CSMA-Based Distributed Scheduling in Multi-hop MIMO Networks under SINR Model

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***Abstract*—We study the problem of distributed scheduling in multi-hop MIMO networks. We first develop a "MIMO-pipe" model that provides the upper layers a set of rates and SINR requirements, which capture the rate-reliability tradeoff in MIMO communications. The main thrust of this study is then dedicated to developing CSMA-based MIMO-pipe scheduling under the SINR model. We choose the SINR model over the extensively studied matching or protocol-based interference models because it more naturally captures the impact of interference in wireless networks. The coupling among the links caused by the interference makes the problem of devising distributed scheduling algorithms particularly challenging. To that end, we explore CSMA-based MIMO-pipe scheduling, from two perspectives. First, we consider an idealized continuous time CSMA network. We propose a dual-band approach in which control messages are exchanged instantaneously over a channel separate from the data channel, and show that CSMA-based scheduling can achieve throughput optimality under the SINR model. Next, we consider a discrete time CSMA network. To tackle the challenge due to the coupling caused by interference, we propose a "conservative" scheduling algorithm in which more stringent SINR constraints are imposed based on the MIMO-pipe model. We show that this suboptimal distributed scheduling can achieve an efficiency ratio bounded from below.**

I. Introduction

In this paper, we study distributed scheduling in multi-hop networks with MIMO links, where each node is equipped with an antenna array. There has been a tremendous body of work on the multiple-input multiple-output (MIMO) technology from an information theoretic perspective and phyical layer communications. For single-user wireless channels, it has been shown that using the MIMO technique can lead to dramatic improvements, in terms of capacity and link reliability [1], [2]. Recent studies have explored the fundamental tradeoffs and relations between the different gains in single-user MIMO systems [3]. In stark contrast to the simpler single-user settings, however, there has been little work on exploring the MIMO technique in multi-hop wireless networks, and it remains an open problem to obtain a rigorous understanding of the tradeoffs between the possible MIMO gains in multi-hop networks.

Leveraging MIMO gains in multi-hop networks is intimately related to link scheduling, because the intrinsic rate-reliability tradeoff hinges heavily on the SINR values of the MIMO links that interfere with each other (see, e.g., [4], [5]). We will take two steps to explore optimal scheduling in MIMO multi-hop networks:

1) Develop a link abstraction that can capture the rate-reliability tradeoff for MIMO communications;
2) Obtain a clear understanding of throughput-optimal scheduling under the SINR model[1], and use this as a basis for studying distributed MIMO link scheduling.

We first develop an appropriate "MIMO-pipe" model that provides an abstraction of the rate-reliability tradeoff in MIMO communications, while keeping the scheduling complexity low for scheduling. Clearly, a naive MIMO model that chooses the highest rate for a given SINR is not optimal for MIMO scheduling since it may prevent other links from being simultaneously active and degrade the overall throughput. Instead, our proposed MIMO-pipe model consists of a set of feasible rates and the corresponding SINR requirements. We should note that in general, the possible number of network states is enormous. Therefore, it is desirable for scheduling to be distributed and adaptive using local information only. An excellent candidate for distributed scheduling is to use Carrier Sense Multiple Access (CSMA)-based distributed scheduling (see [7], [8], [9], [10], [11] and the references therein).

In CSMA-based scheduling, nodes first sense the channel activity before attempting transmissions, and only when the channel is sensed to be idle can the nodes continue with data transmissions. When the channel is detected to be busy, the nodes need to wait for a random amount of backoff time before (re-)attempting the transmission. Due to its simplicity, CSMA and its variants have been widely opted in practical MAC protocols (e.g., IEEE 802.11). It has been shown in [7], [10], [12] that under an idealized CSMA model where the random backoff times are assumed to be continuous, optimal distributed scheduling can be found by adapting the CSMA parameters to meet the traffic demand within the capacity region. In practical scenarios, the backoff times are discrete and collisions cannot be avoided completely. With this motivation, recent work [8] has proposed an interesting discrete-time scheduling algorithm with separate control slots and data slots and shown that it can achieve throughput

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[1] A scheduling algorithm is said to be throughput optimal if it can achieve every point in the capacity region [6].

optimality under some conditions.

With this insight, we focus on CSMA-based MIMO-pipe scheduling. We should note that the interference models used in the afore-mentioned works are based on matching [13] or protocol-based models where two links cannot transmit simultaneously if one link is within the range (or hops) of the other link. In contrast, *the transmission under the SINR model depends only on the aggregated interference level, but not on which links the interference originates from.* Consider a network with K links, and suppose the scheduler needs to decide whether link i can be turned on. Under the SINR model, this decision depends on the joint activity status of all other links in the network and there are 2^{K-1} possibilities. It is clear that the classical matching approach would not work well here since to appropriately capture the effect of the SINR model would require exponential complexity [14]. In a nutshell, the SINR model induces intrinsic and possibly global coupling, making it challenging to carry out distributed scheduling. In general, it has been an open problem on how to develop provably efficient distributed scheduling algorithm under the SINR model (even for the SISO case), and a principal goal of this study is to take some steps in this direction.

We will explore CSMA-based MIMO-pipe scheduling, from both continuous time and discrete time perspectives. We summarize below the main contributions in this study.

1) We take a bottom-up approach to develop the MIMO pipe model, which consists of multiple stream configurations, each with a feasible rate and the corresponding SINR requirement. In this model, the tradeoff between diversity and multiplexing of MIMO communications can be captured to the selection of the MIMO configurations. Simply put, each stream configuration is treated as a virtual link with a rate and SINR requirement, and each MIMO link is mapped to multiple virtual links with different SINR requirements.

2) We consider CSMA-based MIMO-pipe scheduling first in a continuous time network. To realize the message passing for exchanging interference tolerance information, we propose a dual band approach where control signals are exchanged instantaneously over a channel separate from the data channel, and show that CSMA-based scheduling can achieve throughput optimality under the SINR model. Further, we characterize the optimal backoff parameters corresponding to different stream configurations.

