is due to Han and Kobayashi [7]. Simply put, the HK scheme involves splitting the transmitted information of both users into two parts: private information to be decoded only at own receiver and common information that can be decoded at both receivers. By decoding the common information, part of the interference can be canceled off, while the remaining private information from the other user is treated as noise. It is known that the evaluation of the HK rate region is complex, and following [6], we make some simplifying assumptions. We fix the decoding order at each of the relays, i.e., common messages are decoded first and the private message is decoded last. Indeed, this decoding order has been shown to achieve the best known symmetric rates under moderate interference conditions [6].

Let each source has a transmit power constraint Γ. Source $S_i,\ \ i = 1, 2,$ draws its message $V_i \in \{1, \ldots, 2^{N_1 R_i}\}$ and splits it into $(V_{p,i}, V_{c,i})$, where $V_{p,i} \in \{1, \ldots, 2^{N_1 R_{p,i}}\}$, $V_{c,i} \in \{1, \ldots, 2^{N_1 R_{c,i}}\}$ and $R_i = R_{c,i} + R_{p,i}$. Source S_i generates two codebooks: $\mathcal{C}_{p,i}$, with $|\mathcal{C}_{p,i}| = 2^{N_1 R_{p,i}}$, to carry private message to the intended relay, and $\mathcal{C}_{c,i}$, with $|\mathcal{C}_{c,i}| = 2^{N_1 R_{c,i}}$, to carry common message to be decoded by both relays. These codebooks satisfy the power constraints Γ_p and Γ_c respectively, with $\Gamma_p + \Gamma_c = \Gamma$. For a given block length N_1, source S_i chooses codewords $C_{p,i}^{N_1}$ and $C_{c,i}^{N_1}$ from $\mathcal{C}_{p,i}$ and $\mathcal{C}_{c,i}$ respectively, and transmits their superposition : $X_{s,i}^{N_1} = C_{p,i}^{N_1} + C_{c,i}^{N_1}$.

In the decoding process, as mentioned earlier, each relay first decodes both the common messages $C_{c,1}^{N_2}$ and $C_{c,2}^{N_2}$ treating the private message signals as noise, and then decodes its own private message treating the other link's private message as noise. Further, we restrict our analysis to the case, where the common rates of both links are equal, and the private rates of both the links are equal, i.e., $R_{c,1} = R_{c,2}$ and $R_{p,1} = R_{p,2}$.

We can show that [14], the achievable private and common information rates on each $S \rightarrow L$ link are given by

$$R_p = C\left(\frac{h_{sr}^2 \Gamma_p}{\sigma_r^2 + {h_{sr}^c}^2 \Gamma_p}\right)$$

$$R_c = \min\left\{C\left(\frac{{h_{sr}^c}^2 \Gamma_c}{\sigma_r^2 + \|\mathbf{h}_s\|^2 \Gamma_p}\right),\ \ \frac{1}{2} C\left(\frac{\|\mathbf{h}_s\|^2 \Gamma_c}{\sigma_r^2 + \|\mathbf{h}_s\|^2 \Gamma_p}\right)\right\},$$

where, for convenience, we have used $\|\mathbf{h}_s\|^2 = {h_{sr}^c}^2 + h_{sr}^2$. Therefore, the overall information rate per user that can be achieved over $S \rightarrow L$ links is given by

$$I_{\mathsf{SL}} = \max_{\left\{\substack{\Gamma_c, \Gamma_p: \\ \Gamma_c + \Gamma_p = \Gamma}\right\}} R_p + R_c.$$

We briefly review the results for the weak and strong interference regimes in the first hop communications. Under the weak interference (i.e., the noisy-interference) condition, the results in [2], [12], [9] suggest that the sum capacity (hence the symmetric capacity under symmetric conditions) is achieved by each relay decoding its own message, while treating the interference as noise, i.e., no interference cancellation is needed. It follows that $R_c = 0$ and $I_{\mathsf{SL}} = R_p$ in this case. When the first hop has strong (or very strong) interference, each relay can completely decode both the intended and interfering messages [5]. Therefore, each relay has complete information of what the other relay transmits in the second hop. That is, the entire message transmitted is common. We thus have $R_p = 0$ and $I_{\mathsf{SL}} = R_c$ for this case.

IV. Transmission from Relays: The Second-hop Communications

Relays process the data during the first slot before transmitting in the second hop. Relay L_1 has $(V_1, V_{c,2})$, while relay L_2 has $(V_2, V_{c,1})$. Observe that each relay has a piece of common information intended for the other receiver. Therefore, the second hop transmission can be viewed as an interference channel where each transmitter has partial information intended for the other receiver. A principal goal of our study here is to devise relaying schemes for the second hop that effectively harness the side information available.

