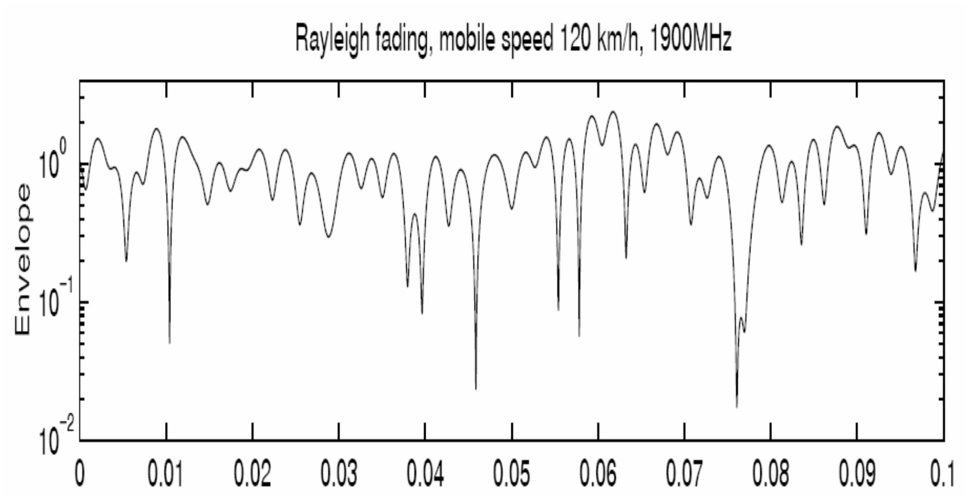
Distributed Opportunistic scheduling (DOS) for Ad-Hoc Communications: An Optimal Stopping Approach

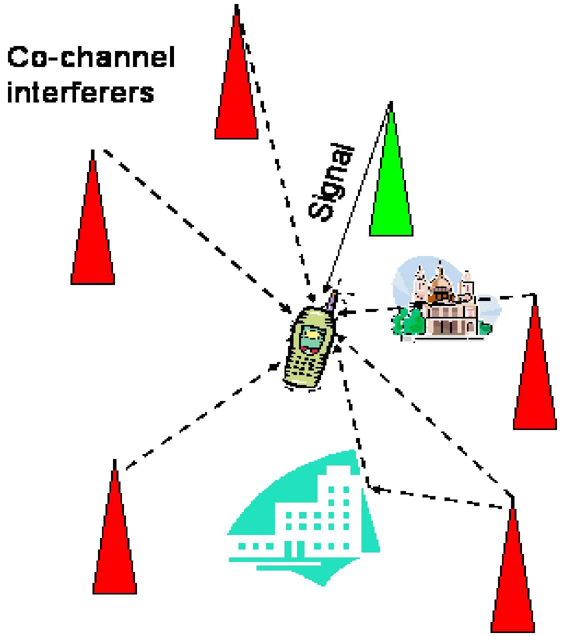
Junshan Zhang
Dept. of Electrical Engineering
Arizona State University

Joint work With Dong Zheng and Weiyan Ge

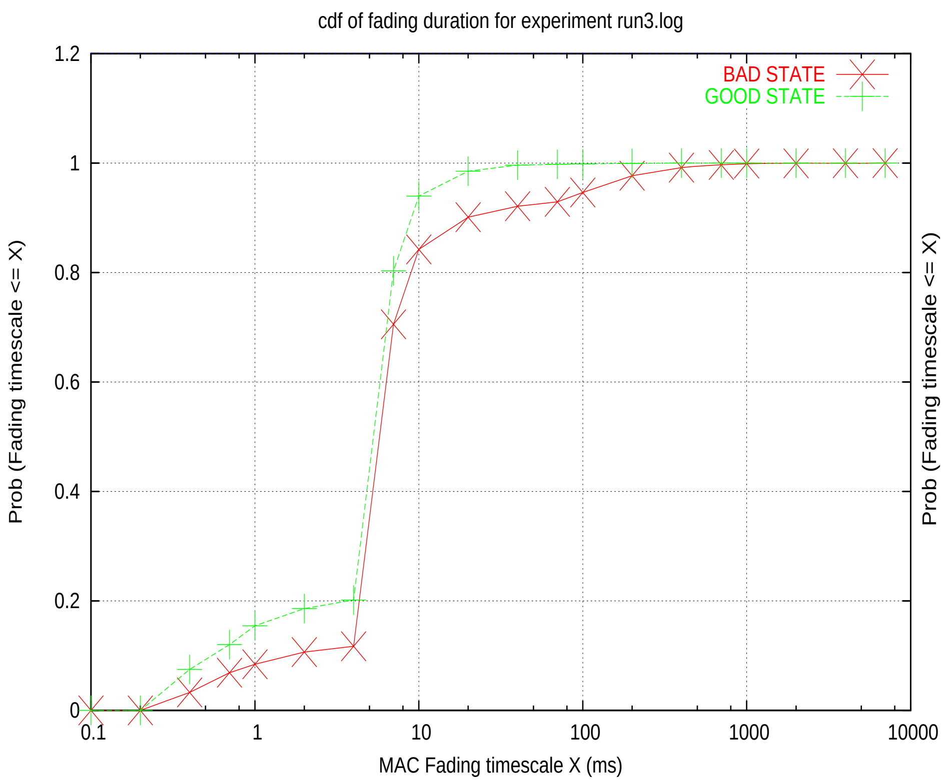
Unique Challenges in Wireless Communications