3) The main thrust is then dedicated to CSMA-based MIMO-pipe scheduling in a discrete time network. To tackle the intrinsic challenge embedded in the "interference aggregation effect" under the SINR model, we devise a distributed scheduling algorithm using a "conservative" interference management strategy. Specifically, we impose a more stringent SINR constraint to ensure that the transitions of the network states only happen in the feasible state region, at the cost of reduced throughput. We then systematically quantify the performance gap between the optimal scheduling and the conservative scheduling approach. We prove that the conservative scheduling algorithm attains at least a fraction (called *efficiency ratio*) of the throughput region, where the efficiency ratio is intimately tied to the *effective interference number*. As expected, the computation of the efficiency ratio needs global information, and we derive a local search algorithm to find a bound on the efficiency ratio.

II. System Setup and Related Work

Consider a multi-hop MIMO network consisting of K links, where each link employs N transmit antennas and N receive antennas. The received signal at the ith receiver denoted as $\mathbf{y}_i$, is given by

$$\mathbf{y}_i = \sqrt{\frac{P}{N d_{ii}^{\alpha}}}\mathbf{H}_{ii}\mathbf{s}_i + \sum_{j \neq i} \sqrt{\frac{P}{N d_{ji}^{\alpha}}}\mathbf{H}_{ji}\mathbf{s}_j + \mathbf{n}_i, \quad (1)$$

where P is the total transmission power at each transmitter, and is assumed to be fixed throughout the paper; $\mathbf{s}_i$ is the $N \times 1$ transmitted signal from the ith transmitter, with normalized power at each antenna array to be 1, in each symbol period; α is the path loss exponent; d_{ji} is the distance from the jth transmitter to the ith receiver; $\mathbf{H}_{ji}$ is the $N \times N$ channel matrix between the jth transmitter to the ith receiver, where the entries of each matrix are i.i.d. complex circular symmetric Gaussian with unit variance. Furthermore, the entries of $\mathbf{H}_{ji}$ are independent from those of $\mathbf{H}_{ji'}$ if $i \neq i'$; $\mathbf{n}_i$ is the additive White Gaussian noise with $\sigma^2 = E[||\mathbf{n}_i^2||]/N$.

The first term in (1) is the desired data signal for link i, while the last two terms are co-channel interference and noise, respectively. As is standard, we assume that the channel matrix $\mathbf{H}_{ii}$ is known at the receiver but unknown at the transmitter of link i (CSI at the receiver) [15]. Moreover, in practical systems, it is difficult, if not impossible, to obtain the MIMO channel matrices $\{\mathbf{H}_{ji}, j \neq i\}$ from the interferers, simply because the signals are not intended for the desired link and it is infeasible to estimate and track these complex matrices. With this insight, we impose the assumption that *for each individual MIMO link, the interfering signals are unknown to its receiver and its transmitter*. Based on the above signal model, it is clear that different from single-user MIMO systems, multi-hop networks are interference-limited, and MIMO communications are intimately tied to the SINR values that are coupled across the links.

A. SINR Model versus Protocol Model

In general, two links can coexist with each other if they can simultaneously transmit successfully. An interference model specifies the link coexistence constraint. We say that the network is in a *feasible state* if the set of active links satisfy the *coexistence constraint* of the interference model. Let M be the set of all vectors of feasible states corresponding to the network. We use binary vector $\mathbf{x}^i = \{0,1\}^K$ to indicate the link states in a feasible state $i \in M$. It follows that $x_k^i = 1$, if link k is active in state i; $x_k^i = 0$ otherwise. With some abuse of notation, we also treat $\mathbf{x}^i$ as the set of active links in state i, i.e., $k \in \mathbf{x}$ if $x_k^i = 1$.

Clearly, different interference models yield different link coexistence constraints and hence different feasible states. Roughly speaking, existing interference models can be classified into two categories, namely the protocol model and the

SINR model. Under the protocol model, the transmission of link l is deemed successful if no other links within a certain transmission range are active, i.e., there is no collision. Due to its simplicity, the protocol model has been widely used. Under the protocol model, the coexistence relationship between two links is determined by their geometry only, and hence is "static".

In contrast, under the SINR model, whether a transmission is successful or not depends on its own channel condition as well as the level of the cochannel interference. A transmission of a link is said to be successful if its SINR is greater than a pre-determined threshold. Thus, unlike protocol or matching-based models, the SINR model can adequately capture the intrinsic probabilistic nature of wireless communications. The SINR model, built upon recent advances in PHY-layer communication research, opens a new avenue for more efficient resource allocation in wireless networks.

As noted before, one significant challenge under the SINR model is that multiple links can transmit successfully through a common channel, even if they "interfere" with each other, which is drastically different from that under the protocol model. Furthermore, link relationship is not only based on their distance, but also depends on the status of the neighboring links, and therefore is "dynamic". *In essence, the link coexistence relationship is multi-lateral and dynamic (in contrast to binary and static under the protocol model). To track this dynamics of the link coexistence relationship, conflict graphs for link scheduling would have to be constructed dynamically incurring extremely high complexity.* That is to say, links scheduling under the SINR model is much more challenging.

In principle, every link in the network can contribute interference to an active receiver. However, as shown in recent study on statistical distribution of co-channel interference (see, e.g., [16],[17]), the aggregated interference from transmitters beyond certain distance (so called "close-in" radius) can be safely neglected. With this insight, we choose the close-in radius to be the K-hop distance (e.g., K can be 10). We should caution that this is different than the k-hop interference model under the protocol (matching) based model where only one link can be active at one time within a k-hop distance, whereas under the SINR model, multiple active links could coexist simultaneously. In this setup, we still use $N(l)$ to denote the set of links that cause interference to link l.

B. CSMA-based Scheduling under Protocol Model

We provide below a brief review of [7], [8], the most related works to our study here.

