Large-System Performance Analysis of Blind and Group-Blind Multiuser Receivers

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***Abstract*—We present a large-system performance analysis of blind and group-blind multiuser detection methods. In these methods, the receivers are estimated based on the received signal samples. In particular, we assume binary random spreading, and let the spreading gain N, the number of users K, and the number of received signal samples M all go to infinity, while keeping the ratios K/N and M/N fixed. We characterize the asymptotic performance of the direct-matrix inversion (DMI) blind linear minimum mean-square error (MMSE) receiver, the subspace blind linear MMSE receiver, and the group-blind linear hybrid receiver. We first derive the asymptotic average output signal-to-interference-plus-noise ratio (SINR) for each of these receivers. Our results reveal an interesting "saturation" phenomenon: The output SINR of each of these receivers converges to a finite limit as the signal-to-noise ratio (SNR) of the desired user increases, which is in stark contrast to the fact that the output SINR achieved by the exact linear MMSE receiver can get arbitrarily large. This indicates that the capacity of a wireless system with blind or group-blind multiuser receivers is not only *interference-limited*, but also *estimation-error limited*. We then show that for both the blind and group-blind receivers, the output residual interference has an asymptotic Gaussian distribution, independent of the realizations of the spreading sequences. The Gaussianity indicates that in a large system, the bit-error rate (BER) is related to the SINR simply through the Q function.**

***Index Terms*—Blind multiuser detection, group-blind multiuser detection, large-system analysis.**

I. Introduction

In this paper, we present a large-system performance analysis of blind and group-blind linear multiuser detection techniques for the following canonical discrete-time K-user synchronous code-division multiple-access (CDMA) channel

$$\boldsymbol{r}[i] = \sum_{k=1}^{K} A_k b_k[i] \boldsymbol{s}_k + \boldsymbol{n}[i] \tag{1}$$

$$= \boldsymbol{S}\boldsymbol{A}\boldsymbol{b}[i] + \boldsymbol{n}[i], \qquad i = 1, 2, \ldots, M \tag{2}$$

where $A_k > 0$, $b_k[i] \in \{+1, -1\}$, and $\boldsymbol{s}_k$ are the received amplitude, data bit, and unit-energy spreading signature sequence of the kth user, respectively, and

$$\boldsymbol{n}[i] \sim \mathcal{N}(\boldsymbol{0}, \eta \boldsymbol{I}_N)$$

is the additive white Gaussian noise, with $\boldsymbol{I}_n$ denoting an $n \times n$ identity matrix. These are collected in vector forms as

$$\boldsymbol{S} = [\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K]$$
$$\boldsymbol{A} = \mathrm{diag}\{A_1, \ldots, A_K\}$$

and

$$\boldsymbol{b}[i] = [b_1[i], \ldots, b_K[i]]^T.$$

In a direct-sequence spread-spectrum system with spreading gain N, the signature sequence of the kth user is of the form

$$\boldsymbol{s}_k = \frac{1}{\sqrt{N}} [s_{1,k}\, s_{2,k} \cdots s_{N,k}]^T, \quad s_{n,k} \in \{+1, -1\}. \tag{3}$$

A number of recent works [2]–[4], [6], [5], [8], [13], [14], [16], [20], [24], [25], [27], [28], [32], [36], [35], [37] have analyzed the asymptotic performance of various CDMA receivers for systems with random spreading sequences, when the number of users K grows without bound and the ratio of the number of users K to the spreading gain N is kept fixed. In particular, a capacity analysis of large CDMA networks with linear multiuser receivers is provided in [24], and a fundamental result—the Tse–Hanly equation—is obtained, the solution to which is the output signal-to-interference-plus-noise ratio (SINR) of the linear minimum mean-square error (MMSE) multiuser receiver. Generalization of the results in [24] to more complex systems with asynchrony, multipath fading, and antenna diversity can be found in [4], [13], [25]. The optimal near–far resistance for synchronous dual-rate CDMA systems is studied in [3]. The performance of a reduced-rank linear filtering receiver using the linear MMSE criterion is evaluated in [8], and an important conclusion is that the rank needed to achieve a desired SINR does not scale with the system size. In [16], a random matrix model is introduced to describe the spatial and temporal multipath propagation between transmitting and receiving antenna arrays, and this model enables the calculation of SINR for large antenna systems. In [27], a large system analysis for a single-class system is carried out to derive the Shannon-theoretic performance, where all users have the same received powers and the same rate requirements. The impact of fading on the admissibility and network capacity is studied in [34], [35]. The Gaussianity of the output interference of linear MMSE receivers in a large network is established in

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[37], via exploiting random matrix theory and martingale limit theory.

On the other hand, motivated by the recent development of blind and group-blind multiuser detection techniques [7], [29], [30], [19], [31], the analytical performance assessment for these multiuser detection methods has been developed in [10]–[12]. where the effect of estimation error (due to the finite number of received signals, based on which these receivers are estimated) on the receiver performance. These studies on blind receiver performance analysis (referred to later as BRPA) differs from the works on large-system performance analysis (LSPA) mentioned earlier in the following perspectives.

- LSPA treats the spreading sequences as independent and identically distributed (i.i.d.) random variables with zero mean and unit variance; whereas in BRPA, the spreading sequences are considered as deterministic vectors.
- LSPA considers the performance of exact linear multiuser receivers; whereas BRPA treats the performance of blind and group-blind multiuser receivers estimated from the received signals.
- LSPA considers the asymptotic performance when both the number of users K and the spreading gain N tend to infinity; BRPA focuses on the case where both K and N are fixed, while considering the performance as a function of the sample size M.

In this paper, we focus on large-system asymptotic performance analysis for blind and group-blind multiuser detection methods. In particular, we assume binary random spreading, and we let N, K, and M go to infinity while keeping the ratios $\alpha_K \triangleq \frac{K}{N}$ and $\alpha_M \triangleq \frac{M}{N}$ fixed. (We note that similar approaches have been adopted in recent work [32], [33].) We also assume that $\alpha_K \le 1$, and that α_M is reasonably large. In such a scenario, we characterize the asymptotic performance of various blind and group-blind multiuser receivers. We summarize our main findings as follows.

1) As the processing gain increases, the output SINR of each receiver considered here converges with probability one to some deterministic limit, independent of the spreading signatures.

2) Based on the limiting SINR of the blind linear MMSE receivers, we find that as the signal-to-noise ratio (SNR) of the desired user A_1^2/η increases, the output SINR achieved by the direct-matrix-inversion (DMI) blind receiver converges to α_M, and that achieved by the subspace blind receiver approaches α_M/α_K; this is in sharp contrast to the fact that the asymptotic SINR achieved by the exact linear MMSE receiver can get arbitrarily large. This observation reveals a "saturation" phenomenon of the SINR achieved by the blind receivers. That is, beyond certain input SNR, the estimation error dominates and hence determines the output SINR. This indicates that the capacity of a wireless system with blind multiuser receivers is not only *interference-limited*, but also *estimation-error limited*. Furthermore, the effect of estimation error can be quantified, as shown in Section III.

3) The group-blind linear receiver outperforms the subspace blind linear receiver because the asymptotic SINR of the former is strictly higher than that of the latter. Moreover, the capacity of a wireless system with group-blind linear receivers is also estimation-error limited.

4) We also establish that the output residual interference of these receivers is asymptotically Gaussian, for almost every realization of the spreading signatures. Therefore, the asymptotic bit-error rate (BER) is related to the asymptotic SINR simply through the Q function. As a side result, we obtain that the estimation error of these receivers is asymptotically orthogonal to the spreading signature sequences.