We briefly outline the encoding procedure that relays employ for the second-hop transmission. Relay L_i, depending on its forward channel conditions, splits its decoded message into three parts $(U_{p,i}, U_{c,1}, U_{c,2})$, and generates three codebooks: $\mathcal{X}_{p,i}$ with $|\mathcal{X}_{p,i}| = 2^{N_2 T_p}$, to carry the private message $U_{p,i}$ intended for its own receiver D_i, and $\mathcal{X}_{c,i}$ with $|\mathcal{X}_{c,i}| = 2^{N_2 T_{c,i}}$, to carry the common messages which are relayed cooperatively. For a given block length N_2, relays use codewords $X_{p,i}^{N_2}$, $X_{c,1}^{N_2}$ and $X_{c,2}^{N_2}$ to transmit $U_{p,i}, U_{c,1}$, and $U_{c,2}$ respectively. We assume that each relay has a power constraint P, and we denote by P_p and P_c, the powers allocated to the transmission of private and common messages respectively. All codebooks are generated from i.i.d Gaussian symbols with the statistics being specific to the transmission scheme employed.

In what follows, we focus on the moderate interference case for the second hop where $h_{rd} > h_{rd}^c$. Further, for convenience, we define $\mathbf{h}_1 = [h_{rd}, h_{rd}^c]^\dagger, \mathbf{h}_2 = [h_{rd}^c, h_{rd}]^\dagger$ and $\|\mathbf{h}\|^2 = h_{rd}^2 + {h_{rd}^c}^2$.

A. A Naive Relaying Scheme Based on HK Strategy

Consider a naive scheme where each relay discards the part of information not intended to it (which could be treated as side information for cooperative communication), i.e., L_1 discards $U_{c,2}$, and L_2 discards $U_{c,2}$. Clearly, this strategy is appealing only when relays have little or no common information, i.e., $R_c \to 0$. For the given index, the relay draws common and private codewords with the power constraint, $E[X_{p,i}^2[n]] \le P_p$ and $E[X_{c,i}^2[n]] \le P_c$, and transmits the sum given by $X_{r,i}^{N_2} = X_{p,i}^{N_2} + X_{c,i}^{N_2}$.

As before, the decoding order at the receiver is fixed with the decoding of the common message followed by that of private message. The private information rate (T_p), the common information rate and, the total information rate $I_{\mathsf{LD}}^{\mathsf{HK}}$, on each $L \to D$ link can then be found, using the approach in the Section III. Since this scheme does not exploit the information available in common messages, it is clear that this scheme is suboptimal in general. In the following, we present a few alternative approaches in which relays exploit the common message information.

B. Distributed Beamforming of Common Messages

Different from the above scheme where relays discard the "interfering part" of their common message, the two relays exploit the knowledge available to carry out distributed beamforming of the common messages. For the given message set, each relay draws private and common codewords with the constraints $E[|X_{p,i}|^2] \le P_p,\ i = 1, 2,$ and $E[|X_{c,i}|^2] \le 1,\ i = 1, 2,$ respectively. Then, relay L_i performs pre-filtering to transmit a linear combination of the codewords:

$$X_{r,i}^{N_2} = X_{p,i}^{N_2} + a_{i,1}X_{c,1}^{N_2} + a_{i,2}X_{c,2}^{N_2},\ i = 1, 2,$$

where $a_{i,j}$'s are the beamforming coefficients satisfying the transmit power constraint $a_{i,1}^2 + a_{i,2}^2 \le P_c,\ i = 1, 2.$ Depending on the choice of coefficients $a_{i,j}$'s, the methods of beamforming may vary. Next, we explore two variants of beamforming.

1) Coherent Beamforming (CBF): In CBF, relays coherently relay common messages to the receivers, which extract the required signal by using successive interference cancellation. For this scheme, we set $a_{1,1} = a_{2,2} = \sqrt{P_{cm}}$ and $a_{1,2} = a_{2,1} = \sqrt{P_{cm}}$. Observe that P_{cm} is the portion of the power used by each relay to relay the intended part of its common message, while P_{cu} is the power used to relay the "interfering part" as the side information. Considering the transmission of only common messages, we equivalently model the channel as

$$Y_{d,1}[n] = \theta X_{c,1}[n] + \eta X_{c,2}[n] + \zeta_{d,1}[n], \qquad (3)$$

for $n = N_1 + 1, \ldots, N_2$, where, for convenience, we have defined $\theta = \left(h_{rd}\sqrt{P_{cm}} + h_{rd}^c\sqrt{P_{cu}}\right); \eta = \left(h_{rd}\sqrt{P_{cu}} + h_{rd}^c\sqrt{P_{cm}}\right).$ $\zeta_{d,1}[n]$ is the equivalent noise at D_1 in decoding the common messages, with $\zeta_{d,1}[n] \sim \mathcal{N}(0, \sigma_\zeta^2)$, where $\sigma_\zeta^2 = \|\mathbf{h}\|^2 P_p + \sigma_d^2$. Similarly, for the second receiver D_2,

$$Y_{d,2}[n] = \theta X_{c,2}[n] + \eta X_{c,1}[n] + \zeta_{d,2}[n], \qquad (4)$$

for $n = N_1 + 1, \ldots, N_2$, where $\zeta_{d,2}[n] \sim \mathcal{N}(0, \sigma_\zeta^2)$.