Assuming the protocol model, an "ideal" CSMA-based scheduling is proposed in [7] for a continuous time network. The feasible states of the ideal CSMA network can be modeled as a continuous-time Markov chain, where transitions only occur between these feasible states that differ from each other by one link status. It follows that the stationary distribution of feasible states $\mathbf{x}^i$ can be characterized by

$$p(\mathbf{x}^i) = \prod_{k\in\mathbf{x}^i} R_k \Big/ \sum\nolimits_l \big(\prod_{k\in\mathbf{x}^l} R_k\big). \tag{2}$$

An adaptive CSMA scheduling algorithm is also developed that adjusts link parameter R_k based on local queue information only.

The algorithm in [8] (cf. [9]) assumes a synchronized time-slotted scheduling, where each time slot consists of a control slot and a data slot. It has been shown that the network states can be modeled as a discrete-time Markov chain, and the corresponding stationary distribution can also be written in a product form.

III. MIMO-pipe Modeling: Rates, SINR, and Interference Tolerance Levels

A first key step in our study on MIMO scheduling is to develop a PHY-based tractable model that captures the rate-reliability tradeoff for a single MIMO link, which we call the "MIMO-pipe model".

In MIMO networks, every MIMO link can offer stream multiplexing by opening up multiple spatial data streams in the same frequency channel, and achieve spatial multiplexing gain. However, given the number of antennas and the total transmission power at each node,[2] the greater the number of data streams there are at each MIMO link, the lower the reliability and the interference tolerance capability per stream. Accordingly, the required average SINR per receive antenna [15], called SINR requirement, is more stringent. In the following, we will elaborate further, the tradeoff between stream multiplexing gain and interference tolerance capability (determined by SINR requirements). Based on the fundamental requirements for reliable communications, we consider a few configurations for stream multiplexing, and develop the corresponding rate and the SINR requirement.

We assume for now that the transmission power is equally split among the transmit antennas. Since *there is no interference information at the transmitter, we set the transmission rate of each stream to be the same, denoted as R_s*. Our intuition is that the rates for data streams depend on the SINR values and hence on the interference, which is however unknown and time-varying. So it is reasonable to set the same rate for difference streams. The SINR requirement of stream r at configuration j, can be in general given by

$$\beta_{jr} = f(j, r, H, P_e), \tag{3}$$

which depends on the channel matrix H and the BER requirement P_e for reliable communication, and the function f hinges on the physical-layer techniques, such as coding and modulation.

Due to self-interference cross data streams on the same MIMO link, the SINR values of different streams can be different. To guarantee the decodability of all data streams,

[2]In this study, the transmission power is assumed to be fixed. Dynamic power control is beyond the scope of this paper.

the SINR requirement of configuration j should be set as $\beta_j = \max\{\beta_{j1}, \beta_{j2}, ..., \beta_{j\Theta(j)}\}$, i.e., the highest SINR requirement corresponding to the bottleneck stream. Such bottleneck stream usually has the least number of transmit antennas. It is therefore reasonable to consider configurations in which transmit antennas are equally divided for each streams. For example, for the 4×4 MIMO link, we consider three configurations: 1-transmit antenna per stream, 2-transmit antennas per stream, and 4-transmit antennas per stream, with data rates $4\mathcal{R}_s, 2\mathcal{R}_s, \mathcal{R}_s$, and SINR requirement $\beta_1 > \beta_2 > \beta_3$, respectively. Furthermore, it can determine how much more interference it can tolerate at each configuration without violating the SINR requirement. The interference power that can be tolerated at configuration j is called the *interference tolerance level*, denoted as T_j. The relationship between interference tolerance (reliability) and rate is shown in Fig. 1. *We emphasize that the stream configurations here correspond to a few points on the rate-reliability tradeoff curve, and that the rates are set to multiplications of the basic rate to reflect the multiplexing gain. More generally, one can choose any points on the rate-reliability tradeoff cruve and develop a corresponding abstraction of the MIMO link.*

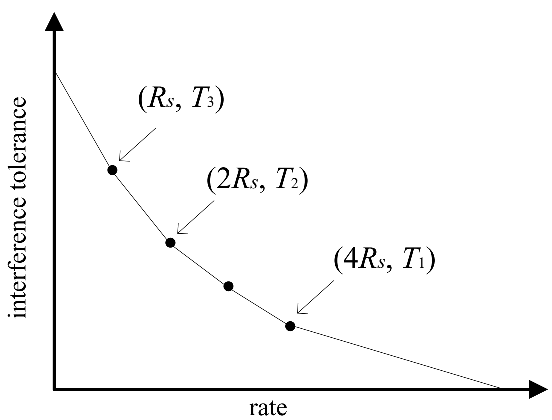


Fig. 1. rate-reliability tradeoff for a MIMO link with 4 antennas

Without loss of generality, suppose that each link has J configurations, and for configuration j, $j \in [1...J]$, there are $\Theta(j)$ date streams. If each stream rate is R_s, then the link rate $\mathcal{R}_j$ is $\mathcal{R}_s\Theta(j)$. The interference tolerance for link i at configuration j is T_{ij}. Accordingly, associated with each link, there are multiple pairs of rates and interference tolerance levels, which can be calculated off-line at the receiver.

Clearly, the configuration with more data streams (higher multiplexing degree) can achieve a higher data rate, but in the meanwhile, fewer transmit antennas are assigned to each stream which results in a lower interference tolerance level. Once a link chooses a higher rate configuration, it would not be able to co-exist with many nearby links. So there exists an intrinsic tradeoff between link scheduling and the link configurations, in order to achieve optimal network throughput, and a key part of MIMO-pipe scheduling is to determine what probability one link should activate each configuration with.

IV. CSMA-based MIMO-pipe Scheduling: A Continuous-time Perspective

In this section, we study CSMA-based scheduling from a continuous time perspective under SINR model.

A. A Dual Band Approach

The first step is to design an interference management mechanism so that the network state can still associate with a Markov chain under SINR model. Observe that the network state dynamics can be captured by a continuous time Markov chain, with each state in the Markov chain corresponding to a feasible state, provided that the operation of the network satisfies the following conditions:

(C1) State transitions take place from one feasible state to another feasible state where there is only one link state in difference between the two state vectors.