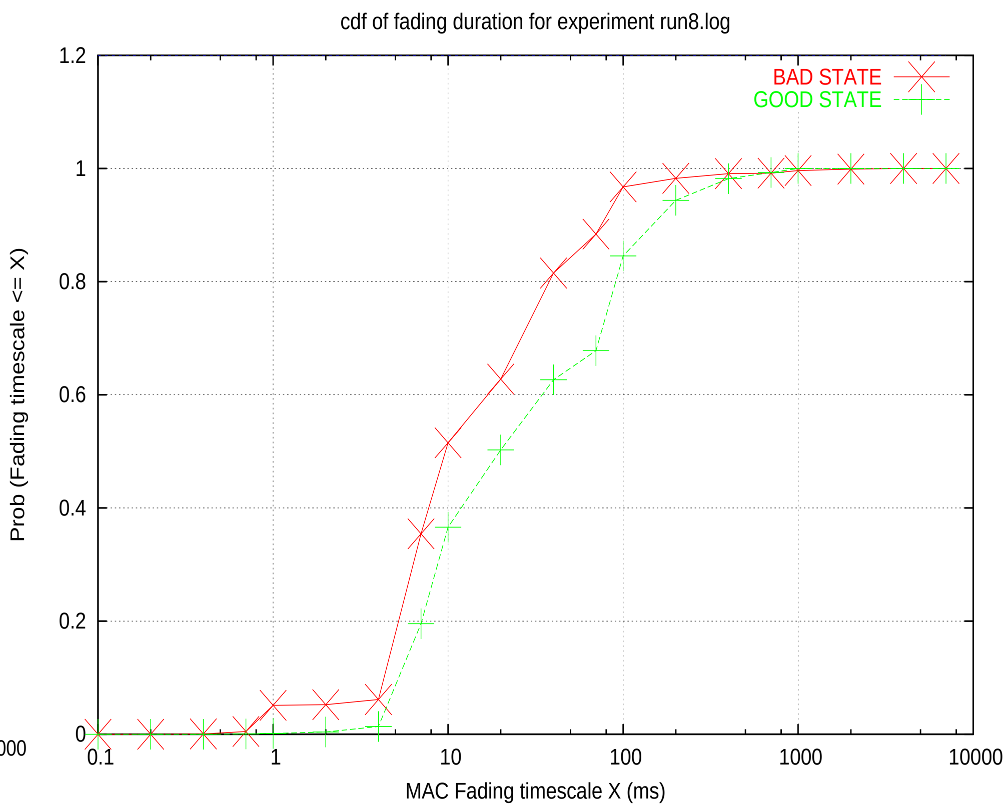
- Channel fading; co-channel interference





Time Scales of Channel Variation and MAC Interference Variation





- Measurement data [Aguayo-Bicket-Biswas-Judd-Morris 04] [Cao-Raghunagthan-Kumar 06]

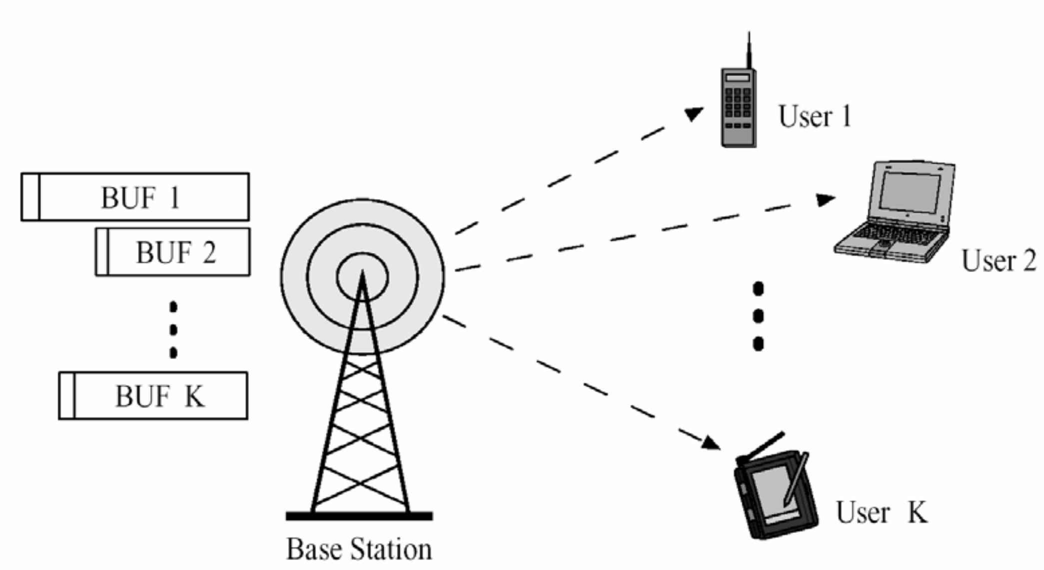
Unified PHY/MAC Design

- Traditional wisdom treats link losses due to fading separately from those incurred by interference;
  - MAC layer: scheduling used to resolve interference
  - PHY layer: coding/modulation, diversity schemes
- However, fading can often adversely affect MAC layer!
- Indeed, time scales of channel variation and MAC variation are of the same order.

➡ This calls for channel-aware scheduling!

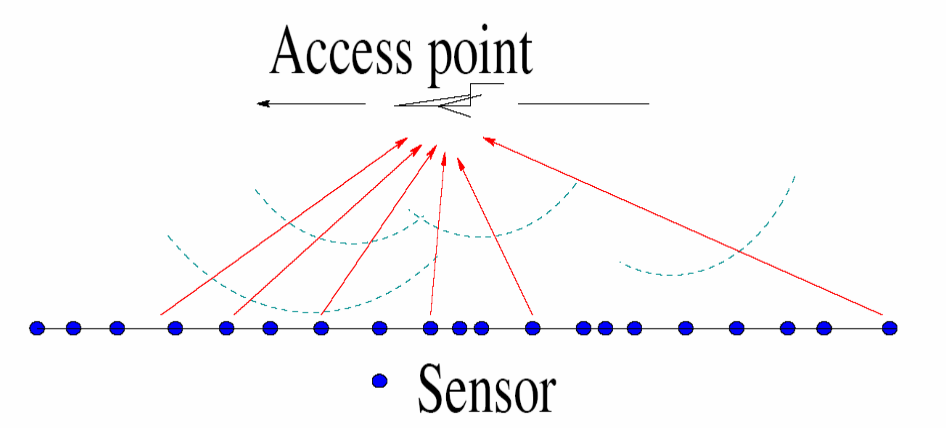
Centralized Opportunistic Scheduling for Downlink Transmission