Consider the transmission of common information. We observe that (3) represents a virtual MAC formed by $(X_{c,1}, X_{c,2}, Y_{d,1})$, and (4) represents another virtual MAC formed by $(X_{c,1}, X_{c,2}, Y_{d,2})$. The achievable common information rate region is the intersection of these two MACs. The following proposition gives the optimal power allocation vector for the common information transmission.

Proposition 1: *The maximum common information rate due to CBF is achieved by relays allotting equal power to both useful and interfering part of their common messages. That is, $P_{cm} = P_{cu} = P_c/2$. Furthermore, the maximum common rate achieved on each $L \to D$ link is given by*

$$T_c = \frac{1}{2} C\left(\frac{\left(h_{rd}^c + h_{rd}\right)^2 P_c}{\|\mathbf{h}\|^2 P_p + \sigma_d^2.}\right). \qquad (5)$$

The proof can be found in [14].

Next, consider the transmission of the private message. Since the common messages are decoded first and the private message is decoded later by canceling the interference due to the common messages, the information rate of private message on each $L \to D$ link is given by $T_p = C\left(\frac{h_{rd}^2 P_p}{\sigma_d^2 + {h_{rd}^c}^2 P_p}\right)$.

The maximum rate that can be achieved in the second hop due to CBF is

$$I_{\mathsf{LD}}^{\mathsf{CBF}} = \max_{\substack{P_c, P_p: \\ P_c + P_p = P}} T_p + T_c;\ \text{s. t. } T_c \le R_c.$$

We observe that this scheme is in similar spirit to the relay cooperation scheme considered in [13, Sec. IV]

2) Zero-Forcing Linear Beamforming (ZFBF): In CBF, each receiver needs to decode all the common messages. This might restrict the rate of common information when the interference is moderately low. This problem can be circumvented by subtracting the known interference due to interfering part of the common message at the relays, and this is called zero forcing beamforming (ZFBF). ZFBF consists of inverting the channel matrix by the relays to create orthogonal channels between the relays and the receivers. It can be shown that for ZFBF, $a_{i,j}$'s are given by

$$a_{ij} = \begin{cases} \frac{h_{rd}\sqrt{P_c}}{\sqrt{h_{rd}^2 + {h_{rd}^c}^2}} & i = j \\ \frac{-h_{rd}^c\sqrt{P_c}}{\sqrt{h_{rd}^2 + {h_{rd}^c}^2}}. & i \ne j \end{cases}$$

With the fixed decoding order where, common message is decoded first treating the private messages as noise, the private information rate (T_p) and the common information rate (T_c) on each $L \to D$ link due to beamforming can be given by [14]

$$T_p = C\left(\frac{h_{rd}^2 P_p}{\sigma_d^2 + {h_{rd}^c}^2 P_p}\right); \qquad (6)$$

$$T_c = C\left(\frac{\left(h_{rd}^2 - {h_{rd}^c}^2\right)^2 P_c}{\|\mathbf{h}\|^2 (\sigma_d^2 + \|\mathbf{h}\|^2 P_p)}\right). \qquad (7)$$

Thus the rate that can be achieved per user in the second hop due to ZFBF is

$$I_{\mathsf{LD}}^{\mathsf{ZFBF}} = \max_{\substack{P_c, P_p: \\ P_c + P_p = P}} T_p + T_c;\ \text{s. t. } T_c \le R_c.$$

The beamforming strategies discussed above exploit the common information available. However, both schemes have some disadvantages. CBF requires each receiver to decode the interfering message before it decodes its own, and accordingly rate of common information is determined by the weakest link; whereas in the case of ZFBF, relays incur power loss due to the channel inversion (or interference cancellation), and this may be of interest only in the moderately low interference regime.

C. DPC for Mitigating Interference due to Common Messages

As noted above, a key challenge in the second hop is to exploit the availability of side information at one relay regarding a part of information transmitted by the other relay. From the perspective of transmitting common information only, the second hop can be modeled as a *vector broadcast channel*, except that it has the interference due to private messages. MIMO broadcast channels have been well studied and their capacity regions have been characterized in [3],[16] by exploiting "Dirty Paper Coding" proposed by Costa [4]. Essentially, DPC involves pre-cancellation of interference at the transmitter without incuring any power. It has been shown that DPC achieves the capacity for Gaussian MIMO broadcast channels. Therefore, it is clear that DPC's performance in the second hop is superior when entire information at relays is common. In our setting, relays employ the following steps for the cooperative transmission in the second-hop.