(C2) The duration at each feasible state is exponentially distributed.

for ease of exposition, we first study distributed scheduling in SISO networks. The exponential distribution of the backoff time can guarantee a single link state change during each transition. Furthermore, to ensure that the network state evolution stays in feasible states under the SINR model, it suffices that any new transmission meets the following requirements: 1) the transmission itself can tolerate the interference from other active links; and 2) the transmission would not cause the on-going transmissions of other links to fail.

To meet the above requirements, we propose to use the following interference management mechanism. First, each link, based on the current interference level it experiences, calculates how much more interference it can further tolerate without disturbing its transmission, denoted as T_i for link i. No link can launch a new transmission if its interference tolerance is negative. For the second requirement, we assume that each link has the network topology information so it can calculate how much interference it would incur to its neighbors. Such topology information can be obtained before scheduling (see, e.g., [14],[18]). Specifically, each active link updates its inference tolerance level and broadcasts the updated interference tolerance level to nearby links, whenever it starts transmission, or it suffers "new" interference from its neighbors. In this way, each link always knows *how much interference the nearby active link can tolerate currently*. A link cannot begin its transmission if its interference cannot be tolerated by any nearby active link, which can in turn ensure that the second requirement be met.

The above interference management approach involves frequent exchanges of control messages. To ensure that the data transmissions would not collide with the control signal, we separate the data channel and the control channel for each signal. We assume that the transmissions of control signal can be completed instantaneously and do not collide, as is the case in [7]. It is in this sense that we call the proposed algorithm an ideal CSMA-based scheduling under the SINR model. Clearly, the network can still be modeled as continuous time Markov chain, and the stationary distribution and link throughput share the same forms as that under the protocol model.

B. CSMA-based MIMO-pipe Scheduling

In SISO networks, each link transmits only one data stream, so it has only two states (on or off). Hence, it suffices to use a binary vector $\mathbf{x}$ to represent the state of each link. In contrast, in the MIMO-pipe model, each link has multiple configurations with different transmission rates and SINR requirements. Hence, to describe a feasible state in MIMO network, we also need to specify the configuration of each active link. We use two additional K-dimension vectors, besides the vector $\mathbf{x}^i \in \{0,1\}^K$, to describe the feasible state in MIMO networks. Assume each link has J configurations. Vector $\mathbf{z}^i \in \{j\}^K, j \in [1...J]$ denotes which configuration each active link chooses at feasible state i. Since different configurations have different rates, vector $\mathbf{c}^i \in \{\Theta(j)\}^K, j \in [1...J]$ is needed to denote the transmission rates, where $\Theta(\cdot)$ is the mapping from the configuration index to the corresponding normalized transmission rate.

Each link can randomly choose a feasible configuration as long as it satisfies the SINR requirement. Here we denote each configuration as a "virtual link," and each virtual link has its unique mean back-off time. Specifically, $\forall i, j$, virtual link j of link i has mean back-off time $1/R_{ij}$. And each data transmission time is exponentially distributed with mean 1. Then, the feasible states of the MIMO network can be captured by a continuous-time Markov chain. For the MIMO case, we can derive the stationary distribution of feasible states $\mathbf{x}^i$ as

$$p(\mathbf{x}^i) = \frac{\prod\limits_{k \in \mathbf{x}^i} x_k^i R_{kj}}{\sum_l \big(\prod\limits_{k \in \mathbf{x}^l} x_k^l R_{kj}\big)} = \frac{\exp(\sum_{k=1}^K x_k^i r_{kj})}{\sum_l \exp(\sum_{k=1}^K x_k^l r_{kj})}, \quad (4)$$

where $j = z_k^i$ and $r_{kj} = \log(R_{kj})$. The normalized throughput (service rate) of link k is given by

$$\theta_k(\mathrm{r}) = \sum\nolimits_i \Theta(z_k^i) \cdot p(\mathbf{x}^i). \quad (5)$$

The next key step is to optimize the back-off time of each virtual link, so that the corresponding adaptive CSMA algorithm can converge to the throughput optimal one. Specifically, assume that the traffic load at link k is represented by the normalized arrival rate $\lambda_k \geq 0$. A central problem here is to use local information to adapt the back-off time of each MIMO link and its configurations so as to meet the throughput requirement of each link, i.e., $\theta_k \geq \lambda_k$. As in [7], we first consider the following entropy maximization problem:

$$\begin{array}{ll} \max & -\sum_i u_i \log u_i \\ s.t. & \sum_i u_i \cdot c_k^i \geq \lambda_k, \\ & u_i \geq 0, \ \sum_i u_i = 1. \end{array} \quad (6)$$

In the MIMO network, i corresponds to each feasible state associated with the network. If this problem is feasible, the optimal point u^* satisfies the constraint $\sum_i u_i^* \cdot c_k^i \geq \lambda_k$. On the other hand, if we can let the stationary distribution of any feasible state i equals the optimal value u_i^*, $p(i) = u_i^*$, then each link of network can also satisfies the throughput requirement $\theta_k \geq \lambda_k$.

The Lagrangian of (6) can be written as

$$L = -\sum\nolimits_i u_i \log u_i + \sum\nolimits_k y_k \Big(\sum\nolimits_i u_i \cdot c_k^i - \lambda_k\Big) + \mu\Big(\sum\nolimits_i u_i - 1\Big), \quad (7)$$

where y and μ are dual variables. Combine the KKT condition with the requirement $p(\mathbf{x}^i) = u_i^*$, we obtain that

$$u_i^* = \frac{\exp(\sum_{k=1}^K y_k c_k^i)}{\sum_l \exp(\sum_{k=1}^K y_k c_k^l)} = p(\mathbf{x}^i) = \frac{\exp(\sum_{k=1}^K x_k^i r_{kj})}{\sum_l \exp(\sum_{k=1}^K x_k^l r_{kj})}, \quad (8)$$

and hence

$$y_k c_k^i = x_k^i r_{kj}, \qquad z_k^i = j x_k^i, \ c_k^i = \Theta(z_k^i). \quad (9)$$

It follows that the mean back-off time of any configuration satisfies

$$1/R_{kj} = \exp(-y_k \Theta(j)). \quad (10)$$

Then, the throughput-optimal scheduling algorithm can be developed along the same line as in [7].