The rest of this paper is organized as follows. In Section II, we provide some background material on blind and group-blind linear multiuser receivers, and the analytical performance results of these receivers obtained in [10]–[12]. In Section III, we summarize the main results of this paper, namely, the large-system performance of the blind and group-blind linear multiuser receivers, in terms of asymptotic SINR, distribution of residual interference, and BER. In Section IV, we provide some numerical examples to illustrate the theoretical results. The detailed proofs of the main results in this paper can be found in Section V. Finally, some technical theorems and lemmas that are useful in proving the main results are collected in the Appendix.

II. Background

Consider the signal model (1). A linear receiver for user 1 is a (deterministic) vector $\boldsymbol{w}_1 \in \mathbb{R}^N$ such that $b_1[i]$ is demodulated according to

$$\hat{b}_1[i] = \operatorname{sign}\left\{\boldsymbol{w}_1^T \boldsymbol{r}[i]\right\}. \tag{4}$$

Suppose that an estimate $\hat{\boldsymbol{w}}_1$ of the linear receiver $\boldsymbol{w}_1$ is obtained from the received signals $\{\boldsymbol{r}[i]\}_{i=1}^M$. Denote

$$\Delta\boldsymbol{w}_1 = \hat{\boldsymbol{w}}_1 - \boldsymbol{w}_1. \tag{5}$$

Then, the average SINR at the output of the estimated linear receiver $\hat{\boldsymbol{w}}_1$ is given by

$$\begin{aligned}\operatorname{SINR}(\hat{\boldsymbol{w}}_1) &\stackrel{\triangle}{=} c\, \frac{\mathbb{E}\left[\mathbb{E}\left[\hat{\boldsymbol{w}}_1^T \boldsymbol{r}[i] \middle| b_1[i]\right]^2\right]}{\mathbb{E}\left[\operatorname{var}\left(\hat{\boldsymbol{w}}_1^T \boldsymbol{r}[i] \middle| b_1[i]\right)\right]} \\ &= \frac{A_1^2\left(\boldsymbol{w}_1^T \boldsymbol{s}_1\right)^2}{\sum_{k=2}^K A_k^2\left(\boldsymbol{w}_1^T \boldsymbol{s}_k\right)^2 + \eta\|\boldsymbol{w}_1\|^2 + \operatorname{tr}\left(\boldsymbol{C}_w \boldsymbol{C}_r\right)}\end{aligned} \tag{6}$$

with

$$\boldsymbol{C}_r \triangleq \mathbb{E}\left[\boldsymbol{r}[i]\boldsymbol{r}[i]^T\right] = \sum_{k=1}^K A_k^2 \boldsymbol{s}_k \boldsymbol{s}_k^T + \eta \boldsymbol{I}_N \tag{7}$$

$$= \boldsymbol{S}\boldsymbol{A}^2\boldsymbol{S}^T + \eta \boldsymbol{I}_N \tag{8}$$

and $\boldsymbol{C}_w = \mathbb{E}[\Delta \boldsymbol{w}_1 \Delta \boldsymbol{w}_1^T]$. For the following estimated linear receivers, namely, a) DMI blind linear MMSE receiver [7], [26], b) subspace blind linear MMSE receiver [31], and c) subspace group-blind linear hybrid receiver [29], it is shown in [10]–[12] that *for fixed number of users* K, *fixed spreading gain* N, *and fixed set of spreading sequences* $\{\boldsymbol{s}_k\}_{k=1}^K$, we have

$$\sqrt{M}\,(\hat{\boldsymbol{w}}_1 - \boldsymbol{w}_1) \to \mathcal{N}(\boldsymbol{0}, \boldsymbol{C}_w), \quad \text{in distribution, as } M \to \infty.$$

The limiting error covariance $\boldsymbol{C}_w$ has been found in closed-form for each of the above receivers, based on which the average output SINR can then be computed according to (6) [11], [12].

A. Blind Linear MMSE Receivers

Consider again the signal model (1). The linear MMSE receiver for user 1 is defined as

$$\boldsymbol{w}_1 = \min_{\boldsymbol{w}\in\mathbb{R}^N} \mathbb{E}\left[\left(\frac{1}{\mu} b_1[i] - \boldsymbol{w}^T \boldsymbol{r}[i]\right)^2\right] = \frac{A_1}{\mu} \boldsymbol{C}_r^{-1} \boldsymbol{s}_1 \tag{9}$$

where μ is some positive constant. Because the linear detection rule (4) is invariant to a positive scaling, the linear receiver in (9) is invariant to the positive constant A_1/μ. For simplicity, we choose $\mu = A_1$ so that $\boldsymbol{w}_1 = \boldsymbol{C}_r^{-1}\boldsymbol{s}_1$.

In blind multiuser detection, the linear MMSE receiver is estimated from the received signals $\{\boldsymbol{r}[i]\}_{i=1}^M$, with the prior knowledge of the desired user's signature sequence, say $\boldsymbol{s}_1$. Let the eigendecomposition of $\boldsymbol{C}_r$ in (8) be

$$\boldsymbol{C}_r = \boldsymbol{U}_s \boldsymbol{\Lambda}_s \boldsymbol{U}_s^T + \eta \boldsymbol{U}_n \boldsymbol{U}_n^T \tag{10}$$

where $\boldsymbol{\Lambda}_s = \mathrm{diag}\{\gamma_1, \ldots, \gamma_K\}$ contains the largest K eigenvalues of $\boldsymbol{C}_r$; $\boldsymbol{U}_s = [\boldsymbol{u}_1, \ldots, \boldsymbol{u}_K]$ contains the eigenvectors corresponding to the largest K eigenvalues in $\boldsymbol{\Lambda}_s$; $\boldsymbol{U}_n = [\boldsymbol{u}_{K+1}, \ldots, \boldsymbol{u}_N]$ contains the $(N-K)$ eigenvectors corresponding to the smallest eigenvalue η of $\boldsymbol{C}_r$. The linear MMSE receiver for user 1 is then given by [7], [31]

$$\boldsymbol{w}_1 = \boldsymbol{C}_r^{-1}\boldsymbol{s}_1 \tag{11}$$
$$= \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} \boldsymbol{U}_s^T \boldsymbol{s}_1. \tag{12}$$

Corresponding to the above two forms of the linear MMSE receiver, there are two approaches to its blind implementation. In the DMI method, the autocorrelation matrix $\boldsymbol{C}_r$ is replaced by the corresponding sample estimate. That is,

$$\hat{\boldsymbol{C}}_r = \frac{1}{M}\sum_{i=1}^M \boldsymbol{r}[i]\boldsymbol{r}[i]^T \tag{13}$$

$$\hat{\boldsymbol{w}}_1 = \hat{\boldsymbol{C}}_r^{-1}\boldsymbol{s}_1. \tag{14}$$

In the subspace method, the eigencomponents $\boldsymbol{\Lambda}_s$ and $\boldsymbol{U}_s$ are replaced by the corresponding eigenvalues and eigenvectors of the sample autocorrelation matrix $\hat{\boldsymbol{C}}_r$. That is,

$$\hat{\boldsymbol{C}}_r = \frac{1}{M}\sum_{i=1}^M \boldsymbol{r}[i]\boldsymbol{r}[i]^T$$
$$= \hat{\boldsymbol{U}}_s \hat{\boldsymbol{\Lambda}}_s \hat{\boldsymbol{U}}_s^T + \hat{\boldsymbol{U}}_n \hat{\boldsymbol{\Lambda}}_n \hat{\boldsymbol{U}}_n^T \tag{15}$$

$$\hat{\boldsymbol{w}}_1 = \hat{\boldsymbol{U}}_s \hat{\boldsymbol{\Lambda}}_s^{-1} \hat{\boldsymbol{U}}_s^T \boldsymbol{s}_1. \tag{16}$$

For large M, the average output SINRs of the above two forms of the blind linear MMSE receiver are characterized by the following result [11], [12].