1) DPC for Common Messages: The General Case: Relays collaborate to encode the common messages sequentially. Suppose at any time the common message pertaining to one user is encoded, DPC is performed on the common message of the other user treating the interference due to the first user's common message as the known information. Consider the general case, where relays L_1 and L_2 allocate $P_{p,1}$ and $P_{p,2}$ for the transmission of their private messages. Then, it is clear that the sum power budget is $2P-P_{p,1}-P_{p,2}$ for transmitting common messages. While the second-hop is analogous to a vector BC, we should point out that, relay L_1 has to use only $P-P_{p,1}$ and relay L_2 has to use only $P-P_{p,2}$ to transmit the common information. We will develop a time sharing mechanism to overcome this difficulty. Relays employ $2\times N_2$ vector codewords $\mathbf{X}_{c,1}^{N_2}$ and $\mathbf{X}_{c,2}^{N_2}$, whose elements are i.i.d Gaussian vectors with the covariance constraint $E\left[\mathbf{X}_{c,i}[n]\mathbf{X}_{c,i}^{\dagger}[n]\right]\preceq\Sigma_i,\ i=1,2.$, to carry common messages $U_{c,1}$ and $U_{c,2}$ respectively. Depending on the encoding order of the common information, DPC is performed using following strategies.

•**Encoding strategy** π_1: For private messages, set $P_{p,1}=p_1^*$ and $P_{p,2}=p_2^*$. Relays first encode $U_{c,2}$ using $\mathbf{X}_{c,2}^{N_2}$, then generate $\mathbf{X}_{c,1}^{N_2}$ by encoding $U_{c,1}$ and performing DPC treating $\mathbf{h}_1^{\dagger}\mathbf{X}_{c,2}^{N_2}$, the interference at D_1, as the known non-causal side information. Due to DPC, the interference $\mathbf{h}_1^{\dagger}\mathbf{X}_{c,2}^{N_2}$ disappears at D_1, and $\mathbf{h}_2^{\dagger}\mathbf{X}_{c,1}^{N_2}$ appears as an independent noise at D_2. Relays choose covariance matrices so as to maximize the sum-rate, i.e, $\Sigma_1=\Sigma_1^{\pi_1}, \Sigma_2=\Sigma_2^{\pi_1}$; where $\left\{\Sigma_1^{\pi_1},\Sigma_2^{\pi_1}\right\}$ is the solution to (8).

•**Encoding strategy** π_2: For private messages, set $P_{p,1}=p_2^*$ and $P_{p,2}=p_1^*$. Relays encode $U_{c,1}$ in $\mathbf{X}_{c,1}^{N_2}$, then generate $\mathbf{X}_{c,2}^{N_2}$ by encoding $U_{c,2}$ and then performing DPC treating $\mathbf{h}_2^{\dagger}\mathbf{X}_{c,1}^{N_2}$ as the known side information. As before, the optimal sum-rate, $T_{\pi_2}(p_1^*,p_2^*)$, is obtained by solving (8) swapping indexes 1 and 2, with the corresponding covariance matrices being $\{\Sigma_1,\Sigma_2\}=\left\{\Sigma_1^{\pi_2},\Sigma_2^{\pi_2}\right\}$.

•**Time sharing** : Observe that the power allocation for common messages at each relay, corresponding to the strategy π_j, is determined by $P_{c,1}^{\pi_j}\triangleq\left[\Sigma_1^{\pi_j}+\Sigma_2^{\pi_j}\right]_{1,1}$; $P_{c,2}^{\pi_j}\triangleq\left[\Sigma_1^{\pi_j}+\Sigma_2^{\pi_j}\right]_{2,2}$. To meet the power constraint at each relay, relays perform timesharing by using π_1 for the first half of the slot and π_2 for the second half. Note that, due to symmetry in channel conditions, relay L_i obeys the average power constraint per slot: $0.5*(P_{c,i}^{\pi_1}+P_{c,1}^{\pi_i}+p_1^*+p_2^*)=P$.

2) DPC on private message: Observe that when dirty paper coding is applied, each receiver can decode the part of common information intended only to it. Further, due to the non-linearity of DPC[3], the receiver cannot reconstruct, and therefore, cannot subtract the effect of common codewords from the received signal in decoding its own private message. However, relay L_i generates the private codeword $X_{p,i}^{N_2}$ by encoding $U_{p,i}$ and performing DPC treating interference $\mathbf{h}_i^{\dagger}(\mathbf{X}_{c,1}^{N_2}+\mathbf{X}_{c,2}^{N_2})$ as the known side information. Therefore, due to DPC, each individual receiver can decode its private message as though there was no interference due to common messages.

3) Transmission from the Relays: The codewords transmitted by relays is represented in vector form by $\mathbf{X}_r^{N_2}=\mathbf{X}_{c,1}^{N_2}+\mathbf{X}_{c,2}^{N_2}+\mathbf{X}_p^{N_2}$, where $\mathbf{X}_p^{N_2}=\left[X_{p,1}^{N_2},X_{p,2}^{N_2}\right]^{\dagger}$.

4) Decoding of Messages at Receivers: Receivers decode their common messages by treating the interference due to the private messages as noise. When the encoding order π_1 is used, receiver D_2 decodes $\mathbf{X}_{c,2}^{N_2}$ with no interference from $\mathbf{X}_{c,1}^{N_2}$, and receiver D_1 decodes $\mathbf{X}_{c,1}^{N_2}$ treating interference due to $\mathbf{X}_{c,2}^{N_2}$ as noise. The decoding order for the common messages is reversed under π_2. Since private message is protected against the interference due to common messages, each receiver can decode its private message as though there was no interference due to common messages. However, in decoding its own private information, the private information of the other user is still an interference. For the general case, a closed-form solution for the optimal power allocation is still unknown. However, under the symmetric power allocation for the private message set $P_{p,1}=P_{p,2}=P_p$, we have the following proposition that explicitly characterizes the achievable rate.