V. CSMA-based MIMO-pipe Scheduling: A Discrete Time Perspective

In the following, we turn our attention to distributed MIMO-pipe scheduling in a synchronized time-slotted network.

A. CSMA-based Conservative MIMO-pipe Scheduling

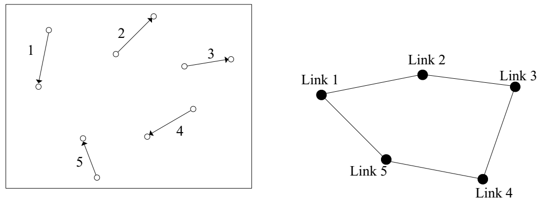
Recall that under the protocol model, the coexistence relationship between two links is static and does not depend on the states of other links, i.e., the two links are either conflicting or can coexist. As a result, each link can tell, based on local information, if it can coexists with its neighboring links. In contrast, under the SINR model, a link l can tolerate several subsets of its neighbors, as long as the aggregated interference from these links is lower than T_l. That is to say, the coexistence relationship between two links is not static but depends on the states of the neighboring links. As a result, the classical matching approach would not work well under the SINR model.

For ease of exposition, again we first consider SISO networks. To tackle the above challenge, we impose a more stringent requirement for link coexistence beyond the nominal SINR constraint. Specifically, for each link l, we rank its neighboring links in an ascending order based on the interference they incur to link l. We partition the neighboring set $N(l)$ into two disjoint sets $N_a(l)$ and $N_b(l)$, where $N_a(l)$ contains all the neighboring links (starting from the link incurring the lowest interference to the highest) such that their aggregated interference to link l is no greater than T_l, and $N_b(l) = N(l) \backslash N_a(l)$. For convenience, we call $N_a(l)$ the "tolerable set" and $N_b(l)$ the "intolerable set".

With the help of the above notation, we impose the following more stringent coexistence constraint:

Conservative coexistence constraint for SISO links: $\forall k \in N(l)$ and $\forall l \in N(k)$, links l and k can coexist if and only if $k \in N_a(l)$ and $l \in N_a(k)$.

Thanks to this new coexistence condition, we can now depict a conflict graph G for the network, where each vertex



(a) An example network with 5 links

(b) The conflict graph for example network in (a)

Fig. 2. A example network and the associated conflict graph.

TABLE I
TOLERABLE SET AND INTOLERABLE SET

link	tolerable links	intolerable links
1	3, 4	2, 5
2	4, 5	1, 3
3	1, 5	2, 4
4	1, 2	3, 5
5	2, 3	1, 4

corresponds to a link, and there is an edge between two vertexes if they conflict with each other. Moreover, only the links in $N_a(l)$ are allowed to transmit simultaneously with l, and the aggregated interference from $N_a(l)$ is guaranteed to be lower than T_l, so the interference aggregation effect is taken care of. Building on this model, we next develop a suboptimal distributed scheduling. Note that the above conservative constraint amounts to a more stringent SINR requirement, and it is in this sense we call the suboptimal scheduling as "conservative scheduling". Fig 2(a) depicts an example network with 5 links under the conservative SINR constraint. The network topology and interference tolerance levels are predetermined. The tolerable sets and the conflicting set of each link are shown in table I. According to the conservative coexistence constraint, only the following link pairs can coexist: (1, 3), (1, 4), (2, 4), (2, 5), (3, 5). The corresponding conflict graph to this network is shown in Fig. 2(b).

Next, we generalize the study to the MIMO-pipe scheduling, and use the notion of "virtual link" introduced in the continuous time case. Let $\mathcal{V}(l)$ be the set of virtual links corresponding to link l, and $l_j \in \mathcal{V}(l)$ be the virtual link corresponding to the jth configuration of link l. For a network with K links and J configurations for each link, we use K-dimension vector $z \in \{j\}^K, j \in [1...J]$ to denote the configuration of each link at a feasible state, i.e. the state of each virtual link. It follows that $l_j \in z(t)$ if link l chooses configuration j in the slot t and $z_l(t) = j$. Similarly, the decision schedule $m(t)$ is a K-dimension integer vector too.

For virtual link $l_i \in \mathcal{V}(l)$, it has a unique interference tolerance level T_{lj}. Each virtual link l_i has a unique tolerable set $N_a(l_i)$ and an intolerable set $N_b(l_i)$ as previous defined.

Conservative coexistence constraint for MIMO pipes:

- At each slot, only one virtual link in $\mathcal{V}(l)$ can be active.
- $\forall k \in N(l)$ and $\forall l \in N(k)$, the virtual links l_i and k_j can coexist if and only if $l_i \in N_a(k_j)$ and $k_j \in N_a(l_i)$.

We next devise CSMA-based MIMO link scheduling by using conservative coexistence constrainst for MIMO pipes. Note that another key challenge is in exchanging control messages and resolving their "collisions" (cf. [8]). Any link l_i includes the information of $N_b(l_i)$ in its control message, so that the links in $N(l)$ can check the coexistence with l_i after receiving this control message. Note that under the protocol model, each link has the knowledge of the states of its conflicting links through the message exchange, and the collision between control messages is easy to tell. In contrast, as noted before control messages may be decoded successfully under the SINR model even if they "collide". To resolve this issue, we use the conservative SINR approach again. Specifically, each link can discover the "collision" based on the SINR level when decoding its control signal, and it will not be included in the decision schedule if a "collision" is detected. The proposed CSMA-based MIMO-pipe scheduling algorithm is summarized in Algorithm 1.

