Traffic Aided Opportunistic Scheduling for Wireless Networks: Algorithms and Performance Bounds

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Abstract

In multiuser wireless networks, opportunistic scheduling can improve the system throughput and thus reduce the total completion time. In this paper, we explore the possibility of reducing the completion time further by incorporating traffic information into opportunistic scheduling. More specifically, we first establish convexity properties for opportunistic scheduling with file size information. Then, we develop new traffic aided opportunistic scheduling (TAOS) schemes by making use of file size information and channel variation in a unified manner. We also derive lower bounds and upper bounds on the total completion time, which serve as benchmarks for examining the performance of the TAOS schemes. Our results show that the proposed TAOS schemes can yield significant reduction in the total completion time. The impact of fading, file size distributions, and random arrivals and departures, on the system performance, is also investigated. In particular, in the presence of user dynamics, the proposed TAOS schemes perform well when the arrival rate is reasonably high.

Key words: Completion time, Cross-layer, Opportunistic Scheduling, Wireless networks.

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1 Introduction

Recent years have witnessed a tremendous growth in the demand for ubiquitous information access. Intense demands in wireless networks can induce possibly large delay, and degrade the system performance. Therefore, it is of great interest to investigate the problem of minimizing the total (or average) *completion time* which consists of both processing time and waiting time (delay) [8].

In wireline networks, the problem of reducing the completion time has been studied extensively. A general review on wireline scheduling algorithms can be found in [12]. In [16], a "shortest remaining processing time first (SRPT)" scheduling scheme is presented. In recent studies [6], [8] (see also the references therein), the authors develop traffic size based scheduling schemes for connection control at Web servers, and show that these schemes can yield significant reduction in the average completion time. Simply put, by properly exploiting the file size information in scheduling (e.g., picking the user with the shortest file), the overall system performance can be improved.

In wireless networks, opportunistic scheduling (see, e.g., [1], [2], [3], [4], [13], [14], [15], [17], [18], [20], [22]) offers a promising solution to reducing the completion time. Opportunistic scheduling originates from a holistic view. Roughly speaking, in a multiuser wireless network, at each moment it is likely there exists a user with "good" channel conditions. By picking the instantaneous "on-peak" user for data transmission, opportunistic scheduling can utilize the wireless resource efficiently and thus improve the overall system throughput dramatically. Hence, the total completion time can be shortened, due to the enhancement of the system throughput.

We note that "picking the user with the shortest file" and "picking the user on the channel peak" may not coincide. As a result, these two approaches may lead to conflicting scheduling. Then, a natural question to ask is "what is the optimal scheme in the sense of minimizing the total completion time?" It is clear that the completion time is correlated across the users, i.e., one user's processing time is related to the delay of the other users. The tight coupling among the transmissions of different users, together with the channel variation in wireless networks, makes the task more challenging.

In this paper, we take a cross-layer design approach to reducing the completion time by exploiting both the file size information and channel state information in a unified manner. Our contributions can be summarized as follows. We first establish general convexity properties for opportunistic scheduling with file size information, which provides a basis for devising scheduling schemes (see also [10]). Then, building on the insights gained from two existing scheduling schemes, namely the wireless shortest remaining processing time first (W-SRPT) scheme and "riding on the channel peak" scheme, we develop new traffic aided opportunistic scheduling (TAOS) schemes, including TAOS-1, TAOS-1a, TAOS-1b, and TAOS-2. Roughly speaking, the TAOS-1 scheme can be viewed as a generalization of the well-known "proportional fair scheduling" by taking into account the file size information; and the TAOS-2 scheme is a locally optimal scheme (we will elaborate on this further in Section 3.2). The third key contribution consists of a lower bound and an upper bound on the total completion time, which serve as benchmarks for examining the performance of the TAOS schemes. The results show that the TAOS schemes can reduce the completion time of the entire system significantly. We then extend the study to the cases with random arrivals and departures. As expected, the proposed TAOS schemes yield significant reduction of the total completion time when the arrival rate is high.

In related work, a framework for scheduling in wireless networks is presented in [14]. The authors of [11] study scheduling in power-controlled CDMA data networks, assuming that the channel remains static during the entire transmission of each user. An interesting work by Tsybakov studies file transmission over wireless fading channels in [21]. More specifically, a recursive method using dynamic programming is used to compute the expected completion time, based on which optimal and suboptimal scheduling algorithms are devised accordingly. One key difference between [21] and our study here lies in the fact that [21] considers the average performance and focuses on the expected completion time, whereas we are interested in an (arbitrary) "sample" case and study the completion time for a given set of files. Needless to say, different performance metrics used would lead to different scheduling strategies. A recent work [18] proposes a Foreground-Background (FB) scheduling algorithm for the heavy-tailed data traffic, and develops a new hierarchical approach using both proportional fair opportunistic scheduling ([20], [22]) and FB scheduling. The authors of [2] have investigated the scheduling for possible simultaneous

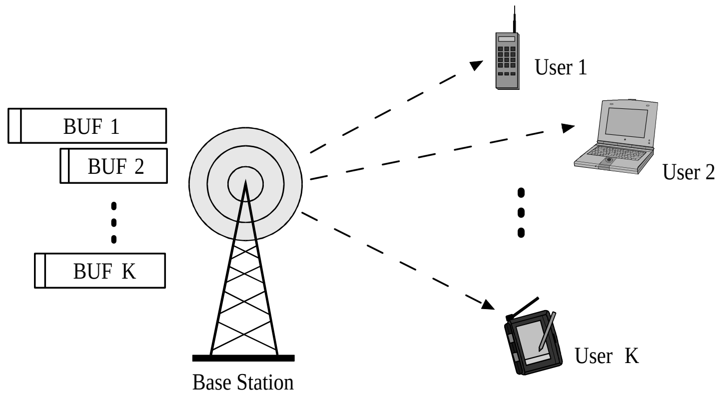

Fig. 1. Downlink transmissions in a cellular system

transmissions, taking into account both channel conditions and delay constraints.