Proposition 2: [Symmetric Case] *Under symmetric power allocation $P_{p,1}=P_{p,2}=P_p$, the common information rate per link is*

$$T_c=\frac{1}{2}C\left((h_{rd}^2-{h_{rd}^c}^2)^2\frac{P_c^2}{\sigma_\zeta^4}+\frac{2P_c}{\sigma_\zeta^2}\left\|\mathbf{h}\right\|^2\right), \qquad (9)$$

where $P_c=P-P_p$ and $\sigma_\zeta^2=\sigma_d^2+\left\|\mathbf{h}\right\|^2P_p$. The private information rate per link is given by $T_p=C\left(\frac{h_{rd}^2P_p}{\sigma_d^2+{h_{rd}^c}^2P_p}\right)$. The achievable rate per user in the second hop is given by

$$I_{\mathrm{LD}}^{\mathrm{DPC}}=\max_{\left\{\substack{P_c,P_p:\\ P_c+P_p=P}\right\}} T_p+T_c;\ s.\ t.\ T_c\leq R_c.$$

Proof: Refer to [14].

[3]DPC is often likened to pre-subtraction of the interference at the transmitter. This analogy, however, is not complete. In order to avoid violating the power constraints, special coding needs to be performed, rendering the DPC operation non-linear

$$T_{\pi_1}(p_1^*, p_2^*) = \max_{\Sigma_1,\Sigma_2} \left(C\left(\frac{\mathbf{h}_2^\dagger \Sigma_2 \mathbf{h}_2}{\sigma_d^2 + h_{rd}^2 p_2^* + {h_{rd}^c}^2 p_1^*} \right) + C\left(\frac{\mathbf{h}_1^\dagger \Sigma_1 \mathbf{h}_1}{\sigma_d^2 + \mathbf{h}_1^\dagger \Sigma_2 \mathbf{h}_1 + h_{rd}^2 p_1^* + {h_{rd}^c}^2 p_2^*} \right) \right) \tag{8}$$
$$\text{subject to} \quad \mathsf{tr}\,(\Sigma_1 + \Sigma_2) \leq 2P - P_{p1} - P_{p2}$$

D. Layered Coding with DPC

Among the schemes discussed above, no single scheme is superior for all the ranges of common information available. For example, when we focus just on the transmission of common information, the relays can form a virtual two-antenna transmitter and perform transmission using DPC which is known to be the best strategy in such cases. On the other hand, when all the information available at the relays are private, it is beneficial to perform Han-Kobayashi type of coding at the relays. Clearly, a better strategy is to exploit the capabilities inherent in both HK coding and DPC. In light of these observations, we propose a novel marriage of DPC and HK coding. Layered coding consists of coding over different tiers.

1) Splitting the Private Message for Interference Cancellation: Relay L_i decomposes its private message set $U_{p,i}$ further into sub-messages $U_{p,i} = (U_{pp,i}, U_{pc,i})$, where $U_{pp,i} \in \{1, \ldots, 2^{N_2 T_{pp}}\}$ and $U_{pc,i} \in \{1, \ldots, 2^{N_2 T_{pc}}\}$. We call $U_{pp,i}$ and $U_{pc,i}$ as "sub-private" and "public" messages (to distinguish from the terminology, "common" and "private" message) respectively. For a given message index $U_{p,i} = (U_{pp,i}, U_{pc,i})$, relay L_i draws a sub-private message codeword $X_{pp,i}^{N_2}$ to carry $U_{pp,i}$, and a public message codeword $X_{pc,i}^{N_2}$ to carry $U_{pc,i}$. The sub-private and the public message codewords are generated by i.i.d Gaussian symbols with average power constraint P_{pp} and P_{pc} respectively.

2) DPC for Common Messages: Relay L_i draws two vector codewords, one $\mathbf{X}_{c,1}^{N_2}$ to carry $U_{c,1}$ and $\mathbf{X}_{c,2}^{N_2}$ to carry $U_{c,2}$. Vector codewords are chosen with a covariance constraint $E\left[\mathbf{X}_{c,i}[n]\mathbf{X}_{c,i}^\dagger[n]\right] \preceq \Sigma_i,\ i = 1, 2$, such that $[\Sigma_1 + \Sigma_2]_{1,1} = [\Sigma_1 + \Sigma_2]_{2,2} = P_c$. Relays jointly perform DPC for relaying the common information. The DPC based relaying and the design of optimal covariance matrices Σ_1 and Σ_2 are performed as detailed in the Section IV-C.