We note that under the SINR model, throughput-optimal scheduling requires construction of dynamic conflict graphs to take into account the multi-lateral and dynamic link coexistence relationships, which would incur extremely high complexity. With this insight, we have taken a conservative scheduling approach instead, and imposed the conservative SINR constraint for MIMO pipes. In this way, the coexistence relationship between two virtual links "becomes" static again, greatly simplifying the construction of the corresponding conflict graph.

Observe that in Algorithm 1, each virtual link can make decisions on its transmission state distributedly. As in the protocol model [8], the network state $z(t)$ can be modeled as a discrete-time Markov chain, since the transition probability depends on the selection probability of decision schedule m and the activation probability of each virtual link. It takes the following form:

$$p(\mathbf{z}, \mathbf{z}') = \sum_{m \in A(\mathbf{z}, \mathbf{z}')} a(m) \prod_{l_\alpha \in a} \bar{p}_{l_\alpha} \cdot \prod_{k_\beta \in b} p_{k_\beta} \cdot \prod_{i_\gamma \in c} p_{i_\gamma} \cdot \prod_{j_\theta \in d} \bar{p}_{j_\theta}, \tag{11}$$

where $A(\mathbf{z}, \mathbf{z}')$ denotes the set of decision schedule m which include links differ in $\mathbf{z}$ and $\mathbf{z}'$. For all virtual links included in m with no conflicting links active in the previous slot, they can be classified into four sets: set a denotes the virtual links active in $\mathbf{z}$ and inactive in $\mathbf{z}'$; set b denotes the virtual links inactive in $\mathbf{z}$ and active in $\mathbf{z}'$; set c denotes the virtual links which keep active in two states; and set d denotes the virtual links which keep inactive in two states. It can be verified the

[3]INTENT message has the similar definitions as in [8]. The index of links in $N_b(l_i)$ is included in the INTENT message, so any link k_j receiving this INTENT message can examine if l_i and k_j can coexist.

Algorithm 1 Distributed schedule algorithm under MIMO-pipe model

Initialization: Find $N_a(l_i)$ and $N_b(l_i)$ for every virtual link l_i.

Selection of decision schedule m

1) Virtual link l_i selects a random back-off time uniformly in [1, W_l], and begins back-off.
2) Virtual link l_i stops the back-off timer when one of the following two conditions is valid: (1) l_i hears an INTENT message[3] from virtual link k_j, and link l_i and k_j are conflicting links, or (2) other virtual links in $\mathcal{V}(l)$ send INTENT messages.
3) After l_i finishes the backoff, it sends INTENT message to announce its intention to be included in the decision schedule.
4) If there is a "collision" among the INTENT messages, l_i will not be included in $m(t)$ in this control slot.

Setup of the transmission state

Any link l_i in m can change its transmission state if both of the following conditions are satisfied:

- $\forall k \in N(l)$, if virtual link k_j is active in previous data slot, l_i and k_j can coexist conservative SINR constraint.
- no virtual link in $N(l)$ is active in previous data slot

If the above conditions are valid, l_i will change its state as follows: let $z_l(t) = i$ with activation probability p_{l_i}, and $z_l(t) = 0$ with probability $\bar{p}_{l_i} = 1 - p_{l_i}$.

If either condition is not satisfied, then $z_l(t) = z_l(t-1)$

Data transmission

- If $z_l(t) = i$, l will transmit using configuration i in the data slot.
- If $z_l(t) = 0$, l will not transmit in the data slot.

stationary distribution of feasible state $\mathbf{z}$ is given by:

$$p(\mathbf{z}) = \prod_{l_i \in \mathbf{z}} \frac{p_{l_i}}{\bar{p}_{l_i}} \Bigg/ \sum_{\tilde{\mathbf{z}} \in M} \prod_{l_i \in \tilde{\mathbf{z}}} \frac{p_{l_i}}{\bar{p}_{l_i}}. \quad (12)$$

As in [8], each p_{l_i} can be adapted using local queue information.

B. Efficiency Ratio of Conservative MIMO-pipe Scheduling

It is clear that the conservative scheduling can only achieve a fraction of the throughput region for the SINR-based scheduling. Next, we characterize the efficiency ratio achieved by the conservative SINR-based scheduling. More specifically, we want to characterize what is the maximum value $0 \le \gamma \le 1$ such that for any feasible arrival rate λ in the ideal case, $\gamma\lambda$ is achievable by the conservative scheduling.

It has been proved in [8] that the adaptive CSMA-based scheduling can achieve throughput optimality. It means that the throughput region of the algorithm is the convex hull of the set of feasible states. To compare the feasible arrival rate region under the SINR case and the conservative SINR case, it suffices to compare the convex hull formed by feasible states from these two constraints.

For notation convenience, let $\mathcal{S}$ and $\mathcal{C}$ denote the sets of the feasible states under the nominal SINR model and the conservative SINR model, respectively. For a MIMO-pipe model with K links, we use a K-dimension vector to denote the feasible state, where each element is the link transmission rate at corresponding state. For each feasible state $s \in \mathcal{S}$, there exists a subset $C \subset \mathcal{C}$ such that the set of the active virtual links in s, can be "covered" by the union of the sets of the active virtual links for the feasible states in C, i.e.,

$$\begin{aligned} &\{l \in 1, 2, \cdots, K : s_l = r \text{ in State } s\} \\ &\subset \bigcup_{c \in C} \{l \in 1, 2, \cdots, K : c_l = r \text{ in State } c\} \end{aligned} \quad (13)$$

Note that there may exist multiple different subsets $C \subset \mathcal{C}$ that "covers" the set of the active links of s. Nevertheless, we will show that only the subsets with the least cardinality are closely related to the efficiency ratio.

To this end, let $V_k \subset \mathcal{C}$ be the *minimal covering set* for state s_k in the sense that 1) V_k satisfies (13), and 2) for any other subset $V \subset \mathcal{C}$ that satisfies (13), we have that the cardinality of V_k is no larger than that of V, i.e., $|V_k| \le |V|$. Worth noting is that the minimal covering set is not unique, but the cardinality of the minimal covering set is unique.