The rest of this paper is organized as follows. In the next section, we introduce system models and give a brief review of two existing scheduling schemes. In Section 3, we present general properties and traffic aided opportunistic scheduling schemes. In Section 4, we derive performance bounds. In Section 5, we illustrate the performance gains of these scheduling schemes with numerical examples. The conclusions are given in Section 6.

2 Background

We consider the downlink and assume that the transmissions of data users within a cell are time division multiplexed (TDM). Simply put, at each time slot, all users utilize pilot signaling to estimate the channel condition and feed the information back to the base station. Based on the channel conditions and file size information, an opportunistic scheduler at the base station arranges the transmissions, aiming to minimize the completion time of the whole system.

For simplicity, we consider the transmission of one backlogged file for every user (cf. [23]). (In the case when there are multiple backlogged files for one user, we assume that this user finishes one file before transmitting the next. We will address the case with random arrivals and departures in Section 5.3.) In this context, we use “file” and “user” interchangeably. Without loss of generality, we call the user with the shortest initial file size, user 1, and so on, i.e., $F_k = F_{(k)}$, where F_k denotes the initial file size of user k, and $F_{(k)}$ denotes the kth order statistic of the initial file sizes. A simple diagram for this system is given in Fig. 1.

2.1 Channel Model

In a wireless channel, the received signal can be expressed as

$$y = gs + w, \tag{1}$$

where s denotes the transmitted signal, g denotes the channel gain (which can be time-varying), and w denotes the white Gaussian noise. We assume that the time-varying fading is due to distance-related attenuation and small-scale fading. Specifically, we consider two kinds of fading channels: Rayleigh fading and Rician fading [19]. Let $R(t)$ denote the instantaneous data rate supported by the fading channel at time slot t. For simplicity, we assume ideal coding, and thereby achievable channel capacity.

2.2 Completion Time

As is standard [8], we define the completion time of file k, denoted ψ_k, as the duration from the moment the file arrives at the system to the moment the file departs the system completely. Accordingly, the total (system) completion time is defined as

$$\Psi = \sum_{k=1}^{K} \psi_k, \tag{2}$$

where K is the number of files. It is worth pointing out that in many practical wireless systems, the transmission time of each file is on the order of seconds or even minutes, which is much greater than the slot duration and the coherent time of fading; and this is an assumption we impose throughout.

2.3 Wireless SRPT and "Riding on the Channel Peak"

In this paper, we aim to reduce the total completion time by making use of file size information and channel state information in a unified manner. In what follows, we first recapitulate two scheduling schemes, i.e., the wireless SRPT (see also [16]) scheme and "riding on the channel peak" scheme, which can be viewed as two extreme cases utilizing the traffic information only or channel conditions only.

2.3.1 Wireless SRPT

Suppose that the scheduler at the base station utilizes only the knowledge of the average data rates and file size information of mobile users. The wireless SRPT scheduler picks the user with the shortest (*expected*) remaining processing time (see also [12], [16]), i.e.,

$$k^* = \arg\min_k \left(\frac{X_k(t)}{E[R_k]} \right), \tag{3}$$

where $X_k(t)$ denotes the residual backlogged file size of user k at time t, and $E[R_k]$ denotes the average data rate of user k.

2.3.2 "Riding on the Channel Peak"

If the scheduler utilizes only the knowledge of the instantaneous channel conditions, "riding on the channel peak" is then a plausible approach to improving the total system throughput, thereby reducing the completion time. Specifically, at each time slot, the "riding on the peak" scheduler (see, e.g., [4], [9], [15], [20]) arranges the transmission for user k^* with

$$k^* = \arg\max_k \left(\frac{R_k(t)}{E[R_k]} \right), \tag{4}$$

where $R_k(t)$ denotes the instantaneous data rate of user k.

3 Traffic Aided Opportunistic Scheduling

Given a set of users, our goal is to minimize the total completion time. As noted in the Introduction, compared with the simple Round-Robin algorithm, both "picking the user with the shortest file" and "picking the user on the channel peak" can lead to reduced completion time. Then, it is natural to expect that we can improve the performance further by exploiting both the file size information and channel variation. Thus motivated, we devise traffic aided opportunistic scheduling schemes.

3.1 Convexity Properties of Scheduling Schemes

We start with characterizing general properties for opportunistic scheduling with file size information. Let $X_k(t)$ and $R_k(t)$ denote the backlog size and instantaneous data rate of user k at time slot t, respectively. ($X_k(0) = F_k$ is the initial file size of user k.) Suppose that allocating partial time resource within one slot is applicable. Let $\gamma_k(t)$ denote the portion of time resource assigned to user k at time slot t, and define

$$\gamma(t) \triangleq (\gamma_1(t), \cdots, \gamma_K(t))^T, \tag{5}$$

where $(\cdot)^T$ denote the vector transpose. We assume that $R_k(t)$ and $\gamma(t)$ do not change within each slot. By neglecting the edge effect, we define the completion time of user k corresponding to a scheduling function $\gamma(t)$ as

$$\psi_k(\gamma) \triangleq \inf \Big\{ t : \sum_{n=1}^{t} R_k(n)\gamma_k(n)T_s \geq X_k(0) \Big\}, \tag{6}$$

where T_s denotes the slot duration, and t is the time in terms of slots. Then, a scheduling function $\gamma(t)$ is said to be admissible if for $k = 1, \cdots, K$,

$$0 \leq \gamma_k(t) \leq 1, \tag{7}$$

$$\sum_{k=1}^{K} \gamma_k(t) \leq 1, \tag{8}$$

$$\gamma_k(t) = 0 \text{ for all } t \geq \psi_k(\gamma). \tag{9}$$

Note that

$$\sum_{n=1}^{\psi_k(\gamma)} R_k(n)\gamma_k(n)T_s = X_k(0). \tag{10}$$

Let Γ denote the set of all admissible scheduling functions. We have the following propositions on the general properties of opportunistic scheduling with file size information.