Proposition 2.1: For large-signal sample size M, the average output SINR of the blind linear MMSE receiver, denoted $\mathrm{SINR}^{(N)}(\hat{\boldsymbol{w}}_1)$, is well approximated by

$$\frac{A_1^2\left(\boldsymbol{w}_1^T\boldsymbol{s}_1\right)^2}{\mathcal{I}_0 + \frac{N}{M}\left(\mathcal{I}_1 + \mathcal{I}_2 + \mathcal{I}_3\right)} \tag{17}$$

with

$$\mathcal{I}_0 = \sum_{k=2}^K A_k^2\left(\boldsymbol{w}_1^T\boldsymbol{s}_k\right)^2 + \eta\|\boldsymbol{w}_1\|^2 \tag{18}$$

$$\mathcal{I}_1 = \frac{K+1}{N}\,\boldsymbol{w}_1^T\boldsymbol{s}_1 \tag{19}$$

$$\mathcal{I}_2 = -\frac{2}{N}\sum_{k=1}^K A_k^4\left(\boldsymbol{w}_1^T\boldsymbol{s}_k\right)^2\left(\boldsymbol{w}_k^T\boldsymbol{s}_k\right) \tag{20}$$

$$\mathcal{I}_3 = \frac{N-K}{N}\,\tau\eta \tag{21}$$

where

$$\boldsymbol{w}_k = \boldsymbol{C}_r^{-1}\boldsymbol{s}_k = \boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T\boldsymbol{s}_k \tag{22}$$

is the exact MMSE receiver for user k, and

$$\tau = \begin{cases} \boldsymbol{w}_1^T\boldsymbol{s}_1/\eta, & \text{DMI blind receiver} \\ \eta\boldsymbol{s}_1^T\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1} & \\ \quad\cdot(\boldsymbol{\Lambda}_s - \eta\boldsymbol{I}_K)^{-2}\boldsymbol{U}_s^T\boldsymbol{s}_1, & \text{subspace blind receiver.} \end{cases} \tag{23}$$

B. Group-Blind Linear Hybrid Receiver

In group-blind multiuser detection, it is assumed that the receiver has the knowledge of the spreading sequences of some but not all the users. Without loss of generality, assume that the first $\tilde{K}$ users' spreading sequences are known to the receiver, whereas those of the rest $(K-\tilde{K})$ users' are unknown. Denote

$$\tilde{\boldsymbol{S}} = [\boldsymbol{s}_1 \; \boldsymbol{s}_2 \; \cdots \; \boldsymbol{s}_{\tilde{K}}].$$

It is assumed that $\tilde{\boldsymbol{S}}$ has full column rank. Denote $\tilde{\boldsymbol{e}}_k$ as the kth unit vector in $\mathbb{R}^{\tilde{K}}$. The group-blind linear hybrid receiver zero-forces the interference caused by the $\tilde{K}$ known users and suppress that from the remaining $(K-\tilde{K})$ unknown users according to the MMSE criterion. In particular, such a receiver for user 1 is given by the solution to the following constrained optimization problem:

$$\boldsymbol{w}_1 = \arg\min_{\boldsymbol{w}\in\mathbb{R}^N} \mathbb{E}\left[\left(A_1 b_1[i] - \boldsymbol{w}^T\boldsymbol{r}[i]\right)^2\right], \quad \text{s.t. } \boldsymbol{w}^T\tilde{\boldsymbol{S}} = \tilde{\boldsymbol{e}}_1^T. \tag{24}$$

Note that the cost function in (24) parallels that in (9) with $\mu = \frac{1}{A_1}$. (We have chosen $\mu = \frac{1}{A_1}$ for convenience because the detection rule (4) is invariant to positive scaling.) The solution to (24) is given in terms of the signal subspace components as [29]

$$\boldsymbol{w}_1 = \boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{e}}_1. \tag{25}$$

Such a linear group-blind multiuser detector can also serve as the first stage in an iterative nonlinear group-blind multiuser detector [22]. Replacing $\boldsymbol{U}_s$ and $\boldsymbol{\Lambda}_s$ in (25) by $\hat{\boldsymbol{U}}_s$ and $\hat{\boldsymbol{\Lambda}}_s$, respectively, we obtain the estimated group-blind linear hybrid receiver as

$$\hat{\boldsymbol{w}}_1 = \hat{\boldsymbol{U}}_s \hat{\boldsymbol{\Lambda}}_s^{-1} \hat{\boldsymbol{U}}_s^T \tilde{\boldsymbol{S}} \left(\tilde{\boldsymbol{S}}^T \hat{\boldsymbol{U}}_s \hat{\boldsymbol{\Lambda}}_s^{-1} \hat{\boldsymbol{U}}_s^T \tilde{\boldsymbol{S}} \right)^{-1} \tilde{\boldsymbol{e}}_1. \tag{26}$$

For convenience, define the partition of the following $K \times K$ matrix:

$$\boldsymbol{S}^T \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} \boldsymbol{U}_s^T \boldsymbol{S} = \begin{bmatrix} \boldsymbol{\Psi}_{11} & \boldsymbol{\Psi}_{12} \\ \boldsymbol{\Psi}_{12}^T & \boldsymbol{\Psi}_{22} \end{bmatrix} \tag{27}$$

where the dimension of $\boldsymbol{\Psi}_{11}$ is $\tilde{K} \times \tilde{K}$. Denote

$$\boldsymbol{R} \triangleq \boldsymbol{S}^T \boldsymbol{S}$$

and

$$\boldsymbol{\Xi} \triangleq \left[\boldsymbol{A}^{-2} \left(\boldsymbol{R} + \eta \boldsymbol{A}^{-2} \right)^{-1} \boldsymbol{A}^{-2} \boldsymbol{R}^{-1} \boldsymbol{A}^{-2} \right]_{1:\tilde{K},\, 1:\tilde{K}}. \tag{28}$$

The following result gives the average output SINR of the group-blind linear hybrid receiver for given signatures in a finite-size system (i.e., fixed K and N) with large sample size M [11], [12].

Proposition 2.2: For large-signal sample size M, the average output SINR of the estimated group-blind linear hybrid receiver, denoted by $\mathrm{SINR}^{(N)}(\hat{\boldsymbol{w}}_1)$, is well approximated by

$$\frac{A_1^2}{\mathcal{I}_0 + \frac{N}{M} \left(\mathcal{I}_1 + \mathcal{I}_2 + \mathcal{I}_3 \right)} \tag{29}$$

with

$$\mathcal{I}_0 = \sum_{k=1}^{K-\tilde{K}} A_{\tilde{K}+k}^2 \left(\boldsymbol{w}_1^T \boldsymbol{s}_{\tilde{K}+k} \right)^2 + \eta \|\boldsymbol{w}_1\|^2 \tag{30}$$

$$\mathcal{I}_1 = \frac{K - \tilde{K}}{N} \left[\boldsymbol{\Psi}_{11}^{-1} \right]_{1,1} \tag{31}$$

$$\mathcal{I}_2 = -\frac{2}{N} \sum_{k=1}^{K-\tilde{K}} A_{\tilde{K}+k}^4 \left[\boldsymbol{\Psi}_{12}^T \boldsymbol{\Psi}_{11}^{-1} \right]_{k,1}^2 \cdot \left[\boldsymbol{\Psi}_{22} - \boldsymbol{\Psi}_{12}^T \boldsymbol{\Psi}_{11}^{-1} \boldsymbol{\Psi}_{12} \right]_{k,k} \tag{32}$$

$$\mathcal{I}_3 = \frac{N - K}{N} \eta^2 \left[\boldsymbol{\Psi}_{11}^{-1} \boldsymbol{\Xi} \boldsymbol{\Psi}_{11}^{-1} \right]_{1,1} \tag{33}$$

where $\boldsymbol{w}_1$ is given by (25).