Define the *effective interference number* as the maximum of the cardinalities among the minimal covering set for all the feasible states in $\mathcal{S}$, i.e.,

$$N(\mathcal{S}, \mathcal{C}) \triangleq \max_{\{k: s_k \in \mathcal{S}\}} |V_k|.$$

Under the conservative SINR model, any s_k in $\mathcal{S}$ can be decomposed into no more than $N(\mathcal{S}, \mathcal{C})$ states in $\mathcal{C}$, where $N(\mathcal{S}, \mathcal{C})$ depends on the coexistence relationship of links.

***Theorem** 5.1:* The conservative MIMO-pipe scheduling results in an efficiency ratio $\gamma \ge 1/N(\mathcal{S}, \mathcal{C})$.

Proof: For any feasible arrival rate $\lambda = \{\lambda_1, \lambda_2, \cdots, \lambda_K\}^T$ under the SINR model, there exists a state probability vector $\mathbf{P} = \{P_1, P_2, \cdots, P_{|\mathcal{S}|}\}^T$ such that $\sum_{i=1}^{|\mathcal{S}|} P_i = 1$, and

$$\mathbf{P}^T \mathbf{A}^S \ge \lambda, \quad (14)$$

where $\mathbf{A}^S$ is a $|\mathcal{S}| \times K$ matrix, with

$$A_{k,l}^S \triangleq (\text{Transmission rate of link } l \text{ in state } s_k), \forall k, l. \quad (15)$$

To show that $\gamma \ge \frac{1}{N(\mathcal{S}, \mathcal{C})}$, it suffices to show that there exists a state probability vector $\mathbf{Q} = \{Q_1, Q_2, \cdots, Q_{|\mathcal{C}|}\}^T$ such that

$$\mathbf{Q}^T \mathbf{A}^C \ge \gamma\lambda, \quad (16)$$

where $\mathbf{A}^C$ is defined in the same way as $\mathbf{A}^S$ in (15).

We use induction on $|\mathcal{S}|$ to show that (16) is valid for some $\mathbf{Q}$, for any given $\mathbf{P}$ satisfying (14). It is easy to verify when $|\mathcal{S}| = 1$. Assume that the conclusion holds when $|\mathcal{S}| = n$. Now we consider the case $|\mathcal{S}| = n + 1$, pick the state s_k in $\mathcal{S}$ such that $|V_k| = N(\mathcal{S}, \mathcal{C})$. Without the loss of generality, suppose $k = n + 1$.

It follows from (14) that for $l = 1, 2, \cdots, K$,

$$\sum_{i=1}^{n} P_i A_{i,l}^S + P_{n+1} A_{n+1,l}^S \ge \lambda_l, \quad (17)$$

which indicates that for $l = 1, 2, \cdots, K$,

$$\sum_{i=1}^{n} P_i' A_{i,l}^S \ge \lambda_l', \quad (18)$$

where

$$P'_i \triangleq \frac{P_i}{1-P_{n+1}},\ \lambda'_l \triangleq \frac{\lambda_l - P_{n+1}A^S_{n+1,l}}{1-P_{n+1}}. \quad (19)$$

By induction, based on (18), there exists $\mathbf{Q}'$ such that $\sum_{j=1}^{|\mathcal{C}|} Q'_j = 1$, and for $l = 1, 2, \cdots, K$,

$$\sum_{j=1}^{|\mathcal{C}|} Q'_j A^C_{j,l} \geq \gamma' \lambda'_l, \quad (20)$$

where $\gamma' \triangleq \frac{1}{N(\mathcal{S}',\mathcal{C})}$ and $\mathcal{S}' = s_k, k = 1, 2, \cdots, n$. It is clear that, $\gamma' \geq \gamma$, and it follows that

$$\sum_{j=1}^{|\mathcal{C}|} Q'_j A^C_{j,l} \geq \gamma \lambda'_l, \forall l = 1, 2, \cdots, K. \quad (21)$$

Similar, we can find $\mathbf{Q}''$ such that $\sum_{j=1}^{|\mathcal{C}|} Q''_j = 1$, and

$$\sum_{j=1}^{|\mathcal{C}|} Q''_j A^C_{j,l} \geq \gamma A^S_{n+1,l}, \forall l = 1, 2, \cdots, K. \quad (22)$$

Define

$$Q_j \triangleq Q'_j(1-P_{n+1}) + Q''_j P_{n+1}, \forall j = 1, 2, \cdots, |\mathcal{C}|. \quad (23)$$

Observe that $\mathcal{Q} = \{Q_1, Q_2, \cdots, Q_{|\mathcal{C}|}\}^T$ defined above is a state probability vector, i.e.,

$$\sum_j Q_j = (\sum_j Q'_j)(1-P_{n+1}) + (\sum_j Q''_j)P_{n+1} = 1. \quad (24)$$

Furthermore, multiplying (21) with $(1-P_{n+1})$ on both sides yields that

$$\sum_{j=1}^{|\mathcal{C}|} Q'_j(1-P_{n+1})A^C_{j,l} \geq \gamma(\lambda_l - P_{n+1}A^S_{n+1,l}), \forall l = 1, 2, \cdots, K. \quad (25)$$

Further, multiplying (22) with P_{n+1} yields that

$$\sum_{j=1}^{|\mathcal{C}|} Q''_j P_{n+1} A^C_{j,l} \geq \gamma P_{n+1} A^S_{n+1,l}, \forall l = 1, 2, \cdots, K. \quad (26)$$

Adding the above two equations together, we see that $\mathcal{Q}$ defined in (23) satisfies (16), and the proof is concluded. ∎

The above result reveals that the efficiency rate is bounded from below by the reciprocal of the effective interference number. Note that determining the effective interference number requires globe information of all the feasible states in general. In the following, we develop a local search algorithm to find an upper bound on the effective interference number.