3) DPC on Sub-private message: Each relay performs DPC on its sub-private message, $U_{pp,i}$, treating interference $\mathbf{h}_i^\dagger(\mathbf{X}_{c,1}^{N_2} + \mathbf{X}_{c,2}^{N_2})$ as the known side information. Therefore, due to DPC, each individual receiver can decode its sub-private message as though there was no interference due to common messages.

4) Transmission of the codewords: After having computed the codewords for the given message, relays jointly transmit the linear sum of codewords, which is given by

$$\mathbf{X}_r^{N_2} = \mathbf{X}_{pc}^{N_2} + \mathbf{X}_{c,1}^{N_2} + \mathbf{X}_{c,2}^{N_2} + \mathbf{X}_{pp}^{N_2},$$

where $\mathbf{X}_{pc}^{N_2} = [X_{pc,1}^{N_2}, X_{pc,2}^{N_2}]^\dagger$ and $\mathbf{X}_{pp}^{N_2} = [X_{pp,1}^{N_2}, X_{pp,2}^{N_2}]^\dagger$.

5) Decoding at the Receivers: The signal received at the receiver D_i is represented as

$$Y_{d,i}[n] = \mathbf{h}_i^\dagger\left(\mathbf{X}_{pc}[n] + \mathbf{X}_{c,1}[n] + \mathbf{X}_{c,2}[n] + \mathbf{X}_{pp}[n]\right) + Z_{d,i}[n],$$

for $n = N_1 + 1, \ldots, N_2$. Receivers decode messages in the following order. Public messages are decoded first treating $\mathbf{h}_i^\dagger\left(\mathbf{X}_{c,1}^{N_2} + \mathbf{X}_{c,2}^{N_2} + \mathbf{X}_{pp}^{N_2}\right)$, the interference from common messages and private messages, as noise. Once the public messages are decoded, each receiver subtracts their effect $\mathbf{h}_i^\dagger\mathbf{X}_{pc}$, from the received signal $Y_{d,i}$. In the second step, each receiver decodes the part of common message intended to it. In decoding the common message, each receiver treats $\mathbf{h}_i^\dagger\mathbf{X}_{pp}^{N_2}$, the interference due to sub-private messages, as noise. Finally, receivers decode their sub-private messages individually, treating the interference due to the sub-private message of other user as noise. Note that the interference from public messages disappears due to interference cancellation at the receiver, and the interference from common messages is canceled out due to the DPC performed on sub-private messages.

Next, we evaluate the achievable rates due to this scheme. Let a power split (P_{pc}, P_c, P_{pp}) at relays is given such that $P_{pc} + P_c + P_{pp} = P$. Consider the public message part. In decoding public messages, receivers perform successive decoding as detailed in Section III. As noted earlier, the public messages are decoded treating common and sub-private messages as noise. Therefore, equivalent noise at receiver D_i in decoding the public message is given by

$$\Psi_i[n] = \mathbf{h}_i^\dagger\left(\mathbf{X}_{c,1}[n] + \mathbf{X}_{c,2}[n] + \mathbf{X}_{pp}[n]\right) + Z_{d,i}[n].$$

We note that $\Psi[n] \sim \mathcal{N}(0, \sigma_\Psi^2)$, where (cf. [14] for details)

$$\sigma_\Psi^2 = \|\mathbf{h}\|^2(P - Ppc) + \frac{4P_c {h_{rd}^c}^2 h_{rd}^2 (\|\mathbf{h}\|^2 P_{pp} + \sigma_d^2)}{\|\mathbf{h}\|^2(\|\mathbf{h}\|^2 P_{pp} + \sigma_d^2) + P_c h_\Delta^2} + \sigma_d^2.$$

Recalling the details of evaluating common information rate from Section III, we obtain the rate for transmitting public message as

$$T_{pc} = \min\left\{ C\left(\frac{{h_{rd}^c}^2 P_{pc}}{\sigma_\Psi^2}\right), \frac{1}{2}C\left(\frac{\left({h_{rd}^c}^2 + h_{rd}^2\right)P_{pc}}{\sigma_\Psi^2}\right) \right\}.$$

Common messages are decoded treating sub-private messages as noise. Thus from calculations similar to the Section IV-C, we can evaluate the rate of common message as (9)

$$T_c = \frac{1}{2}C\left((h_{rd}^2 - {h_{rd}^c}^2)^2 \frac{P_c^2}{\sigma_p^4} + \frac{2P_c}{\sigma_p^2}\|\mathbf{h}\|^2)\right),$$

where $\sigma_p^2 = \|\mathbf{h}\|^2 P_{pp} + \sigma_d^2$. In decoding sub-private messages, the interference due to the public and common messages is canceled out. Thus, we have the rate for private message as

$$T_{pp} = C\left(\frac{{h_{rd}}^2 P_{pp}}{\sigma_d^2 + {h_{rd}^c}^2 P_{pp}}\right).$$

Thus, the achievable rate per user due to the layered DPC coding can be given by

$$I_{\mathsf{LD}}^{\mathsf{LAY-DPC}} = \max_{\left\{\substack{P_{pc}, P_c, P_{pp}: \\ P_c + P_p = P}\right\}} T_{pc} + T_c + T_{pp};\ \text{s. t. } T_c \leq R_c.$$

We remark that our choice of covariance matrices, in performing DPC for the common messages, is greedy, in the sense that it maximizes the rates for transmission of the common message, without taking into account their effect on the public messages.