III. Main Results

Throughout this paper, we impose the following regularity conditions on the signal amplitudes.

Condition 1: The empirical distribution function of the sequence $\{A_k^2,\, k = 1, \ldots, K\}$ converges weakly to a distribution function F_μ as $N \to \infty$.

Condition 2: The sequence $\{A_k^2,\, k = 1, \ldots, K\}$ is bounded above and below by positive numbers.

Next, a few words on the above conditions. Condition 1 is a standard assumption in the literature on the large system analysis. Condition 2 is applicable to many practical systems of interest, and will be used in proving the almost sure (a.s.) convergence of the SINR.

Denote the eigenvalues of the random matrix $\boldsymbol{S}\boldsymbol{A}^2\boldsymbol{S}^T$ by $\lambda_1, \ldots, \lambda_N$ (they are random, depending on the realization of $\boldsymbol{S}$), and the empirical distribution of the eigenvalues by G_N. We know from Theorem A.1 in Appendix A that G_N converges weakly to some deterministic distribution function G^*, with probability one [21]. Define

$$\beta_0 = \int_0^\infty \frac{1}{\lambda + \eta} \, \mathrm{d}G^*(\lambda). \tag{34}$$

It has been shown that β_0 is the unique positive solution to the following fixed-point equation (cf. [24]):

$$\beta_0 = \frac{1}{\eta + \alpha_K \int_0^\infty \frac{\mu}{1 + \mu\beta_0} \, \mathrm{d}F_\mu(\mu)}. \tag{35}$$

We note that the quantity β_0 can be interpreted as the SINR achieved by unit received power [24], [37].

A. Asymptotic SINR

Our results on the SINR indicate that in a large system with many users and large processing gain, the performance dependence of the blind and group-blind linear receivers on the signature sequences vanishes. More precisely, as noted before, we assume that $\alpha_K \triangleq \frac{K}{N}$ and $\alpha_M \triangleq \frac{M}{N}$ are fixed when $N \to \infty$. We also assume that α_M is reasonably large so that the SINR approximation given by (17) and (29) hold. Note that this assumption is indeed reasonable because in many practical systems the data packet size is much larger than the processing gain. Our results reveal that the limiting output SINR achieved by various blind and group-blind linear receivers is deterministic, and is intimately related to β_0 defined in (34).

The following theorem gives the average output SINR of the blind linear MMSE receivers in the asymptotic regime (as $N \to \infty$).

Theorem 3.1: Suppose Conditions 1 and 2 hold. Assume that α_M is large enough so that Proposition 2.1 holds. Then as $N \to \infty$, for almost every realization of signature sequences, the output SINR achieved by the DMI blind linear MMSE receiver is well approximated by

$$\frac{A_1^2 \beta_0}{1 + \frac{1}{\alpha_M} \left(1 + A_1^2 \beta_0 \right)} \tag{36}$$

and that achieved by the subspace blind MMSE receiver is well approximated by

$$\frac{A_1^2 \beta_0}{1 + \frac{1}{\alpha_M} \left[\alpha_K + c_1 (1 - \alpha_K) \right] \left(1 + A_1^2 \beta_0 \right)} \tag{37}$$

where

$$c_1 = \frac{\beta_0 - e_1 + \eta e_2}{\beta_0 \left[1 + A_1^2 (e_1 + (1 - \alpha_K)/\eta) \right]^2} \tag{38}$$

with

$$e_1 = \lim_{\eta \to 0} \left(\beta_0(\eta) - \frac{1 - \alpha_K}{\eta} \right) \tag{39}$$

$$e_2 = \lim_{\eta \to 0} - \frac{\mathrm{d} \left(\beta_0(\eta) - \frac{1 - \alpha_K}{\eta} \right)}{\mathrm{d}\eta}. \tag{40}$$

As noted before, we focus on the case where $\alpha_K \le 1$ in this paper. It is worth noting that as $\alpha_K > 1$, the subspace blind MMSE receiver is identical to the DMI receiver, and (36) still holds. (It should be pointed out that the result in (36) can be derived from [32, eqs. (27), (53), and (54)], with $1/\alpha_M$ here being replaced by $1/(\alpha_M - 1)$. Clearly, the difference between $1/\alpha_M$ and $1/(\alpha_M - 1)$ is negligible since α_M is assumed to be large.) Indeed, in many practical systems, the packet size M is on the order of hundreds of bits and therefore α_M is reasonably large, indicating that the results in (36) and (37) provide a good approximation of the output SINR.

Several important observations based on these new findings (36) and (37) are in order.

1) It is known that the asymptotic SINR of the exact linear MMSE receiver $\boldsymbol{w}_1$ given by (11) or (12) satisfies [24]

$$\mathrm{SINR}^{(N)}(\boldsymbol{w}_1) \overset{\text{a.s.}}{\to} A_1^2\beta_0. \tag{41}$$

Comparing (41) with (36) and (37), we see that the performance loss in the asymptotic regime due to the estimation error inherent to the blind receiver is captured by the factor

$$\left[1 + \frac{1}{\alpha_M}\left(1 + A_1^2\beta_0\right)\right]^{-1}$$

for the DMI receiver; and by the factor

$$\left[1 + \frac{1}{\alpha_M}\left[\alpha_K + c_1(1-\alpha_K)\right]\left(1 + A_1^2\beta_0\right)\right]^{-1}$$

for the subspace receiver.

2) When the SNR of the desired user A_1^2/η increases (as $\eta \to 0$), by (36), the asymptotic SINR achieved by the DMI blind receiver converges to α_M; and by (37), that achieved by the subspace blind receiver approaches $\frac{\alpha_M}{\alpha_K}$ (since by (38), $c_1 \to 0$ as $A_1^2/\eta \to \infty$), i.e.,

$$\mathrm{SINR}^{(\infty)}(\hat{\boldsymbol{w}}_1) \to \begin{cases} \alpha_M, & \text{DMI} \\ \frac{\alpha_M}{\alpha_K}, & \text{subspace} \end{cases} \quad \text{as } \frac{A_1^2}{\eta} \to \infty. \tag{42}$$

This is in stark contrast to the fact that the asymptotic SINR achieved by the exact linear MMSE receiver given by (41) can get arbitrarily large, as A_1^2/η increases. This observation reveals a "saturation" phenomenon of the SINR achieved by the blind receivers. That is, beyond certain input SNR, the estimation error dominates and hence determines the output SINR, and therefore, the performance gain obtained by increasing the user transmission power diminishes. This saturation phenomenon of SINR indicates that the capacity of a wireless system with blind multiuser receivers is not only *interference-limited*, but also *estimation-error limited*. Furthermore, the effect of estimation error can be quantified as shown earlier.

3) From (36) and (37), it is clear that in a large system, the relative performance of the DMI blind receiver and the subspace blind receiver hinges on the parameter c_1. That is, if $c_1 < 1$, then the subspace receiver outperforms the DMI receiver; otherwise, the DMI receiver outperforms the subspace receiver. Moreover, it is seen from (42) that in the high-SNR region, the asymptotic output SINR of the subspace blind receiver is higher than that of the DMI blind receiver by a factor $1/\alpha_K$ (recall that $\alpha_K \le 1$). Therefore, in the high-SNR region, the subspace blind receiver outperforms the DMI blind receiver. On the other hand, in the low-SNR region, e.g., as $A_1^2 \to 0$, then by (38), if $\eta < \frac{e_1}{e_2}$, then $c_1 < 1$, hence the subspace receiver performs better; and if $\eta > \frac{e_1}{e_2}$, then $c_1 > 1$, hence the DMI receiver performs better.