Proposition 1 *Γ is a convex set. Moreover, for any $\gamma, \gamma' \in \Gamma$ and $\theta \in (0, 1)$,*

$$\psi_k\Big(\theta\gamma + (1-\theta)\gamma'\Big) = \max\Big(\psi_k(\gamma), \psi_k(\gamma')\Big), k = 1, \cdots, K. \tag{11}$$

Proof: It is straightforward to see that Γ is a convex set. Without loss of generality, suppose $\psi_k(\gamma) \leq \psi_k(\gamma')$. Then, for $0 < \theta < 1$,

$$\begin{aligned}
&\sum_{t=1}^{\psi_k(\gamma')} R_k(t)\Big(\theta\gamma_k(t) + (1-\theta)\gamma'_k(t)\Big)T_s \\
&= \theta \sum_{t=1}^{\psi_k(\gamma')} R_k(t)\gamma_k(t)T_s + (1-\theta) \sum_{t=1}^{\psi_k(\gamma')} R_k(t)\gamma'_k(t)T_s \\
&\overset{(a)}{=} \theta X_k(0) + (1-\theta)X_k(0) \\
&= X_k(0),
\end{aligned} \tag{12}$$

where (a) follows from that fact that $\psi_k(\gamma) \leq \psi_k(\gamma')$ and $\gamma_k(t) = 0$ for $t > \psi_k(\gamma)$, which indicates that

$$\sum_{t=1}^{\psi_k(\gamma')} R_k(t)\gamma_k(t)T_s = \sum_{t=1}^{\psi_k(\gamma)} R_k(t)\gamma_k(t)T_s = X_k(0). \tag{13}$$

For convenience, we define

$$\psi_k(\theta) \triangleq \psi_k\Big(\theta\gamma(t) + (1-\theta)\gamma'(t)\Big).$$

Moreover, if $\psi_k(\theta) < \psi_k(\gamma')$,

$$\begin{aligned}
&\sum_{t=1}^{\psi_k(\theta)} R_k(t)\Big(\theta\gamma_k(t) + (1-\theta)\gamma'_k(t)\Big)T_s \\
&= \theta \sum_{t=1}^{\psi_k(\theta)} R_k(t)\gamma_k(t)T_s + (1-\theta) \sum_{t=1}^{\psi_k(\theta)} R_k(t)\gamma'_k(t)T_s \\
&= \theta \sum_{t=1}^{\psi_k(\gamma')} R_k(t)\gamma_k(t)T_s - \theta \sum_{t=\psi_k(\theta)+1}^{\psi_k(\gamma')} R_k(t)\gamma_k(t)T_s \\
&\quad + (1-\theta) \sum_{t=1}^{\psi_k(\gamma')} R_k(t)\gamma'_k(t)T_s - (1-\theta) \sum_{t=\psi_k(\theta)+1}^{\psi_k(\gamma')} R_k(t)\gamma'_k(t)T_s.
\end{aligned} \tag{14}$$

Since

$$\sum_{t=\psi_k(\theta)+1}^{\psi_k(\gamma')} R_k(t)\gamma_k(t)T_s \geq 0,$$

and

$$\sum_{t=\psi_k(\theta)+1}^{\psi_k(\gamma')} R_k(t)\gamma'_k(t)T_s > 0,$$

it follows that

$$\begin{aligned}
&\sum_{t=1}^{\psi_k(\theta)} R_k(t)\Big(\theta\gamma_k(t)+(1-\theta)\gamma'_k(t)\Big)T_s \\
&< \theta \sum_{t=1}^{\psi_k(\gamma')} R_k(t)\gamma_k(t)T_s + (1-\theta)\sum_{t=1}^{\psi_k(\gamma')} R_k(t)\gamma'_k(t)T_s \\
&= X_k(0),
\end{aligned} \tag{15}$$

which contradicts (10). Therefore, for $k = 1, \cdots, K$,

$$\psi_k\Big(\theta\gamma + (1-\theta)\gamma'\Big) = \max\Big(\psi_k(\gamma), \psi_k(\gamma')\Big). \tag{16}$$

■

Proposition 2 *Define* $\Psi(\gamma) \triangleq \sum_{k=1}^{K} \psi_k(\gamma)$*. Then,* $\Psi(\gamma)$ *is a concave function on* Γ*.*

Proof: Since any $\theta \in (0, 1)$,

$$\begin{aligned}
\Psi(\theta\gamma + (1-\theta)\gamma') &= \sum_{k=1}^{K} \psi_k(\theta\gamma + (1-\theta)\gamma') \\
&= \sum_{k=1}^{K} \max\Big\{\psi_k(\gamma), \psi_k(\gamma')\Big\} \\
&\geq \sum_{k=1}^{K} \Big\{\theta\psi_k(\gamma) + (1-\theta)\psi_k(\gamma')\Big\} \\
&= \theta\Psi(\gamma) + (1-\theta)\Psi(\gamma'),
\end{aligned} \tag{17}$$

we conclude that $\Psi(\gamma)$ is a concave function on Γ. ■

Propositions 1 and 2 reveal that the optimal solution γ^* is an extreme point of Γ, i.e., $\gamma_k^*(t) = 0$ or 1, for $k = 1, \cdots, K$. Roughly speaking, assigning the entire time slot to one user while keeping others silent can lead to shorter processing time.

We note that the globally optimal scheduling scheme is highly non-trivial. In what follows, we present several suboptimal opportunistic scheduling schemes, and derive lower bounds and upper bounds on the total completion time. Specifically, building on the insights gained from the two extreme cases studied in Section 2.3, we develop traffic aided opportunistic scheduling schemes, namely TAOS-1, TAOS-1a, TAOS-1b, and TAOS-2, which inherit the merits of both algorithms above.