- Assumption: BS has knowledge of instantaneous channel conditions of all users.
- BS opportunistically picks the user with “good” channel conditions at each slot; [Tse00], [Liu-Chong-Shroff01], [Borst01], [Viswanath-Tse-Laroia02], [Andrews et al 01], …



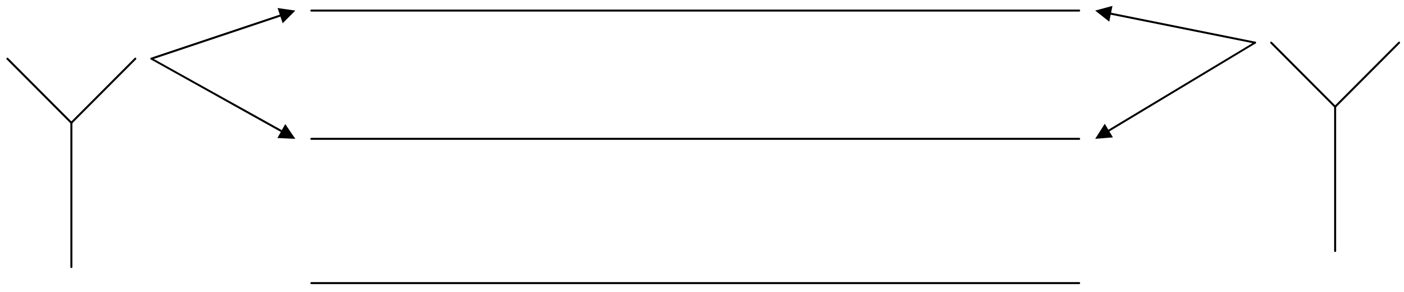
Channel-Aware Aloha

- Many-to-one network model: contention probability is a function of each link's channel condition [Adireddy-Tong05][Qin-Berry03].



Rate-adaptive MAC for Ad-Hoc Networks

- RBAR [Holland-Vaidya-Bahl01], OAR [Sadeghi-Kanodia-Sabharwal-Knightly 02] are perhaps among the first few that exploit channel codtion for rate-adaptive MAC.
  - Adapts the rate based on current channel condition
- MOAR [Kanodia-Sabharwal-Knightly04]: single-link with multi-channels.



Motivation for Channel Aware Distributed Scheduling

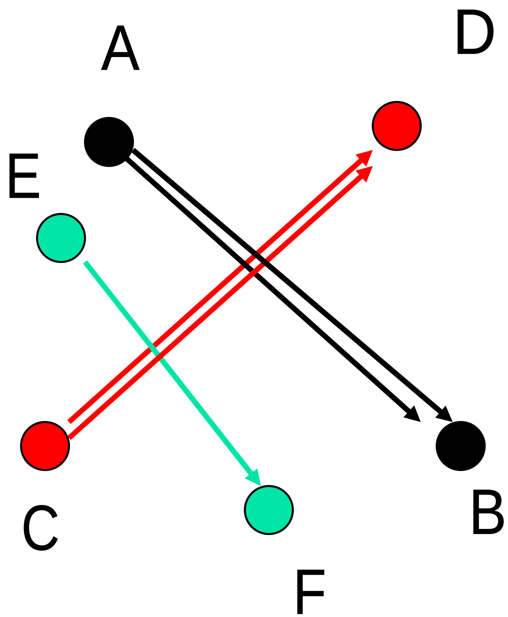
- Rich diversities in wireless communications:
  - Spatial diversity; time diversity; multi-user diversity …
- Open question: How to exploit rich diversities for ad-hoc communications ?
- Challenges in devising channel-aware scheduling for ad-hoc communications:
  - Links have no knowledge of others' channel conditions; even their own channel conditions are unknown before probing.
  - Due to co-channel interference, usually only one link can use the channel in a neighborhood.
  - Q) which link to schedule, and how?