4) Heuristically, we can approximate the output SINR achieved by the blind linear MMSE receivers in a large CDMA system (with many users K and large processing gain N) as follows. For the DMI blind receiver

$$\overline{\mathrm{SINR}} \cong \frac{A_1^2\beta_0}{1 + \frac{N}{M}(1 + A_1^2\beta_0)} \tag{43}$$

and for the subspace blind receiver

$$\overline{\mathrm{SINR}} \cong \frac{A_1^2\beta_0}{1 + \frac{N}{M}\left[\frac{K}{N} + c_1\left(1 - \frac{K}{N}\right)\right](1 + A_1^2\beta_0)}. \tag{44}$$

Our next result gives the expression for the average output SINR of the group-blind linear hybrid receiver in the asymptotic regime (as $N \to \infty$). Define $\alpha_{\tilde{K}} \triangleq \frac{\tilde{K}}{N}$.

Theorem 3.2: Suppose Conditions 1 and 2 hold. Assume that α_M is large enough so that Proposition 2.2 holds. Then as $N \to \infty$, for almost every realization of signature sequences, the output SINR achieved by the group-blind linear hybrid receiver is well approximated by

$$\frac{A_1^2\beta_0}{1 + \frac{1}{\alpha_M}\left[(\alpha_K - \alpha_{\tilde{K}}) + c_1(1-\alpha_K)\right](1 + A_1^2\beta_0)}. \tag{45}$$

We have the following observations.

1) Comparing (41) with (45), it is seen that the performance loss in the asymptotic regime due to the estimation error inherent to the group-blind linear hybrid receiver is captured by the factor

$$\left[1 + \frac{1}{\alpha_M}\left[(\alpha_K - \alpha_{\tilde{K}}) + c_1(1-\alpha_K)\right]\left(1 + A_1^2\beta_0\right)\right]^{-1}.$$

2) Comparing (37) with (45), it is clear that the asymptotic SINR of the group-blind linear hybrid receiver is strictly higher than that of the subspace blind linear MMSE receiver, since $0 < \alpha_{\tilde{K}} \le \alpha_K$. On the other hand, comparing (36) with (45), it is seen that the group-blind linear hybrid receiver outperforms the DMI blind receiver in the asymptotic regime if and only if $c_1 < 1 + \frac{\tilde{\alpha}_K}{1-\alpha_K}$.

3) The asymptotic SINR achieved by the group-blind linear hybrid receiver saturates when the SNR of the desired user, A_1^2/η, increases, i.e.,

$$\mathrm{SINR}^{(\infty)}(\hat{\boldsymbol{w}}_1) \to \frac{\alpha_M}{\alpha_K - \alpha_{\tilde{K}}}, \qquad \text{as } \frac{A_1^2}{\eta} \to \infty. \tag{46}$$

Accordingly, the capacity of a wireless system with group-blind receivers is also estimation-error limited. Comparing (46) with (42) we see that in the high-SNR region, the group-blind receiver achieves the best performance, followed by the subspace blind receiver, which in turn outperforms the DMI blind receiver. On the other hand, in the low-SNR region, e.g., $A_1^2/\eta \to 0$, then by (38), if $\eta < \frac{e_1 \beta_0}{e_2}$, then $c_1 < 1 + \frac{\tilde{\alpha}_K}{1-\alpha_K}$, hence the group-blind receiver outperforms the DMI blind receiver; otherwise, the DMI receiver outperforms the group-blind receiver.

4) Heuristically, we can approximate the average output SINR achieved by the group-blind linear hybrid receiver in a large CDMA system as

$$\overline{\text{SINR}} \cong \frac{A_1^2 \beta_0}{1 + \frac{N}{M}\left[\frac{K-\tilde{K}}{N} + c_1\left(1 - \frac{K}{N}\right)\right]\left(1 + A_1^2 \beta_0\right)}. \tag{47}$$

B. Receiver Output Distribution and BER

Using (5), the output of an estimated linear receiver is given by the following generic form:

$$\begin{aligned} \hat{\boldsymbol{w}}_1^T \boldsymbol{r} &= \boldsymbol{w}_1^T \boldsymbol{SAb} + \boldsymbol{w}_1^T \boldsymbol{n} + \Delta \boldsymbol{w}_1^T \boldsymbol{SAb} + \Delta \boldsymbol{w}_1^T \boldsymbol{n} \\ &= \boldsymbol{w}_1^T \boldsymbol{s}_1 A_1 b_1 + \mathbb{I}_0^{(N)} + \mathbb{I}_1^{(N)} \end{aligned} \tag{48}$$

where $\mathbb{I}_0^{(N)} + \mathbb{I}_1^{(N)}$ is the overall residual interference, with

$$\mathbb{I}_0^{(N)} = \sum_{k=2}^{K} \boldsymbol{w}_1^T \boldsymbol{s}_k A_k b_k + \boldsymbol{w}_1^T \boldsymbol{n} \tag{49}$$

$$\mathbb{I}_1^{(N)} = \Delta \boldsymbol{w}_1^T \boldsymbol{SAb} + \Delta \boldsymbol{w}_1^T \boldsymbol{n}. \tag{50}$$

Note that the term $\mathbb{I}_0^{(N)}$ in the preceding expressions corresponds to the sum of the multiple-access interference (MAI) and noise components at the output of the *exact* linear receiver $\boldsymbol{w}_1$, and $\mathbb{I}_1^{(N)}$ represents the MAI enhancement and the noise enhancement due to the estimation error $\Delta \boldsymbol{w}_1$.

The BER is an important performance measure in communications, and is determined by the distribution of the overall residual interference consisting of the MAI, the estimation error noise, and the ambient noise. For CDMA systems with multiuser receivers, it is of interest to consider a model—the same random signature—is repeated from symbol to symbol (see, e.g., [7], [24], [31], [37]). Accordingly, the more interesting quantity in this case is the conditional distribution of the receiver output given the signatures [36], [37]. Based on (48), one would expect that the output interference distribution depends on the choices of the signatures. As revealed in the following theorem, however, the dependence on the signatures vanishes as the processing gain becomes large. In [37], it is shown that for almost every realization of the signature sequences, the conditional distribution of the residual interference at the output of an exact linear MMSE receiver (i.e., the term $\mathbb{I}_0^{(N)}$ in (48) when $\hat{\boldsymbol{w}}_1$ denotes the blind linear MMSE receiver) converges weakly to the same Gaussian distribution for large K and N, independent of signature sequences. In Section V, we provide a parallel study to establish the Gaussianity of the residual interference at the output of an exact group-blind linear hybrid receiver. To study the distribution of the estimated receiver output (48), it then suffices to characterize the distribution of the interference and noise enhancement term $\mathbb{I}_1^{(N)}$ in (48), and the correlation of $\mathbb{I}_0^{(N)}$ and $\mathbb{I}_1^{(N)}$. As shown in Section V, $\mathbb{I}_1$ is also asymptotically Gaussian, independent of the signature sequences. We note that the techniques required to establish the Gaussianity of $\mathbb{I}_1$ are different from those used in [37].

We have the following result on the distribution of the residual interference at the output of the blind linear MMSE receivers (cf. [37]).

Theorem 3.3: Suppose Conditions 1 and 2 hold. Then the conditional distribution of the overall interference at the output of the blind linear MMSE receiver $\mathbb{I}_0^{(N)} + \mathbb{I}_1^{(N)}$, given the signatures sequences, converges with probability one (as $N \to \infty$) to a Gaussian distribution with zero mean.

It is worth noting that the variances of the limiting Gaussian distributions corresponding to the DMI and subspace blind receivers are different (the variances will be presented in Section V).

Similar results hold for the group-blind linear hybrid receiver, as given by the following theorem.