3.2 Traffic Aided Opportunistic Scheduling

3.2.1 TAOS-1

Clearly, giving high priority to users with either small file sizes or "good" channel conditions would yield reduction in the total completion time. Thus, it is plausible to devise a cost function (or priority function), which increases with the file size and decreases with the instantaneous data rate, and develop cost function based scheduling accordingly. To provide fairness (see [20], [22]), we can also take into account the average throughput in designing the function. Along this line, we construct the cost function as $\frac{F_k U_k(t)}{R_k(t)}$, where F_k denotes the initial backlogged file size of user k, and U_k is the average throughput of user k. The corresponding scheduling scheme, TAOS-1, picks user k^* with

$$k^* = \arg\min_k \left(\frac{F_k U_k(t)}{R_k(t)} \right). \tag{18}$$

The average throughput U_k can be calculated as (see [20], [22])

$$U_k(t+1) = \begin{cases} (1 - \frac{1}{T^c})U_k(t) + \frac{1}{T^c} R_k(t) & k = k^* \\ (1 - \frac{1}{T^c})U_k(t) & k \neq k^*, \end{cases} \tag{19}$$

where T^c is the sliding observation window in terms of the number of slots.

Building on the TAOS-1 scheme above, we have also derived two other sub-optimal schemes: TAOS-1a and TAOS-1b. TAOS-1a does not consider the fairness, and the corresponding cost function is simply $\frac{F_k}{R_k(t)}$. That is, the TAOS-1a scheme schedules the transmission for user k^* with

$$k^* = \arg\min_k \left(\frac{F_k}{R_k(t)} \right). \tag{20}$$

TAOS-1b takes into account the dynamics of the file size by replacing F_k in (20) with $X_k(t)$. Then, we obtain a scheduling scheme totally relying on the instantaneous information at time t. More specifically, the TAOS-1b scheme picks user k^* with

$$k^* = \arg\min_k \left(\frac{X_k(t)}{R_k(t)} \right). \tag{21}$$

3.2.2 TAOS-2

In the above heuristic schemes, the cost functions cannot characterize the completion time. In what follows, we develop an opportunistic scheduling scheme, via devising a cost function directly related to the completion time. Observe that wireless SRPT scheduling utilizes only the average data rate, not the instantaneous channel state information. Then, if the instantaneous channel information is incorporated into wireless SRPT, the scheduling scheme can exploit the dynamics of file size and channel in a more integrated manner, thereby achieving more reduction in the total completion time. Thus motivated, we devise a new TAOS scheme which evolves in two phases. Specifically, the TAOS-2 algorithm can be outlined as follows:

Phase I:

i) Sort all users in the ascending order of $\frac{X_k(t)}{E[R_k]}$, and let $I_k(t)$ denote the rank of user k among the ordered variates.

Phase II:

ii) Compute the cost function for user k as

$$D_k(t) = \Big(I_k(t) - 1\Big) - \Big(M(t) - I_k(t) + 1\Big)\left(\frac{R_k(t)}{E[R_k]} - 1\right), \tag{22}$$

where $M(t)$ denotes the number of remaining users in the system.

iii) Schedule the transmission for user k^* which has the smallest cost function, i.e.,

$$k^* = \arg\min_k \Big(D_k(t)\Big). \tag{23}$$

Worth noting is that this constructive opportunistic scheduling scheme is locally optimal, and we also call it locally optimal TAOS (LO-TAOS). In the following, we elaborate further on this.

Consider the scheduling at the nth slot. Observe that when only the average data rate $\{E[R_i],\ i = 1, \cdots, K\}$ and remaining file size $\{X_i(n),\ i = 1, \cdots, K\}$ are utilized, wireless SRPT would be the best scheduling scheme (see also [12]). We adopt the wireless SRPT criterion in Phase I of the TAOS-2 scheme, and let $Z_0(n)$ denote the corresponding expected system time for the remaining files. In Phase II, building on the rank $\{I_i(n), i = 1, \cdots, K\}$ obtained in Phase I, the scheduler optimizes the scheduling by exploiting the knowledge of the instantaneous data rate $\{R_i(n),\ i = 1, \cdots, K\}$. (Note that $\{R_i(t),\ t \geq n+1\}$ is not available at the nth slot.) If the scheduler picks user k for the transmission, the corresponding expected remaining time in term of slots can be expressed as

$$Z_k(n) = Z_0(n) + D_k(n) + \epsilon_k(n), \tag{24}$$

where $D_k(n)$ is given in (22) and $\epsilon_k(n)$ denotes the correction term if the realization of $\{I_i(n+1), i = 1, \cdots, K\}$ is different from $\{I_i(n), i = 1, \cdots, K\}$ derived in Phase I. Specifically, $\epsilon_k(n)$ can be expressed as

$$\epsilon_k(n) = Y_k(n+1) - Y_0(n+1), \tag{25}$$

where $Y_k(n+1)$ denotes the expected remaining time if the system transmits the remaining files ($t \geq n+1$) according to the order given by $\{I_i(n+1), i = 1, \ldots, K\}$, and $Y_0(n+1)$ denotes the expected remaining time if the system transmits the remaining files ($t \geq n+1$) according to the order of $\{I_i(n), i = 1, \ldots, K\}$. We note that the impact of $\epsilon_k(n)$ is negligible, because the scheduler updates the rank $\{I_i(n), i = 1, \ldots, K\}$ in each slot and the changing rate of the ranks is much slower. Therefore, (24) can be approximated as

$$Z_k(n) \cong Z_0(n) + D_k(n). \tag{26}$$

Note that compared with $Z_0(n)$ in Phase I, the first term of $D_k(n)$ in (22), $I_k(t) - 1$, denotes the "cost" experienced by users whose rank is smaller than $I_k(n)$, and the second term, $\Big(M(t) - I_k(t) + 1\Big)\Big(\frac{R_k(t)}{E[R_k]} - 1\Big)$, represents the "saving" of the other users. Then, by scheduling the transmission for user k^* with the smallest D_k:

$$k^* = \arg\min_k \Big(D_k(n)\Big),$$

the expected remaining time (and thus the expected total completion time) is minimized "locally".