We observe that due to the above mentioned decoding order, apart from private messages, common messages also appear as an interference for public messages. Since, common messages are aired using vector broadcast techniques, the signal power of the common messages at the receivers is enhanced significantly, and therefore, public messages may incur significant amount of interference from common messages. Thus when R_c is large, contribution due to public messages vanishes and thus it is beneficial to perform DPC without resorting to split the private message toward interference cancellation. Indeed, when R_c and hence P_c increases, then a small amount of power $P_1 = P - P_c$ is left to rely private and public messages. Concordant to the results obtained for weak interference channels, when P_1 satisfies $P_1 \leq \frac{\sigma_d^2(h_{rd}-2h_{rd}^c)}{2{h_{rd}^c}^3}$, splitting of the private message is no longer necessary. Thus for decoding of private message, treating other user's private message as noise becomes optimal. In this regime of R_c, layered coding scheme boils down to DPC. When R_c is very small, relatively smaller amount of power is spent on transmitting common messages, and therefore, it is beneficial to perform message splitting of the private message to aid in interference cancellation.

E. Layered Coding with Beamforming

In the previous section, we proposed a layered coding strategy that comprises DPC to relay common messages. However, we note that DPC has high complexity. The nonlinearity inherent in DPC inhibits us from avoiding interference due to the common messages when decoding public messages. This may not be superior always. On the other hand, we also observe that BF schemes discussed above are much easier to implement, and allow receivers to decode the common messages before decoding the private messages. In light of the above observations, we propose a coding scheme based on a marriage of HK and BF schemes. The steps used in the layered coding are as follows:

1) Han-Kobayashi coding for the Private Messages: Relay L_i decomposes its private message set $U_{p,i}$ further into public and sub-private messages $U_{p,i} = (U_{pp,i}, U_{pc,i})$, where $U_{pp,i} \in \{1, \ldots, 2^{N_2 T_{pp}}\}$, $U_{pc,i} \in \{1, \ldots, 2^{N_2 T_{pc}}\}$ and $T_p = T_{pc} + T_{pp}$. For a given private message index $U_{p,i} = (U_{pp,i}, U_{pc,i})$, relay L_i draws a sub-private codeword $X_{pp,i}^{N_2}$ to encode $U_{pp,i}$, and a public codeword $X_{pc,i}^{N_2}$ to encode $U_{pc,i}$. The sub-private and the public codewords are generated using i.i.d Gaussian symbols with the average power constraints P_{pp} and P_{pc} respectively, such that $P_p = P_{pp} + P_{pc}$. Each relay then transmits the sum: $X_{p,i}^{N_2} = X_{pp,i}^{N_2} + X_{pc,i}^{N_2}$. Again, receivers use a specific decoding order, where receiver D_i decodes both $X_{pc,1}^{N_2}$ and $X_{pc,2}^{N_2}$, before decoding $X_{pp,i}^{N_2}$.

2) Beamforming for the Common Messages: Relays encode their common messages and compute $a_{i1}X_{c,1}^{N_2} + a_{i2}X_{c,2}^{N_2}$, where $a_{i,j}$'s are beamforming coefficients which are obtained as detailed in Section IV-B.

3) Transmission of the codewords: After having computed the codewords for the given message, relays jointly transmit the linear sum of the codewords:

$$X_{r,i}^{N_2} = a_{i1}X_{c,1}^{N_2} + a_{i2}X_{c,2}^{N_2} + X_{p,i}^{N_2},$$

where the coefficients a_{ij} s are chosen to maximize the rate with a power constraint of P_c on each relay. Details of computing the coefficients a_{ij}s are given in the Section IV-B.

Next, we evaluate the achievable rate of the layered BF scheme. For decoding the common information, the private message is treated as noise. Thus common information is transmitted at a rate of

$$T_c = \max\{T_c^{ZFBF}, T_c^{CBF}\} \tag{10}$$

where T_c^{CBF} and T_c^{ZFBF} are given by (5) and (7) respectively. Once the common messages are decoded, receivers subtract their contribution from the received signal to decode their private messages. Thus there is no interference from the common messages on the private messages. Thus private information is transmitted at a rate of

$$T_p = \max_{\left\{\substack{P_{pc}, P_{pp}: \\ P_{pc}+P_{pp}=P_p}\right\}} T_{pp} + T_{pc}, \tag{11}$$

where $T_{pp} = C\left(\frac{h_{rd}^2 P_{pp}}{\sigma_d^2 + {h_{rd}^c}^2 P_{pp}}\right)$ and

$$T_{pc} = \min\left\{C\left(\frac{{h_{rd}^c}^2 P_{pc}}{\sigma_d^2 + \|\mathbf{h}\|^2 P_{pp}}\right), \frac{1}{2}C\left(\frac{\|\mathbf{h}\|^2 P_{pc}}{\sigma_d^2 + \|\mathbf{h}\|^2 P_{pp}}\right)\right\}.$$