Talk Outline

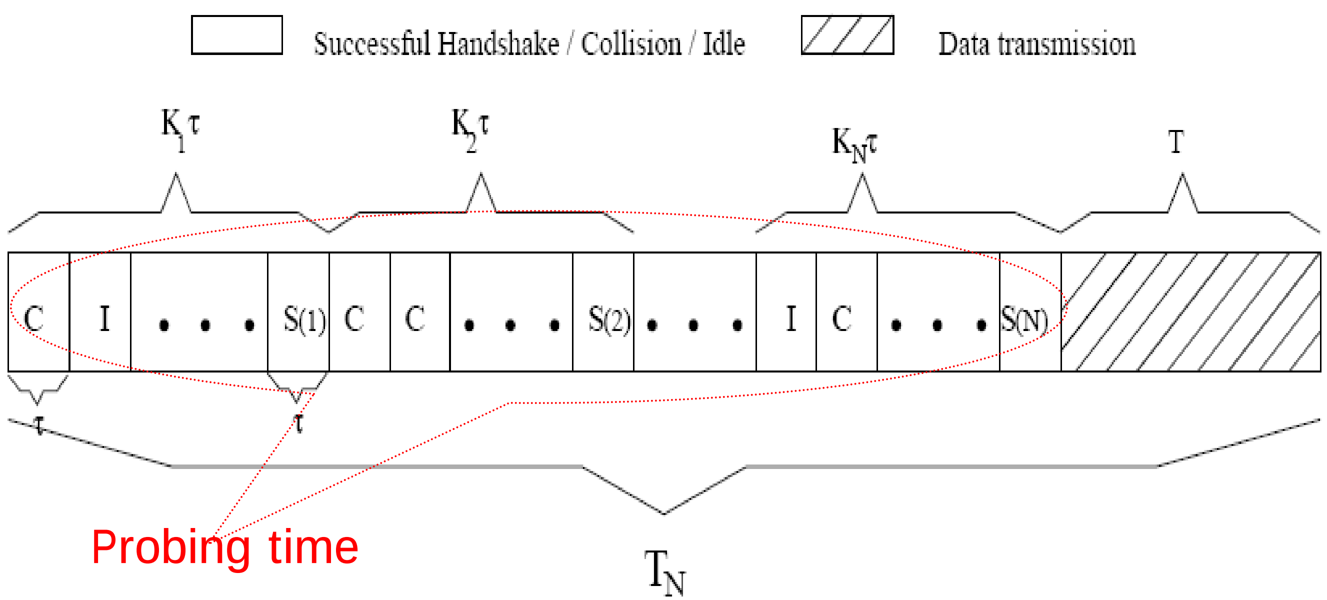
- System model
- Network centric view: a team game approach
  - Homogeneous case: optimal stopping rule for scheduling, optimal rate threshold;
  - Heterogeneous case;
  - Iterative algorithm for computing rate threshold
- User centric view: a non-cooperative game for threshold selection
  - Existence and uniqueness of Nash equalibrium
  - Convergence of best response strategy
  - Stochastic online algorithm for computing thresholds
  - Pricing mechanism to reduce price of anarchy

System Model

- Consider a single-hop network with M links; each link contend probabilistically in mini-slots; let s(n) denote the successful link in n-th round of channel probing.
- Suppose after one successful contention, channel condition is poor. Two options available:
  - Continue data transmission;
  - Or, alternatively, let this link give up this opportunity, and let all links re-contend.
- Intuition: At additional cost, further channel probing can lead to data transmission with better channel conditions.
- In this way, multiuser diversity and time diversity can be exploited in a distributed and opportunistic manner.



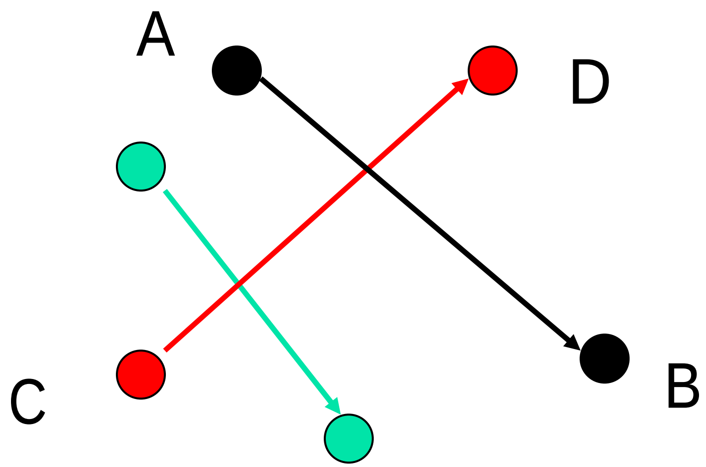
System Model (Cont'd)



- Clearly, there is a tradeoff between throughput gain from better channel conditions and the cost for further channel probing.
- Using optimal stopping theory, we characterize this tradeoff for distributed opportunistic scheduling.

Fundamental Tradeoff Between Channel Probing and Scheduling

- Channel probing is used to resolve interference and estimate channel condition;
- time is the resource 
- Tradeoff between channel probing and scheduling



Assumptions

A1) $\{\mathrm{R}_{n,s(n)}, n = 1, 2, \ldots\}$ are independent.

- Time scale of channel probing is comparable to channel coherence time.
- The probability that one link has two successive successful channel probing is fairly small (when number of links is large):

$$\mathrm{p}_m^2 \prod_{i \neq m} (1 - p_i)^2$$

Network Centric Case: A Team Game View

- Objective: to maximize average network throughput;
- Suppose this scheduling game is carried out L times. Then the average throughput is given by

$$\mathrm{x}_L = \frac{\sum_{l=1}^{L} R_{(N_l)} T}{\sum_{l=1}^{L} T_{N_l}},$$

where T_{N_l} is the l-th realization of the duration of channel probing and transmission time:

$$\mathrm{T}_{N_l} \triangleq \sum_{j=1}^{N_l} K_{(l,j)} \tau + T.$$

Number of contention mini-slots

Maximizing Rate of Return

$$x_L \longrightarrow \frac{E[R_{(N)}T]}{E[T_N]} \quad a.s.$$

The rate of return

- Problem: find optimal stopping policy for maximizing average network throughput:

$$N^* \triangleq \operatorname{argmax}_{N \in Q} \frac{E[R_{(N)}T]}{E[T_N]}, \quad x^* \triangleq \sup_{N \in Q} \frac{E[R_{(N)}T]}{E[T_N]},$$

where $Q \triangleq \{N : N \geq 1, E[T_N] < \infty\}.$

Homogeneous Link Case

Proposition: *a) The optimal stopping rule N^* exists, and is given by*

$$N^* = \min\{n \geq 1 : R_{(n)} \geq x^*\}.$$

A pure threshold strategy

b) The maximum throughput x^* is an optimal threshold, and is the unique solution to

$$E(R_{(n)} - x)^+ = \frac{x\tau}{p_s T},$$

Threshold can be pre-computed

Outline of Proof

- Step 1: establish existence of optimal stopping strategy N(x).

Define reward function $Z_n \triangleq R_{(n)}T - xT_n = R_{(n)}T - xT - x\sum_{j=1}^{n} K_j\tau$.
A key step is to show $E[\sup_n Z_n] < \infty$.