4 Performance Bounds on the Total Completion Time

As noted in the Introduction, different users experience different channels, and the completion time is tightly coupled across the users. This makes it difficult (if not impossible) to find a closed-form solution for the total completion time. Thus, we turn to derive lower bounds and upper bounds on the total completion time. (For completeness, we also analyze the total completion time for the wireless SRPT scheme, which is relegated to Appendix A.) In particular, we first examine a hypothetical case shown in Fig. 2, and characterize the

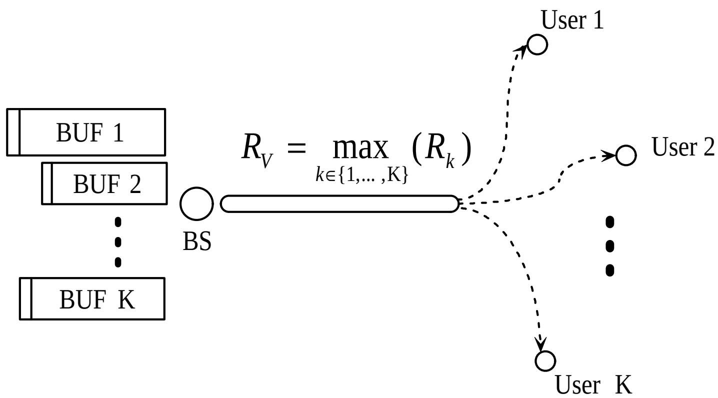

Fig. 2. A hypothetical channel model for the lower bound

corresponding (optimal) total completion time; this serves as a lower bound of all scheduling schemes (we note that this lower bound is not achievable). Next, we derive an upper bound on the total completion time corresponding to the "riding on the channel peak" scheme. Since the TAOS schemes perform better than "riding on the channel peak", this bound is an upper bound on the completion time corresponding to the TAOS schemes.

4.1 A Lower Bound for TAOS

In what follows, we derive a lower bound on the total completion time. Observe that the data rate between the base station and the user scheduled for transmission can not exceed R_{V}, where

$$R_{\mathrm{V}} \triangleq \max_{k\in\{1,\cdots,K\}} (R_k). \tag{27}$$

(Note $R_{\mathrm{V}}(t) = R_{(K)}(t)$, where $R_{(K)}(t)$ is the Kth order statistic of the instantaneous channel capacities of all K users.) We consider a hypothetical case in Fig. 2, where the base station transmits all K files by using R_{V}, regardless of the destination (user). Accordingly, the optimal scheduling scheme in this hypothetical context would be wireless SRPT, which serves as a lower bound. Hence, the lower bound of the total completion time is given by

$$\Psi_{\mathrm{LB}} = \frac{1}{E\big[R_{\mathrm{V}}\big]} \sum_{k=1}^{K} (K-k+1) F_k. \tag{28}$$

With the knowledge of file size distributions, we can also examine the average-case performance. Specifically, the expectation of the total completion time

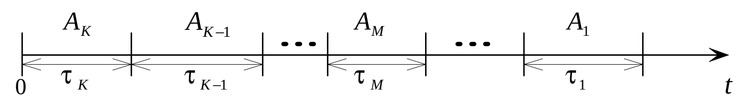

Fig. 3. Transmission dynamics corresponding to "riding on the channel peak"

is

$$E\Big[\Psi_{\mathrm{LB}}\Big] = \frac{1}{E\Big[R_{\mathrm{V}}\Big]} \sum_{k=1}^{K} (K-k+1) E\Big[F_k\Big]. \tag{29}$$

where $E[F_k]$ can be derived by using results on order statistics [7]. ($E[F_k]$ for different file size distributions is given in Appendix B.) Worth pointing out is that this lower bound is not achievable.

4.2 An Upper Bound for TAOS

Now, we examine "riding on the channel peak", and thus find an upper bound for TAOS. Under the assumption that the mobile users have i.i.d. (statistically symmetric) channels, the "riding on the peak" scheduling criterion can be written in the form of

$$k^* = \arg\max_k \Big(R_k(t)\Big). \tag{30}$$

Define $R_{\max} = \max_{k \in B}(R_k)$, where B denote the set of active users remaining in the system. In a system with many users, $R_{\max}$ can be well approximated as (see, e.g., [9])

$$R_{\max}\Big(K\Big) \backsimeq E[R] + \sqrt{2\sigma_R^2 \ln K}, \tag{31}$$

where $E[R]$ and σ_R^2 denote the mean and the variance of the data rate, respectively.

Consider a transmission process in Fig. 3. Let τ_M denote the duration when there are M active users remaining in the system, and A_M denote the system throughput within the period τ_M. Then, $\sum_{k=1}^{K} A_k = \sum_{k=1}^{K} F_k$. Assuming the channel is ergodic, we have that

$$\tau_M = \frac{A_M}{E\Big(R_{\max}(M)\Big)}, \tag{32}$$

where $R_{\max}(M)$ denotes the maximum data rate across M remaining users.

The total completion time is given by

$$\Psi = \sum_{k=1}^{K} \frac{kA_k}{E\Big(R_{\max}(k)\Big)}. \tag{33}$$

Using the results of [7], we get

$$E\Big(R_{\max}(k)\Big) = k\int_0^\infty R[P(R)]^{k-1}p(R)dR, \tag{34}$$

where $P(R)$ and $p(R)$ denotes the cumulative distribution function (CDF) and probability density function (PDF) for the (unordered) data rate R, respectively. Thus,

$$\frac{k}{E\Big(R_{\max}(k)\Big)} = \frac{1}{\int_0^\infty R[P(R)]^{k-1}p(R)dR}. \tag{35}$$

Since $0 \le P(R) \le 1$,

$$\int_0^\infty R[P(R)]^{K-1}p(R)dR = \min_k \Big(\int_0^\infty R[P(R)]^{k-1}p(R)dR\Big). \tag{36}$$

It follows that

$$\frac{K}{E\Big(R_{\max}(K)\Big)} = \max_k \Big(\frac{k}{E\Big(R_{\max}(k)\Big)}\Big). \tag{37}$$

Therefore, we conclude that

$$\Psi \le \max_k \Big(\frac{k}{E\Big(R_{\max}(k)\Big)}\Big)\sum_{k=1}^{K} A_k = \frac{K}{E\Big(R_{\max}(K)\Big)}\sum_{k=1}^{K} F_k. \tag{38}$$

5 Numerical Examples

We illustrate the performance gain of the traffic aided opportunistic scheduling schemes via numerical examples. Using the simple Round-Robin algorithm as the baseline of performance evaluation, we define a *normalized* total completion time as $\Psi_{\text{S}}/\Psi_{\text{R}}$, where Ψ_{S} and Ψ_{R} denote the total completion time of a given scheduling scheme and the Round-Robin algorithm, respectively.