Observe that for any virtual link l_i, the set $L = \{l_i\} \cup \{c | c \subset N(l_i)\}$ satisfies the *nominal SINR constraint*, i.e., the virtual links in L can coexist. We call L a "local feasible state," and clearly virtual link l_i can belong to multiple local feasible states. We use $L(l_i, j)$ to denote the j-th local feasible state of l_i, and $n_v(l_i, j)$ to denote the number of links in $L(l_i, j)$ conflicting with l_i under the *conservative SINR constraint*. For convenience, define

$$n_e \triangleq \max_{l_i} \max_{L(l_i,j)} n_v(l_i, j).$$

It follows that for any virtual link, $n_v(l_i, j)$ would be no greater than n_e. Then, we have the following result.

Algorithm 2 local search algorithm

let $n_e = 0$;
for $l = 1$ to K **do**
  For link l, let $n_l = 0$
  for $i = 1$ to J **do**
    For virtual link l_i, let $n_v(l_i) = 0$
    repeat
      For local feasible state $L(l_i, j)$
      if $n_v(l_i, j) \geq n_v(l_i)$, **then**
        $n_v(l_i) = n_v(l_i, j)$
      end if
    until all local feasible states of l_i has been enumerated
    if $n_v(l_i) \geq n_l$, **then**
      $n_l = n_v(l_i)$
    end if
  end for
  if $n_l \geq n_e$, **then**
    $n_e = n_l$
  end if
end for

***Theorem** 5.2:* The effective interference number is upper bounded by $n_e + 1$, i.e., $N(\mathcal{S}, \mathcal{C}) \leq n_e + 1$.

Due to space limitation, the proof of Theorem 5.2 is omitted (see [19]). Combining Theorems 5.1 and 5.2, we conclude that

$$\gamma \geq \frac{1}{N(\mathcal{S},\mathcal{C})} \geq \frac{1}{n_e + 1}. \quad (27)$$

VI. Numerical Examples

In this section, we illustrate, via numerical examples, the performance by using the proposed CSMA-based MIMO-pipe scheduling in a multi-hop network. Due to space limitation, we present the results for the continuous system only (similar studied can be carried out for the discrete system).

Specifically, we study a network with four 2×2 MIMO links, each with two possible configurations, namely, Configuration 1 (R, β_1) and Configuration 2 $(2R, \beta_2)$ (where $\beta_2 > \beta_1$). The network parameters, such as the transmit power and the distances between the links, are set such that when all the links use Configuration 1, they can transmit simultaneously to achieve a throughput [1, 1, 1, 1]; and if one link chooses Configuration 2, then the other link cannot transmit at the same time.

We study optimal scheduling scheduling under two different MIMO models: one is the naive MIMO transmission approach in which a link always chooses the highest rate to transmit, and the feasible states in this model are: $\{2,0,0,0\}, \{0,2,0,0\}, \{0,0,2,0\}$ and $\{0,0,0,2\}$. The second approach adopts the proposed CSMA-based MIMO-pipe scheduling, which has an additional feasible state $\{1,1,1,1\}$, besides the four feasible states above. The queue behaviors

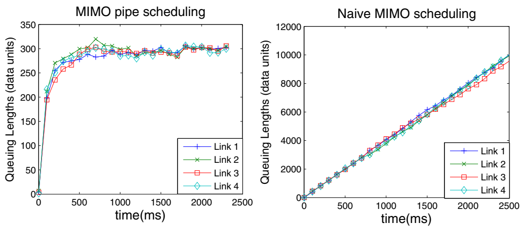


Fig. 3. Queue behavior comparison $\lambda = [1, 1, 1, 1]$.

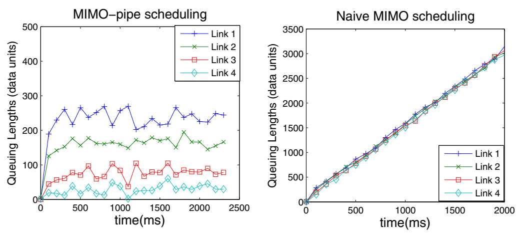


Fig. 4. Queue behavior comparison, with $\lambda = [1.1, 0.8, 0.6, 0.5]$.

with two different arrival rates are depicted in Fig. 3 and 4. It can be observed that the queue length blows up (does not converge) under the naive approach. In contrast, the CSMA-based MIMO-pipe scheduling yields stable queue behavior at both arrival rates, indicating that it can achieve a larger throughput region than that using the naive MIMO scheduling approach.

VII. Conclusion and Future Work

We studied CSMA-based distributed scheduling in multi-hop MIMO networks. There are a few key steps in developing optimal MIMO scheduling, including 1) deriving a PHY-based link abstraction that captures the rate-reliability tradeoff for MIMO communications, and 2) obtaining a clear understanding of throughput-optimal scheduling under the SINR model. To this end, we first developed a MIMO-pipe model that provides the upper layers a set of rates and SINR requirements, which capture the rate-reliability tradeoffs in MIMO communications. We then focused on developing CSMA-based MIMO-pipe scheduling under the SINR model. We observed that throughput-optimal CSMA-based scheduling, under the protocol model, would not work well under the SINR model, due to the dynamic and intrinsic link coupling. To tackle this challenge, we explored CSMA-based MIMO-pipe scheduling in both a continuous-time system and a discrete-time system. Particulary, in the idealized continuous-time CSMA network, we proposed a dual-band approach to facilitate the message passing on interference tolerance levels, and showed that CSMA-based scheduling can achieve throughput optimality under the SINR model. For the more diffiuclt discrete time case, we developed a "conservative" scheduling algorithm in which more stringent SINR constraints are imposed based on the MIMO-pipe model. We showed that an efficiency ratio bounded from below can be achieved by this suboptimal distributed scheduling algorithm.

We believe that the studies here on SINR-based distributed scheduling scratch only the tip of the iceberg. There are still many questions remaining open along this line, and we are currently investigating these issues along this avenue.

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