From Prophet

$$E[\sup_n Z_n] \le E\left[\sup_n \left\{R_{(n)}T - nx\tau\left(\frac{1}{p_s} - \epsilon\right)\right\}\right] + E\left[\sup_n \sum_{j=1}^{n} x\tau\left(\frac{1}{p_s} - \epsilon - K_j\right)\right] - Tx.$$

We use two Maximum Inequalities.

a) Let $\{X_n, n = 1, 2, \ldots\}$ be a sequence of i.i.d. random variables. If $E(X_1^+)^2 < \infty$, then $E[\sup_n (X_n - nc)] \le E(X_1^+)^2/(2c)$, where c is a positive constant.

b) Let $\{X_n, n = 1, 2, \ldots\}$ be a sequence of i.i.d. randomvariables. If $\mu = E[X_1] < 0$ and $\delta^2 = var[X_1] < \infty$. Let $S_n = \sum_{i=1}^{n} X_i$ with $S_0 = 0$ and $M = \sup_{n \ge 0} S_n$. Then $E[M] \le \delta^2/(2|\mu|)$.

Characterizing N* and X*

- Step 2: Define

$$V^*(x) = E[R_{(N(x))}T - xT_{N(x)}] = \sup_{N \in Q} E[R_{(N)}T - xT_N].$$

Suppose after one successful contention, data rate is $R_{(n)}$.

- If skip data transmission, the best we can get is $V^*(x) - xk\tau$.
- By principle of optimality, skip transmission and probe again if

$$R_{(n)}T - xk\tau - xT < V^*(x) - xk\tau;$$

Otherwise, proceed to transmit data.

Characterizing N* and X*

- For every x, there is optimal V*(x); x can be viewed as shadow price per time unit ("time is money").

- Therefore, the optimal stopping rule is

$$\mathrm{N(x)} = \min\{n \geq 1 : \mathrm{R}_{(n)}T - xk\tau - xT \geq V^*(x) - xk\tau\ \}.$$

and V*(x) satisfies the following optimality equation:

$$\mathrm{E}[\max(R_{(n)}T - xK\tau - xT, V^*(x) - xK\tau)] = V^*(x).$$

Characterizing N* and X* (Cont'd)

- V*(x*) = 0 is equivalent to

$$x^* \triangleq \sup_{N \in Q} \frac{E[R_{(N)}T]}{E[T_N]}$$

- V*(x) is decreasing and convex in x, and V*(x*)=0. x* can be found by solving the optimality equation.

Further Remarks

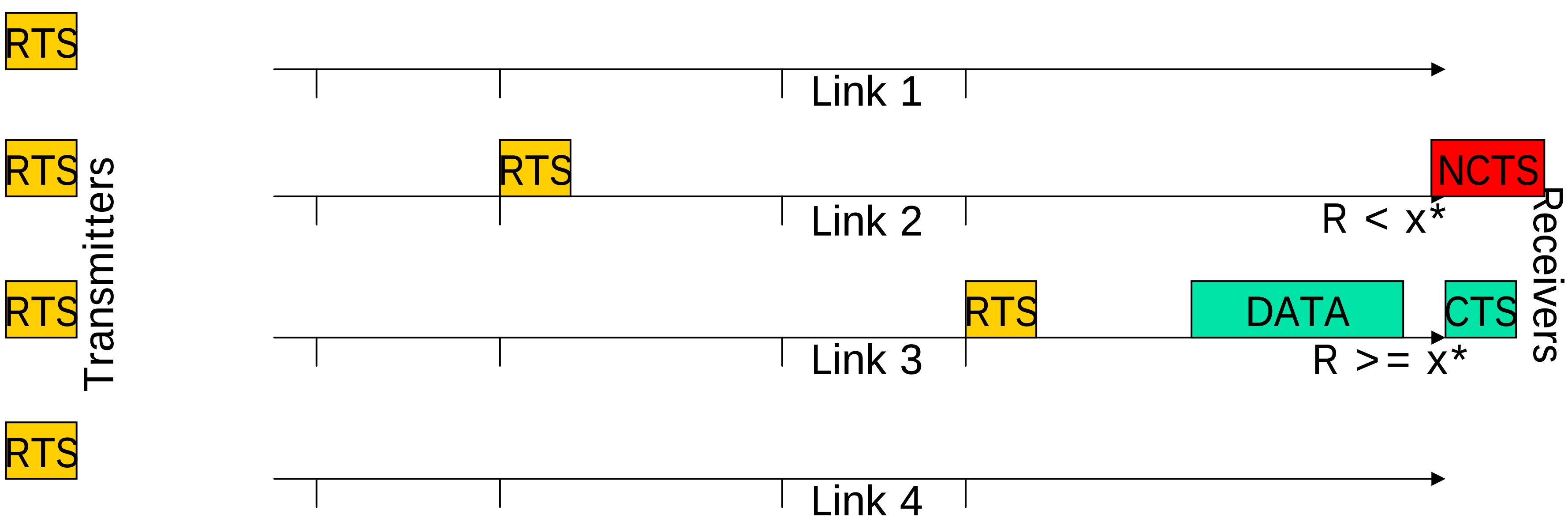
- The optimal stopping rule is a pure threshold policy
- The maximum throughput x* is unique
- The optimal threshold may not be unique
  - In continuous rate case, optimal threshold is unique;
  - In discrete channel case, $\Phi(x^*) = \Phi(\lceil x^* \rceil)$

where $\Phi(\cdot)$ is the average throughput, and $\lceil x^* \rceil$ is the nearest upper quantization level of x^*

Pseudo-Protocol Design

- The optimal distributed scheduling is simple to implement:

if the current rate is larger than the threshold, then transmit data; otherwise, continue probing.