Following [20], we assume that the system bandwidth is 1.25MHz, the slot duration t_s is 1.67ms, and the observation window T^c consists of 1000 slots. In the first example, we assume $K = 20$. All users in the system experience i.i.d.

Table 1
Normalized total completion time

Schemes	W-SRPT	L-bound
Simulation	0.546	0.225
Theoretic	0.545	0.204

Rayleigh fading channels with long-term (average) signal-to-noise ratio (SNR) 0dB, and the maximal Doppler shift $f_m = 10$Hz. Along the line of [5], [24], we assume that the file sizes for Web browsing follow a heavy-tailed (Pareto) distribution with the minimal file size of 50kB and shape parameter $\alpha = 1.2$ (hence, the mean $\mu_F = 300$kB). In Table 1, we observe that the analytical results coincide with that of the Monte Carlo simulation, i.e., the difference between analytical results and simulation results is negligible. Therefore, the following simulation results are reliable for evaluating the performance. Also, we note that the upper bound for TAOS may not always be tight, and the lower bound provides a benchmark on the performance of the TAOS schemes.

5.1 Impact of Fading

Next, we investigate the impact of fading on the performance of TAOS schemes. Suppose that the average SNR is 0dB. Table 2 depicts the normalized total completion time in Rayleigh fading channels. In this example, we assume that the receiver can estimate the channel conditions perfectly, regardless of the Doppler shift. We observe that in such a channel with high dispersion, TAOS-2 achieves the best performance, whereas W-SRPT has the worst performance. The performance of other schemes is between these two above. The physically appealing explanation is that those schemes other than W-SRPT, employ channel variation for opportunistic scheduling. Thus, they achieve significant multiuser diversity gains, even though the gain may vary across different schemes. Therefore, the opportunistic scheduling can outperform W-SRPT in Rayleigh fading channels.

Fig. 4 gives the normalized total completion time in Rician fading channels with respect to the Rice factors. Several observations are in order. First, as would be expected, the total completion time of W-SRPT does not vary with the Rice factor $\mathcal{K}$. In fact, the W-SRPT only utilizes the average data rate,

Table 2
Normalized total completion time of scheduling schemes in Rayleigh Fading Channels

Schemes	W-SRPT	On-peak	TAOS-1	TAOS-1a	TAOS-1b	TAOS-2	L-bound
Simulation	0.548	0.449	0.407	0.371	0.409	0.366	0.225

Table 3
Normalized total completion time of scheduling schemes ($\mathcal{K}$=50dB)

Schemes	W-SRPT	On-peak	TAOS-1	TAOS-1a	TAOS-1b	TAOS-2	L-bound
Simulation	0.551	0.993	0.801	0.551	0.551	0.551	0.541

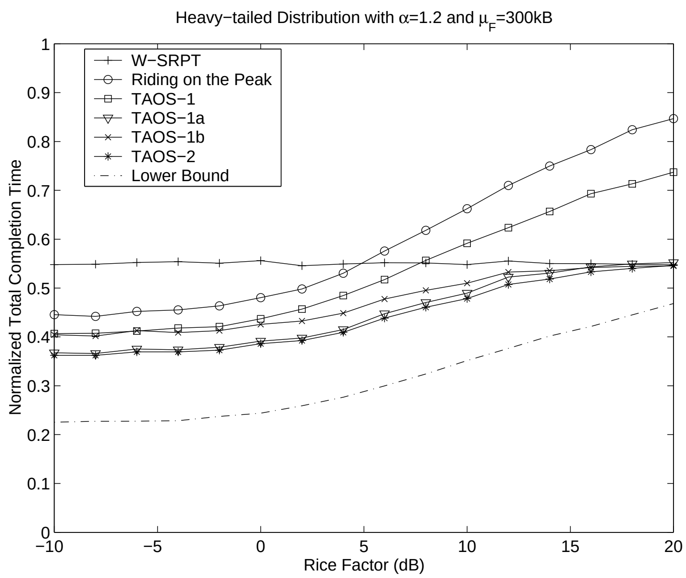

Fig. 4. Normalized total completion time in Rician fading channels

and thus the channel variation has little impact on the W-SRPT scheme. Next, the total completion times corresponding to the "riding on the peak", TAOS-1, TAOS-1a, TAOS-1b, and TAOS-2 schemes, increase with $\mathcal{K}$. Our intuition is as follows: the greater the Rice factor $\mathcal{K}$, the less variation the fading has, and the less *multiuser diversity* (see, e.g., [20], [22]) gains in the system throughput can be exploited. Therefore, as the Rice factor $\mathcal{K}$ increases, more time is needed to complete the transmissions for these three scheduling schemes. Furthermore, after $\mathcal{K}$ is greater than certain values, "riding on the peak" and TAOS-1 perform worse than W-SRPT. Finally, the TAOS-2 always achieves the best performance, which is close to the lower bound, even though

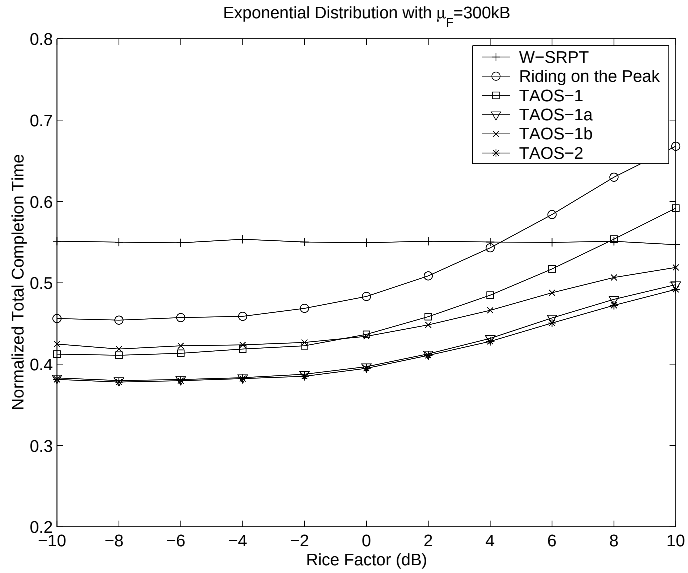

Fig. 5. Normalized total completion time for exponentially distributed file size

in general the lower bound is not achievable.