Thus the achievable rate per user due to the layered BF coding is given by

$$I_{\mathsf{LD}}^{\mathsf{LAY-BF}} = \max_{\left\{\substack{P_c, P_p: \\ P_c+P_p=P}\right\}} T_p + T_c; \text{ s. t. } T_c \leq R_c.$$

We note that though BF scheme is suboptimal to DPC, the ability to subtract common information before proceeding to decode private information may turn out to be advantageous. This is pronounced especially for the low values of R_c under high interference regimes, in which the rate due to CBF is enhanced due to the coherence gain while not causing harm to private message part.

V. Numerical Evaluation

We perform numerical evaluation of the schemes discussed above. The metric of interest is the total achievable symmetric rate per user. For the ease of evaluation, we consider a symmetric interference $L \to D$ link with a link SNR of 20dB, where we define link $\mathsf{SNR} = h_{rd}^2 P/\sigma_d^2$. We are interested in studying the effect of common information availability, R_c at the relays, on the achievable information rate per user I_{LD} over the $L \to D$ link. Define the parameter $\alpha = {h_{rd}^c}^2/h_{rd}^2$. Figs. 2 and 3 show the I_{LD} versus R_c plots for different schemes. The optimal power allocation policies and the corresponding achievable rates of different schemes were determined numerically by exhaustive search. We have the following observations.

- Layered coding with DPC outperforms all other schemes for all the ranges of R_c and for relatively small values of α.

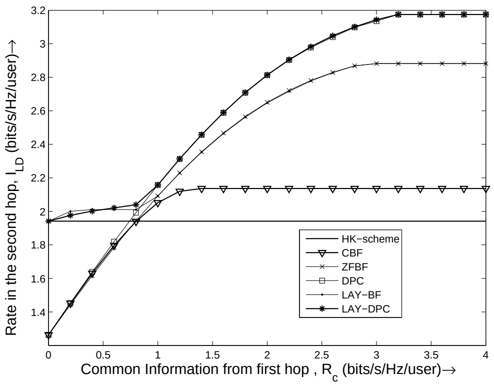

Fig. 2. I_{LD} Vs. R_c curves for $\alpha = 0.2$

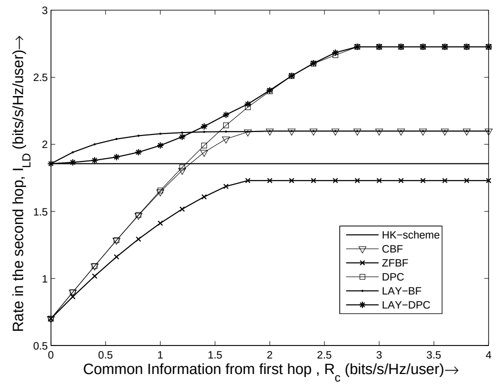

Fig. 3. I_{LD} Vs. R_c curves for $\alpha = 0.6$

This is due to the fact that for relatively smaller values of α's, the interference of common messages on public messages is low and thus enabling efficient message splitting for smaller values of R_c. When R_c is large, layered coding scheme boils down to DPC which is optimal when R_c is large.

• Layered coding with BF is superior to its DPC counterpart for larger values of α in the regime when R_c is small. While both DPC and BF enhance the SNR of the common messages, for layered DPC scheme, the interference due to common messages may drastically affect the rate of public message to an extent where it may become beneficial for relays to allot more power in transmitting private message than on transmitting common messages. However, for layered BF scheme, successive cancellation of interference from common messages at the receivers turns out to be beneficial.

• HK based scheme outperforms DPC and BF schemes when the amount of common information available is relatively low. This indicates that in such cases, it is beneficial to allocate more power for unknown interference cancellation than relaying the common information.

• The performance of layered coding schemes boil down to that of the HK scheme when there is no side information available at the relays, i.e., $R_c = 0$.

• BF is not superior in any regime. For higher values of α, performance of CBF improves whereas the performance of ZFBF becomes worse. When the values of α is low, ZFBF performs better when compared to the CBF. We note that CBF is similar to the relaying scheme considered in [13].

VI. Conclusions

We considered a basic model for the two-hop transmissions of two information flows which interfere with each other. The main focus was on the second hop transmission, where one intrinsic feature is that each relay has access to a part of the information intended for the other destination. Our main contribution in this paper has been the application of the distributed MIMO broadcast techniques to improve the achievable rates in the second hop. In the extreme case, when the relays have complete knowledge of the messages to be transmitted to both destinations, the application is simple. The case where the relays have only partial knowledge of these messages is more challenging. Our best results were obtained using layered approaches, which combine methods from MIMO broadcast (including beamforming and DPC) with Han-Kobayashi, which is the best known approach for non-cooperative communication over the interference channel. Numerical results, at a high SNR of 20 dB, indicate that our approach can provide a substantial rate gain.

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