Heterogeneous Link Case

- Different links have different channel statistics $\{\mathrm{F}_m(r)\}$;
- $\mathrm{R}_{n,s(n)}$ and $\mathrm{R}_{n+1,s(n+1)}$ may follow different distributions.
- Nevertheless, we can treat $\mathrm{R}_{n,s(n)}$ as a compound r.v.

$$\mathrm{P}(\mathrm{R}_{(n)} \leq r) = P(R_{n,s(n)} \leq r) = E\left[P(R_{n,m} \leq r) | s(n) = m\right]$$

Optimal DOS policy:

The maximum network throughput x^* in the heterogeneous case is optimal threshold and is the unique solution to the following fixed point equation

$$x = \frac{\sum_{m=1}^{M} p_{s,m} \int_x^{\infty} r dF_m(r)}{\delta + \sum_{m=1}^{M} p_{s,m} (1 - F_m(x))}.$$

Somewhat surprising, threshold is the same for all links!

Iterative Algorithm for Computing x*

$$\mathrm{x}_{k+1} = \Phi(x_k), \text{ for } k = 0, 1, 2, \ldots,$$

$$\Phi(x) \triangleq \frac{\sum_{m=1}^{M} p_{s,m} \int_x^\infty r dF_m(r)}{\delta + \sum_{m=1}^{M} p_{s,m} (1 - F_m(x))}.$$

Proposition

The iterates generated by the above algorithm converge to x^* *for any non-negative initial value* x_0.

Iterative Algorithm for Computing x*

- Unfortunately, the above iterative algorithm is not pseudo-contraction mapping.
- A counter-example:

Suppose for any m, $f_m(r)$ is given by

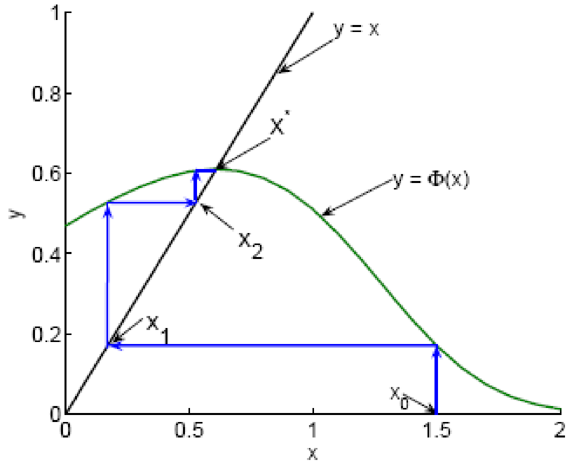
$$f_m(r) = \begin{cases} 0, & x < 0 \\ 0.01, & 0 \leq r < 96 \\ 0.005(r - 94), & 96 \leq r < 98 \\ 0.02(r - 97)^{-3}, & r \geq 98 \end{cases}$$

Let $p_s = 0.99/\text{M}$ and $\delta = 0.05$. The corresponding optimal point $x^* = 72.82$. However,

$$|\Phi(95.5) - x^*| = |45.88 - 72.82| > |95.5 - 72.82|,$$

which violates the condition for pseudo-contraction mapping.

Convergence of Iterative Algorithm



$$\Phi(x) \geq x, \qquad \forall\ x \leq x^*,$$
$$\Phi(x) \leq x, \qquad \forall\ x > x^*.$$

- Key steps:
  - x* is a global optimal point.
  - $\{x_k, k \geq 2\}$ is a monotonically increasing sequence converges to a limit x_∞
  - We show that $x_\infty = x^*$

Numerical Examples

- Define throughput gain over OAR as

$$g(\rho) \triangleq \frac{x^* - x^L}{x^L}.$$

- Consider continuous rate based on Shannon capacity, i.e.,

$$R(h) = \log(1 + \rho h)$$

- Set $\delta = 0.1, p_s = 1/e$

Homogeneous Case

- Optimal throughput for Homogeneous network model:

$$x^* = \frac{\exp\left(\frac{1}{\rho}\right) E_1\left(\frac{\exp(x^*)}{\rho}\right)}{\frac{\delta}{p_s}}.$$

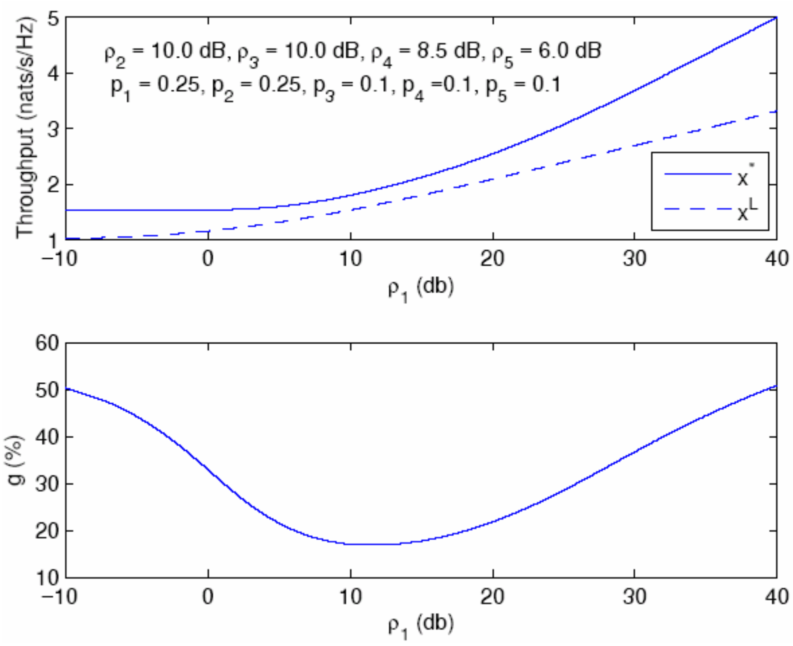
Table 1: $g(\rho)$ for homogeneous network