Table 3 describes the performance of scheduling schemes, when the Rice factor $\mathcal{K} = 50$dB. From Table 3 and Fig. 4, we note that as $\mathcal{K} \to \infty$, i.e., multipath fading diminishes, the performance of TAOS-1a, TAOS-1b, and TAOS-2 converges to that of W-SRPT, whereas "riding on the channel peak" achieves only the same performance as Round-Robin. The performance of TAOS-1 lies in-between. We also observe that as $\mathcal{K} \to \infty$ the lower bound becomes tighter.

5.2 Impact of File Size Distribution

In the following, we examine the impact of network traffic on the performance of scheduling schemes. In particular, we compare the normalized total completion times corresponding to three different file size distributions: the heavy-tailed distribution, the exponential distribution, and the uniform distribution. We assume that they have the same mean $\mu_F = 300$kB. Comparing Fig. 5 and Fig. 6 with Fig. 4 (which is for the heavy-tailed distribution), we observe that our conclusions drawn above for the heavy-tailed distribution case are also applicable to the exponential and uniform distribution cases.

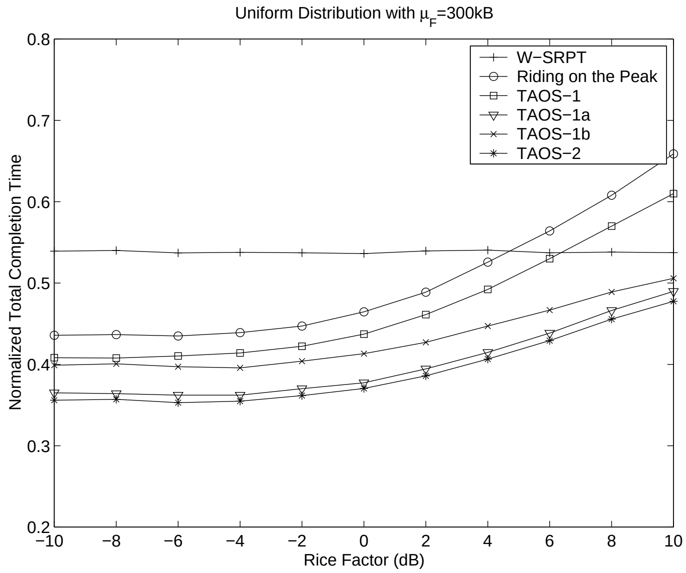

Fig. 6. Normalized total completion time for uniformly distributed file size

5.3 Impact of Random Arrivals and Departures

Next, we extend our study to the cases with random "users" arrivals and departures. We note that the theoretical analysis for the case with user dynamics remains open. In the following, we use simulation to evaluate the performance. Note that in each time slot, the proposed TAOS schemes make use of updated channel and file size information, and arrange the transmissions in an opportunistic manner. Since the TAOS schemes are capable of tracking the user dynamics, the TAOS schemes yield significant gains in the case with random arrivals and departures as would be expected.

Assume that the arrival process is Poisson, all transmissions experience i.i.d. fading, and the file sizes obey the heavy-tailed distribution. Fig. 7 depicts the normalized total completion time with respect to the random arrival rate λ, when $\mathcal{K} = 0$dB. We can see that the higher the arrival rate, the shorter the normalized completion time of TAOS schemes, i.e., the more gains the TAOS schemes can yield. Our intuition is that if the arrival rate is high, at each instance it is likely that more users may join the contention. As a result, TAOS schemes can achieve possibly more multiuser diversity gains, leading to better performance.

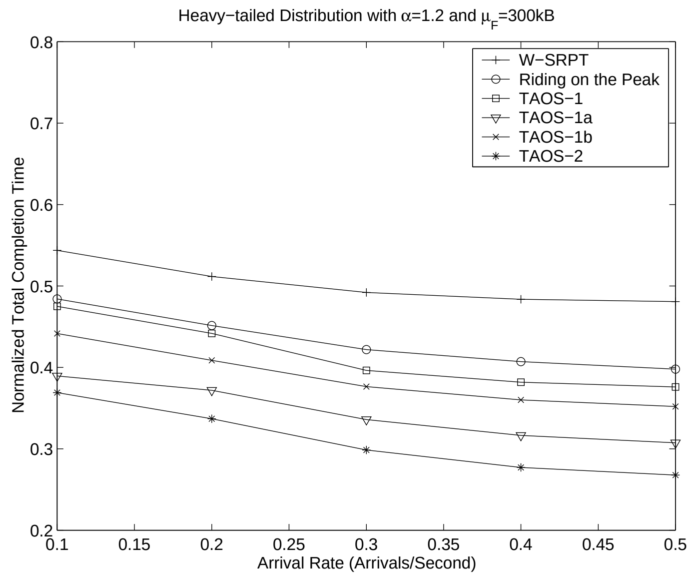

Fig. 7. Normalized total completion time with respect to the arrival rate

5.4 A Comparison of System Throughput

Finally, we investigate the average system throughput corresponding to the TAOS schemes. We use the Round-Robin algorithm as the baseline, and define the throughput gain as $(U_{\mathrm{S}} - U_{\mathrm{R}})/U_{\mathrm{R}}$, where U_{S} and U_{R} denote the average system throughput of a given scheduling scheme and the Round-Robin algorithm, respectively.