Theorem 3.4: Suppose Conditions 1 and 2 hold. Then the conditional distribution of the overall interference at the output of the group-blind linear hybrid receiver $\mathbb{I}_0^{(N)} + \mathbb{I}_1^{(N)}$, given the signatures sequences, converges with probability one (as $N \to \infty$) to a Gaussian distribution with zero mean.

The proofs of Theorems 3.3 and 3.4 are given in Section V. In essence, we show that the estimation error of these linear receivers is asymptotically Gaussian and orthogonal to the signal subspace. It is interesting to note that in a large CDMA network with many users and large processing gain, the estimation error approximately lies in the subspace orthogonal to the signal space. (However, one must be cautioned that the components of the estimation error are correlated, which is evident from the expressions for $\boldsymbol{C}_w$ [cf. (116) and (139)].) We state this important fact in the following proposition.

Proposition 3.1: The estimation error of the blind linear MMSE receiver (DMI or subspace) $\Delta \boldsymbol{w}_1$ is asymptotically orthogonal to the spreading signature sequences, that is,

$$\Delta \boldsymbol{w}_1^T \boldsymbol{s}_k \xrightarrow{\text{a.s.}} 0, \qquad k = 1, \ldots, K. \tag{51}$$

This property also holds for the estimation error of the group-blind linear hybrid receiver.

The proof of Proposition 3.1 is relegated to Appendix B. We have the following remarks in order.

1) In light of Theorems 3.3 and 3.4, assuming antipodal modulation, we conclude that in a large CDMA system, where blind linear MMSE receivers or group-blind linear hybrid receivers are employed, the average BER can be well approximated by

$$\overline{P_e} \cong Q\left(\sqrt{\overline{\text{SINR}}}\right) \tag{52}$$

where $\overline{\text{SINR}}$ is the asymptotic average output SINR of the corresponding to a specific receiver, given by (43) or (44) or (47).

2) We also note that it is not difficult to generalize the preceding result, via the technique *conditional weak convergence* [23], [36], to establish the Gaussianity of the

residual interference at the output of blind linear multiuser receivers in systems where the signatures are complex-valued.

IV. Numerical Examples

In this section, we provide some numerical examples to illustrate the theoretical results outlined in the preceding section.

A. Asymptotic Average Output SINR

We consider the special case when the received powers of all users are the same, i.e., $A_1 = A_2 = \cdots = A_K \triangleq A$. In this case, β_0 in (34) can we written in closed form as (cf. [24])

$$\beta_0 = \frac{1-\alpha_K}{2\eta} - \frac{1}{2A^2} + \sqrt{\frac{(1-\alpha_K)^2}{4\eta^2} + \frac{1+\alpha_K}{2A^2\eta} + \frac{1}{4A^4}}. \quad (53)$$

It is shown in Appendix C that in this case the parameters e_1 and e_2 defined in (39) and (40) are given, respectively, by

$$e_1 = \frac{\alpha_K}{A^2(1-\alpha_K)} \quad (54)$$

$$e_2 = \frac{\alpha_K}{A^6(1-\alpha_K)^3}. \quad (55)$$

To illustrate the convergence of the random SINR to the asymptotic limit in the equal power case for different receivers, Fig. 1 compares the actually realized SINRs from randomly generated spreading sequences to the corresponding asymptotic limits for the three blind receivers. (For the purpose of comparison, we also plot the SINR corresponding to the exact MMSE receiver.) For different spreading lengths, and for each value of α_K, 200 samples of realized SINR for user 1 are obtained from randomly generated normalized ± 1 spreading sequences. We can see that for each receiver, as the processing gain increases, the spread around the asymptotic limit becomes narrower, a phenomenon also observed in [24] for the exact linear MMSE receiver. Moreover, it is seen that the group-blind linear hybrid receiver outperforms the subspace blind linear MMSE receiver, which outperforms the DMI blind linear MMSE receiver at the SNR value considered here.

B. The Output SINR Saturation Phenomenon

We next illustrate the SINR saturation phenomenon due to the estimation error for the DMI/subspace blind and group-blind receivers discussed in this paper. We consider a CDMA system with $N = 32$, $\alpha_M = 10$, $\alpha_K = 0.5$, and $\alpha_{\tilde{K}} = 0.25$ (when the group-blind linear hybrid receiver is employed). In Fig. 2, we plot the average output SINR for each of the three receivers (as well as the exact MMSE receiver) as a function of the input SNR (i.e., $\frac{A^2}{\eta}$), with randomly generated signature sequences, compared with the asymptotic limit. For each SNR value, 200 random realizations of $\boldsymbol{S}$ are generated. In each figure, we also plot the limit SINR value corresponding to $\frac{A^2}{\eta} \to \infty$ (as $\eta \to 0$), as the dashed line. The SINR saturation effect is clearly observed from Fig. 2. In contrast, the SINR corresponding to the exact MMSE receiver grows without bound, as is evident in the figure.

C. Orthogonality Between Estimation Error and Signal Space

Proposition 3.1 reveals that for the three linear receivers considered here, the estimation error $\Delta \boldsymbol{w}_1$ is asymptotically orthogonal to the spreading signatures. Note that (51) is equivalent to the following:

$$\mathbb{E}\left\{ \left\| \boldsymbol{s}_k^T \Delta \boldsymbol{w}_1 \right\|^2 \middle| \boldsymbol{S} \right\} = \boldsymbol{s}_k^T \boldsymbol{C}_w \boldsymbol{s}_k \xrightarrow{\text{a.s.}} \boldsymbol{0}, \qquad k = 1, \ldots, K, \text{ as } N \to \infty. \quad (56)$$

In Fig. 3, we plot the value of $\boldsymbol{s}_k^T \boldsymbol{C}_w \boldsymbol{s}_k$ as a function of N for the blind linear MMSE receiver (DMI or subspace) and the group-blind linear hybrid receiver, with randomly generated spreading sequences. For each value of N, 200 random realizations of $\boldsymbol{S}$ are generated. It is clear from these figures that the $\boldsymbol{s}_k^T \boldsymbol{C}_w \boldsymbol{s}_k$ decreases monotonically as N grows, corroborating (56).

V. Proofs of Main Results

A. Technical Preliminaries

Define

$$\boldsymbol{A}^2 = \mathrm{diag}\left\{A_1^2, A_2^2, \ldots, A_K^2\right\}$$
$$\boldsymbol{C}_{r,1} = \sum_{k=2}^{K} A_k^2 \boldsymbol{s}_k \boldsymbol{s}_k^T + \eta \boldsymbol{I}_N. \quad (57)$$

It is easy to see that the linear MMSE receiver given by (11) or (12) can also be written as

$$\boldsymbol{w}_1 = \frac{1}{1 + A_1^2 \boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} \boldsymbol{s}_1} \boldsymbol{C}_{r,1}^{-1} \boldsymbol{s}_1. \quad (58)$$

Thus,

$$\boldsymbol{w}_1^T \boldsymbol{s}_1 = \frac{1}{1 + A_1^2 \boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} \boldsymbol{s}_1} \boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} \boldsymbol{s}_1. \quad (59)$$

It has been shown in [24] that

$$\boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} \boldsymbol{s}_1 \xrightarrow{\text{a.s.}} \beta_0 \quad (60)$$

where β_0 is given in (34). It follows that

$$\boldsymbol{w}_1^T \boldsymbol{s}_1 \xrightarrow{\text{a.s.}} \frac{\beta_0}{1 + A_1^2 \beta_0}. \quad (61)$$

We need the following lemmas in proving our main results. The proofs of these lemmas are relegated to Appendix B.