ρ	0.5	1	2	5	10
x^*	0.40	0.60	0.90	1.40	1.80
x^L	0.28	0.47	0.73	1.17	1.58
$g(\rho)$	42.8%	27.7%	23.3%	19.7%	13.9 %

Heterogeneous Case

- Heterogeneous network model:

$$x^* = \frac{\sum_{m=1}^{M} p_{s,m} \exp\left(\frac{1}{\rho_m}\right) E_1\left(\frac{\exp(x^*)}{\rho_m}\right)}{\delta}.$$



User-Centric Case: A Non-cooperative Game Perspective

- Goal: Each user chooses threshold to maximize its throughput selfishly
- We cast it as a non-cooperative game:
  - Players: [1, 2, ..., M}
  - action set: $\mathrm{A}_m = \{x_m | 0 \le x_m < \infty\}$
  - Payoff function: throughput ϕ_m

- Average throughput of user m:

$$\phi_m(\mathbf{x}) = \frac{\frac{\int_{x_m}^{\infty} r dF_m(r)}{1 - F_m(x_m)} T}{\frac{\tau + \sum_i p_{s,i}\left(1 - F_i(x_i)\right) T}{p_{s,m}(1 - F_m(x_m))} + T}.$$

Effect channel probing time

Nash Equilibrium

- Can show existence of Nash equilibria
- Some properties of Nash equilibria: component-wise monotonicity, admissible Nash equilibrium

Uniqueness of Nash equilibrium

- In general, the Nash equilibrium is not necessarily unique
  - Example: two links in a network, with the same rate distribution

$$R(r) = \begin{cases} 2\text{Mbps}, & \text{w.p.}0.5 \\ 12\text{Mbps}, & \text{w.p.}0.5 \end{cases}$$

Let $p_s = 0.2$ and $\delta = 0.35$. Then, there exist two Nash equilibria at

$$\mathbf{x} = (1.867, 1.867) \text{ and } \mathbf{x} = (2.18, 2.18)$$

- Uniqueness of Nash equilibrium for homogeneous links
  - Continuous rate over Rayleigh fading: unique Nash equilibrium.
  - General continuous rate case: the Nash equilibrium is unique if

$$xf(x) < \frac{\delta}{p_s(M-1)}.$$

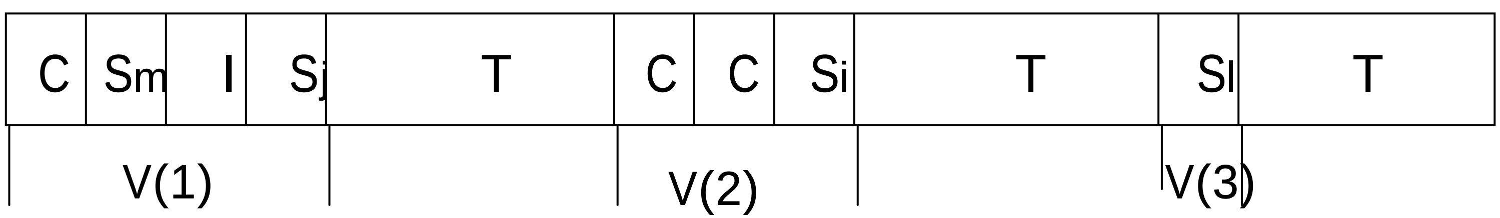
Best Response Strategy and Online Computation Algorithm

- Best Response Strategy: every user updates using best response strategy
  - Use iterative alg. at smaller time scale to find best response.
  - Can show convergence for any initial condition if Nash equilibrium is unique . (not concave, not supermodular game; tricky)
  - Requires global information
- Online Computation Algorithm
  - Each link updates its threshold asynchronously
  - Based on local observations
  - Converge to Nash equilibrium

Stochastic Online Computation Algorithm

- Duration between updates V(k) is a Geometric RV:

$$E[V(k)|\mathcal{F}(k)] = 1/\sum_{i=1}^{M} p_{s,i}(1 - F_i(x_i(k)))$$



Online Computation Algorithm

- Define

$$\widetilde{g_m}(k) \triangleq v(k)\left[p_{s,m}\int_{x_m(k)}^{\infty} r dF_m(r) - \delta x_m(k)\right] - x_m(k).$$

- Updating algorithm

$$x_m(k+1) = [x_m(k) + a_m(k)\,[\widetilde{g_m}((k))]\,I\{k \in N^m\}]_0^b.$$

- Basic features:
  - Asynchronism
  - Stochastic perturbation
  - Projection

Technical Conditions

- Condition 1: Unique Nash equilibrium
- Condition 2: $a_i(k) = a(i, \sum_{j=1}^{k} I\{j \in N^i\}),$

$$\sum_{k=1}^{\infty} a(i,k) = \infty, \quad \text{and} \quad \sum_{k=1}^{\infty} a(i,k)^2 < \infty,$$

and for 0<t<1,

$$\lim_{k\to\infty} \frac{\sum_{j=1}^{\lfloor tk \rfloor} a(i,j)}{\sum_{j=1}^{k} a(i,j)} = 1, \qquad \lim_{k\to\infty} \frac{\sum_{j=1}^{k} a(i,j)}{\sum_{j=1}^{k} a(l,j)} > 0.$$

Convergence of Online Algorithm

- Convergence: Under conditions 1 & 2, the threshold iterates of the asynchronous stochastic algorithm converge to the optimal threshold x* almost surely.