Fig. 8 depicts the throughput gain of the TAOS schemes in Rician fading channels, where $K = 20$. As expected, the throughput gain corresponding to the TAOS schemes lies in between that of the "riding on the channel peak" scheme and that of the W-SRPT scheme. More specifically, the throughput gain of the TOAS-2 scheme is about 15% lower than that of the "riding on the channel peak" scheme. This loss in throughput is due to the fact that the goal of TAOS schemes is set to minimize the total completion time. Another observation is that TAOS-2 achieves higher performance than the TAOS-1 schemes, in terms of both the system throughput and the completion time.

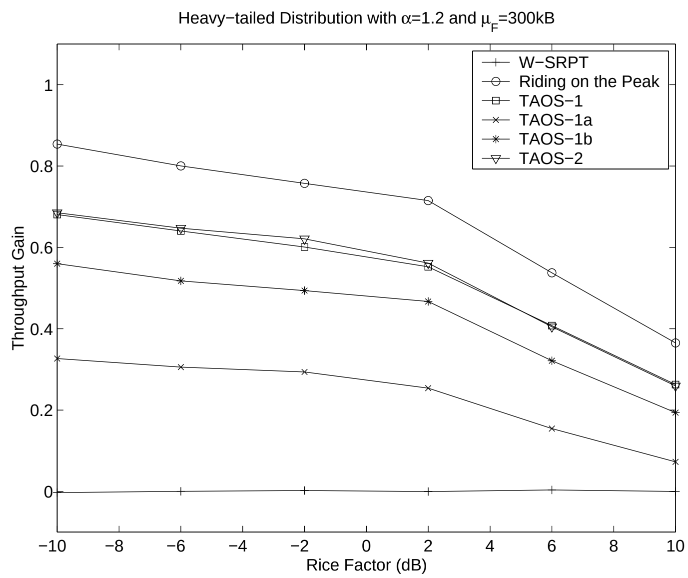

Fig. 8. Throughput gain in Rician Fading Channels

6 Conclusions

In this paper, we study opportunistic scheduling in multiuser wireless networks. The main goal is to minimize the total completion time (which consists of both the processing time and the waiting time). We first establish general properties of scheduling schemes, and the identified convexity properties provide a basis for investigating opportunistic scheduling. Then we take a cross-layer optimization approach and develop new traffic aided opportunistic scheduling schemes, namely TAOS-1, TAOS-1a, TAOS-1b, and TAOS-2. The TAOS schemes make use of both the file size information and channel variation, in a unified manner. As expected, they can yield significant reduction of the total completion time. We also derive lower bounds and upper bounds on the total completion time, which serve as benchmarks for examining the performance of the TAOS schemes.

Next, we investigate the impact of fading, file size distributions, and random arrivals and departures on the system performance. Our results show that the more channel variation, the more gains the TAOS schemes can achieve. This conclusion holds well for different file size distributions. We also observe that with random arrivals and departures, the higher arrival rate, the more reduction of the total completion time the TAOS schemes would yield. Therefore, the proposed TAOS schemes can perform well in heavy-loaded wireless networks. Finally, we examine the throughput performance of the TAOS schemes and the tradeoff between the throughout and the total completion time.

Acknowledgement

The authors thank Dr. Sem Borst for careful reading and valuable comments on an earlier draft of this paper.

A Total Completion Time of Wireless SRPT

We assume that the wireless channels are i.i.d. across the users. It is clear that if wireless SRPT is utilized, the file with shortest processing time would be served first until its transmission is complete. In many practical wireless systems, the transmission time of each file is on the order of seconds or even minutes, whereas the coherent time of fading is on the order of milliseconds. As a result, it is sensible to calculate the completion time based on the average data rate. Also, we can neglect the edge effect, and analyze the performance of scheduling schemes in continuous rather than discrete time (see also [3]). The completion time of user k is given by

$$\psi_k = \frac{1}{E[R]} \sum_{i=1}^{k} F_i, \tag{A.1}$$

where $E[R]$ denotes the average data rate. Therefore, the total completion time is

$$\Psi = \sum_{k=1}^{K} \psi_k = \frac{1}{E[R]} \sum_{k=1}^{K} (K-k+1) F_k. \tag{A.2}$$

It follows that

$$E[\Psi] = \sum_{k=1}^{K} E[\psi_k] = \frac{1}{E[R]} \sum_{k=1}^{K} (K-k+1) E[F_k]. \tag{A.3}$$

Similar analysis is applicable to asymmetric users in Wireless SRPT.

B Expectation of Ordered File Sizes

Assuming the file size distribution is known, we compute the mean of the kth order statistic for different file size distributions. Recall that the file sizes are i.i.d., and we user F_k to denote the kth order statistic [7].

a) *Exponential Distribution*: If the file size is exponentially distributed, i.e.,

$$Pr(F < x) = 1 - e^{-\lambda x}. \tag{B.1}$$

The mean of the kth order statistic can be computed as

$$E[F_k] = \frac{1}{\lambda} \sum_{i=0}^{k-1} \frac{1}{K-i}. \tag{B.2}$$

b) *Uniform Distribution*: Assume that the file size is uniformly distributed within the range of 0 to a. Then the mean of the kth order statistic is

$$E[F_k] = K \binom{K-1}{k-1} \int_0^1 au \cdot u^{k-1} (1-u)^{K-k} du = \frac{a \cdot k}{K+1}. \tag{B.3}$$

c) *Heavy-tailed Distribution*: Suppose that the file size can be modelled as Pareto distribution with the form

$$Pr(F < x) = 1 - \left(\frac{b}{x}\right)^{\alpha}, \ 1 < \alpha < 2. \tag{B.4}$$

The mean of the rth order statistic is

$$\begin{aligned} E[F_k] &= K\binom{K-1}{k-1}\int_0^1 b(1-u)^{-1/\alpha}u^{k-1}(1-u)^{K-k}du \\ &= bK\binom{K-1}{k-1}B(k, K-k-1/\alpha+1), \end{aligned} \tag{B.5}$$

for $(K - k - 1/\alpha + 1) > 0$, where $B(\cdot)$ denotes the Beta-function.

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