Lemma 5.1: Suppose $X_n(\eta) \xrightarrow{\text{a.s.}} X(\eta)$ as $n \to \infty$. Assume that $\lim_{\eta \to 0} X_n(\eta)$ exists for any n and $\lim_{\eta \to 0} X(\eta)$ exists as well. Then

$$\lim_{\eta \to 0} X_n(\eta) \xrightarrow{\text{a.s.}} \lim_{\eta \to 0} X(\eta).$$

Our next lemma gives the asymptotic limit expression of $\boldsymbol{\Psi}_{11}^{-1}$ as $N \to \infty$, where $\boldsymbol{\Psi}_{11}$ is given in (27).

Lemma 5.2: As $N \to \infty$

$$\boldsymbol{\Psi}_{11}^{-1} \xrightarrow{\text{a.s.}} \mathrm{diag}\left\{ \frac{1 + A_1^2 \beta_0}{\beta_0}, \ldots, \frac{1 + A_{\tilde{K}}^2 \beta_0}{\beta_0} \right\}.$$

B. Proof of Theorem 3.1

The proof of Theorem 3.1 boils down to evaluating the interference power. In what follows, we calculate the interference

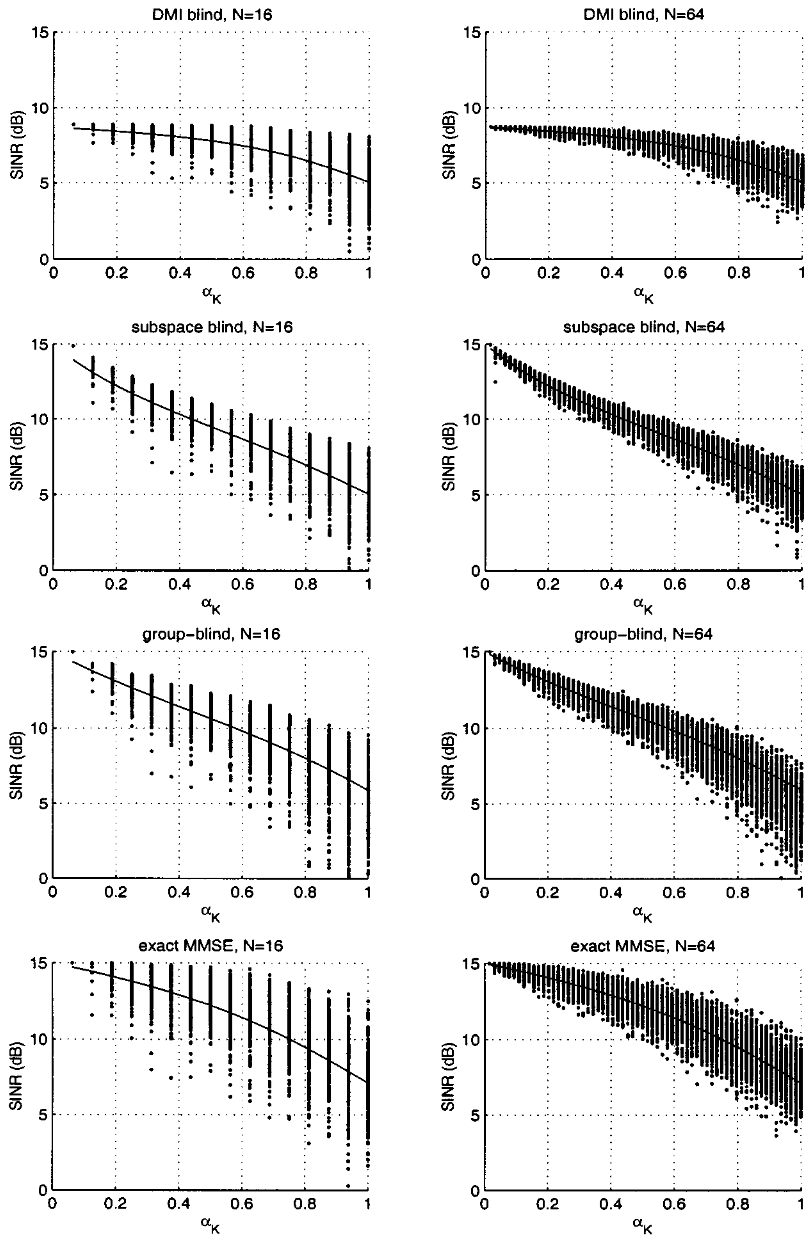


Fig. 1. The average output SINR of four linear receivers (DMI blind MMSE, subspace blind MMSE, group-blind hybrid receiver, and exact MMSE) with randomly generated signature sequences, compared with the asymptotic limits (shown as the dashed lines) in the equal-power regime, for $\alpha_M = 10$, $\frac{A^2}{\eta} = 15$ dB, with $N = 16$ and $N = 64$.

power (i.e., the denominator in (17)) term by term under the assumption of random signatures.

First, we have that

$$\mathcal{I}_0 = \boldsymbol{w}_1^T \left(\sum_{k=2}^{K} A_k^2 \boldsymbol{s}_k \boldsymbol{s}_k^T + \eta \boldsymbol{I}_N \right) \boldsymbol{w}_1 \quad \text{[by (18)]}$$
$$= \frac{1}{(1 + A_1^2 \boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} \boldsymbol{s}_1)^2} \boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} \boldsymbol{s}_1 \quad \text{[by (58)]}$$
$$\xrightarrow{\text{a.s.}} \frac{\beta_0}{(1 + A_1^2 \beta_0)^2}. \quad \text{[by (59)]} \tag{62}$$

Also, it is clear from (19) and (61) that

$$\mathcal{I}_1 = \frac{(K+1)\boldsymbol{w}_1^T \boldsymbol{s}_1}{N} \xrightarrow{\text{a.s.}} \frac{\alpha_K \beta_0}{1 + A_1^2 \beta_0}. \tag{63}$$

Next we evaluate $\mathcal{I}_2$ in (20). Using Condition 2, we assume that the sequence $\{A_k^2,\, k = 1, \ldots, K\}$ is bounded above and below by d_1 and d_2, respectively. Analogously to (61), we have

$$\boldsymbol{w}_k^T \boldsymbol{s}_k \xrightarrow{\text{a.s.}} \frac{\beta_0}{1 + A_K^2 \beta_0} \le \frac{1}{A_K^2} \le \frac{1}{d_2}. \tag{64}$$

Observe that

$$\sum_{k=1}^{K} A_k^4 \left(\boldsymbol{w}_1^T \boldsymbol{s}_k \right)^2$$
$$= \boldsymbol{w}_1^T \left(\sum_{k=1}^{K} A_k^4 \boldsymbol{s}_k \boldsymbol{s}_k^T \right) \boldsymbol{w}_1$$