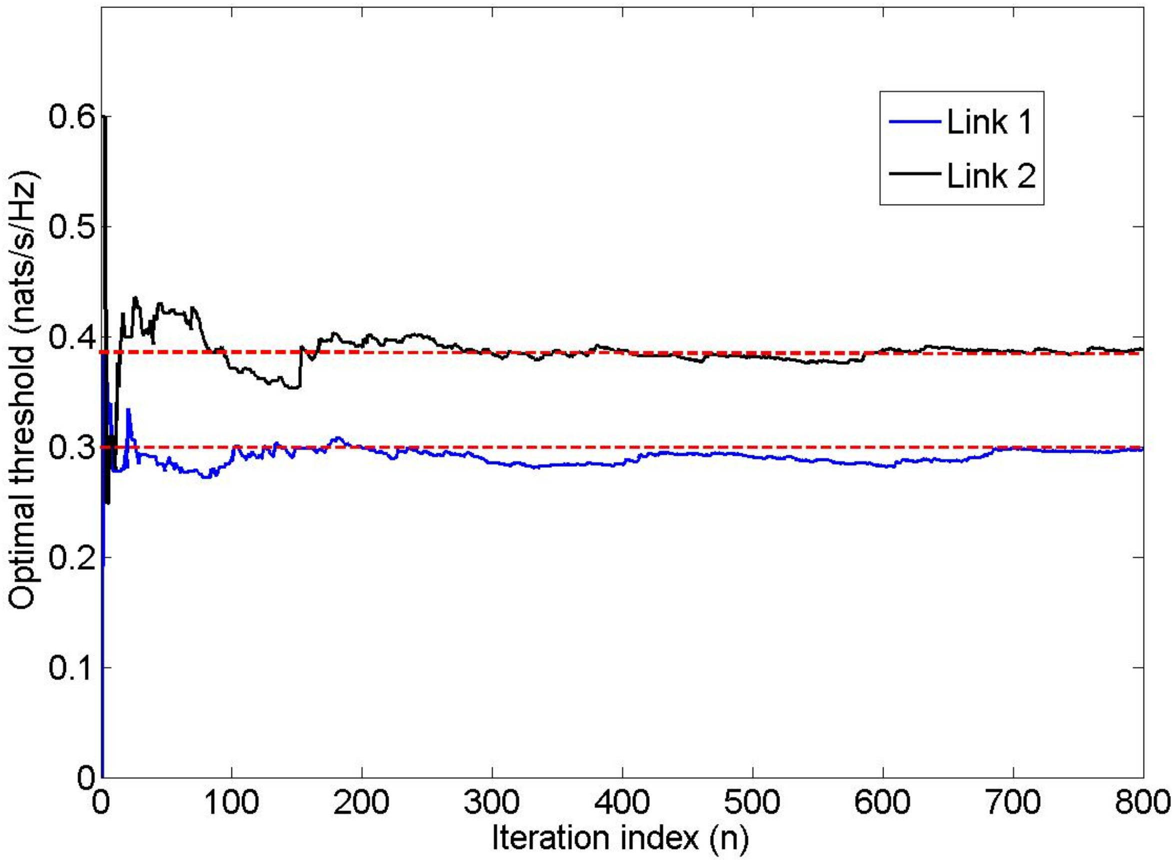
- Sketch of proof:
  - Step 1: Decompose the update into synchronous term, asynchronous term, projection term, and stochastic perturbation term; and show that the effect of last three terms diminishes when k goes to infinity.
  - Step 2: Define linear interpolated function, and show convergence by using Arzela-Ascoli Theorem.
  - Step 3: Show the original sequence of iterates converges.

Numerical Example

Convergence behavior of the best response strategy

Link index	x_0	x_1	x_2	x_3	x_4	x^*
Link 1 ($\rho_1 = 3dB$)	1.00	0.3601	0.2928	0.2999	0.3000	0.30
Link 2 ($\rho_2 = 5dB$)	1.00	0.1081	0.3863	0.3879	0.3882	0.39

Simulation Results



Convergence of the proposed online algorithm

Pricing Mechanism

- Efficiency loss due to non-cooperation.
- By introducing pricing function, players are "encouraged" to obey a social behavior so as to reduce the price of anarchy
- Pricing function

$$c_m(\mathbf{x}) = \alpha \frac{p_{s,m}(1-F_m(x_m))}{\delta+\sum_{i=1}^{M} p_{s,i}(1-F_i(x_i))}.$$

Usage-based pricing

- New non-cooperative game

$$(\mathbf{G}') \quad \max_{x_m} \phi_m(\mathbf{x}) - c_m(\mathbf{x}), \; m = 1, 2, \ldots, M.$$

Numerical Examples

Number of links	1	2	3	4	5
x^*_{co} (nats/s/Hz)	0.586	0.664	1.085	1.217	1.364
x^*_{nco} (nats/s/Hz)	0.586	0.624	0.994	1.043	1.127
η	100%	94.0%	91.6%	85.7%	82.6%
$x^*_{pricing}$ (nats/s/Hz)	0.586	0.650	1.055	1.170	1.293
η'	100%	97.9%	97.2%	96.1%	94.8%

$$\eta \triangleq x^*_{nco}/x^*_{co} \qquad \eta' = x^*_{pricing}/x^*_{co}$$

Conclusions

- We explore distributed opportunistic scheduling for ad hoc communications to exploit rich diversities in wireless communications.
- Particularly, we study distributed opportunistic scheduling from two views: team game and non-cooperative game.
- Optimal stopping theory is used to characterize optimal scheduling strategies.
- Ongoing work: study optimal policy under noisy channel estimation; exploit multi-receiver diversity; make use of queuing information; multi-hop setting ...

Research Thrust I:
Cross-layer Optimization and Design

1. Self-similarity of multi-access interference for wireless resource allocation
2. MIMO ad-hoc Networks
3. Cooperative sensor networks
4. Stochastic network utility maximization
5. Channel aware distributed scheduling for ad-hoc communications
6. Complex network view of ad-hoc/sensor networks

Research Thrust II: Cooperative Relaying and Sensing

1. Capacity bounds of MIMO relay channel
2. Power allocation in wireless relay networks
3. Scaling laws of wideband sensory relay networks
4. Joint communication and sensing

Channel Aware Distributed Scheduling

Thank you !