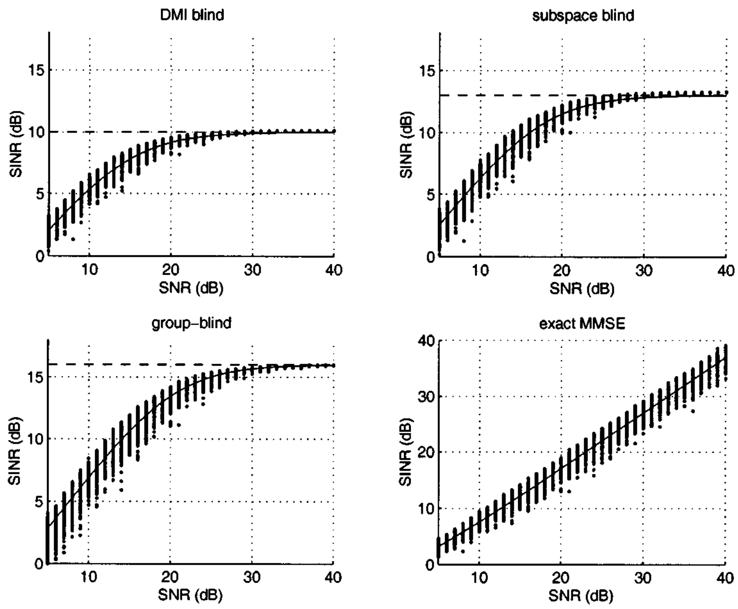
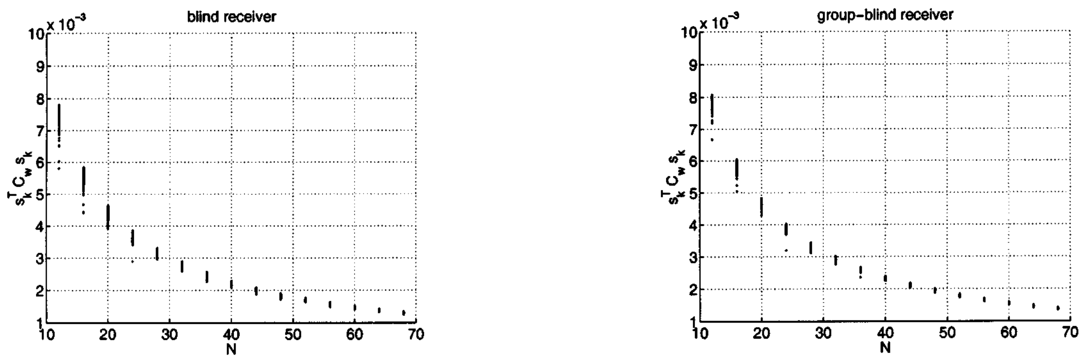


Fig. 2. The output SINR saturation effect of the blind linear receiver. $N = 32$, $\alpha_M = 10$, and $\alpha_K = 0.5$. The solid lines are the given by (36), (37), and (45), and the dashed lines are given by (42) and (46).



(a) (b)

Fig. 3. Illustration of the orthogonality between the estimation error $\Delta \boldsymbol{w}_1$ and the signal space $\boldsymbol{S}$ for the blind and group-blind linear receivers. (a) (DMI or subspace) blind MMSE receiver, $\alpha_M = 10$, $\alpha_K = 0.5$, and $\frac{A^2}{\eta} = 15$ dB. (b) Group-blind hybrid receiver, $\alpha_M = 10$, $\alpha_K = 0.5$, $\alpha_{\tilde{K}} = 0.25$, and $\frac{A^2}{\eta} = 15$ dB.

$$
\begin{aligned}
&= \frac{1}{\left(1 + A_1^2 \boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} \boldsymbol{s}_1\right)^2} \boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} \left(\sum_{k=1}^{K} A_k^4 \boldsymbol{s}_k \boldsymbol{s}_k^T \right) \\
&\quad \cdot \boldsymbol{C}_{r,1}^{-1} \boldsymbol{s}_1 \quad \text{[by (58)]} \\
&= \frac{1}{\left(1 + A_1^2 \boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} \boldsymbol{s}_1\right)^2} \\
&\quad \cdot \left[\left(A_1^2 \boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} \boldsymbol{s}_1 \right)^2 + \boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} \left(\sum_{k=2}^{K} A_k^4 \boldsymbol{s}_k \boldsymbol{s}_k^T \right) \boldsymbol{C}_{r,1}^{-1} \boldsymbol{s}_1 \right].
\end{aligned} \tag{65}
$$

By Condition 2, we use [9, p. 470] to obtain that[1]

$$
\begin{aligned}
&\boldsymbol{C}_{r,1}^{-1} \left(\sum_{k=2}^{K} A_k^4 \boldsymbol{s}_k \boldsymbol{s}_k^T \right) \boldsymbol{C}_{r,1}^{-1} \\
&\preceq d_1 \boldsymbol{C}_{r,1}^{-1} \left(\sum_{k=2}^{K} A_k^2 \boldsymbol{s}_k \boldsymbol{s}_k^T \right) \boldsymbol{C}_{r,1}^{-1} \\
&\preceq d_1 \boldsymbol{C}_{r,1}^{-1} \left(\sum_{k=2}^{K} A_k^2 \boldsymbol{s}_k \boldsymbol{s}_k^T + \eta \boldsymbol{I}_N \right) \boldsymbol{C}_{r,1}^{-1} \\
&\preceq d_1 \boldsymbol{C}_{r,1}^{-1}
\end{aligned} \tag{66}
$$

which implies that

$$
\begin{aligned}
\boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} \left(\sum_{k=2}^{K} A_k^4 \boldsymbol{s}_k \boldsymbol{s}_k^T \right) \boldsymbol{C}_{r,1}^{-1} \boldsymbol{s}_1 &\le d_1 \boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} \boldsymbol{s}_1 \\
&\xrightarrow{\text{a.s.}} d_1 \beta_0. \quad \text{[by (60)]}
\end{aligned} \tag{67}
$$

It then follows by combining (64), (65), and (67) that

$$
\frac{\sum_{k=1}^{K} A_k^4 \left(\boldsymbol{w}_1^T \boldsymbol{s}_k\right)^2 \left(\boldsymbol{w}_k^T \boldsymbol{s}_k\right)}{N} \xrightarrow{\text{a.s.}} 0 \tag{68}
$$

which indicates that

$$
\mathcal{I}_2 \xrightarrow{\text{a.s.}} 0. \tag{69}
$$

[1]By matrix inequality $\boldsymbol{A} \succeq \boldsymbol{B}$ we mean that $\boldsymbol{A} - \boldsymbol{B}$ is positive semidefinite.

In the following, we calculate the limits of $\tau\eta$ in (23). For the DMI blind receiver, we have by (61)

$$\tau\eta = \boldsymbol{w}_1^T \boldsymbol{s}_1 \xrightarrow{\text{a.s.}} \frac{\beta_0}{1 + A_1^2 \beta_0}. \quad (70)$$

For the subspace receiver, we know that

$$\begin{aligned}\frac{\tau}{\eta} &= \boldsymbol{s}_1^T \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} (\boldsymbol{\Lambda}_s - \eta \boldsymbol{I}_K)^{-2} \boldsymbol{U}_s^T \boldsymbol{s}_1 \\ &= \boldsymbol{s}_1^T \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} \boldsymbol{U}_s^T \boldsymbol{U}_s (\boldsymbol{\Lambda}_s - \eta \boldsymbol{I}_K)^{-2} \boldsymbol{U}_s^T \boldsymbol{s}_1. \quad (71)\end{aligned}$$

It is straightforward to see that

$$\begin{aligned}\boldsymbol{s}_1^T \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} \boldsymbol{U}_s^T &= \boldsymbol{s}_1^T \left(\boldsymbol{C}_r^{-1} - \frac{1}{\eta} \boldsymbol{U}_n \boldsymbol{U}_n^T \right) \\ &= \frac{\boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1}}{1 + A_1^2 \boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} \boldsymbol{s}_1}. \quad (72)\end{aligned}$$

For convenience, define

$$\boldsymbol{C}_{r,2} = \boldsymbol{C}_r - \eta \boldsymbol{U}_s \boldsymbol{U}_s^T \quad (73)$$

$$\boldsymbol{C}_{r,3} = \boldsymbol{C}_{r,2} - A_1^2 \boldsymbol{s}_1^T \boldsymbol{s}_1^T. \quad (74)$$

After some algebra, we have that

$$\begin{aligned}\boldsymbol{U}_s (\boldsymbol{\Lambda}_s - \eta \boldsymbol{I}_K)^{-2} \boldsymbol{U}_s^T \boldsymbol{s}_1 &= \left(\boldsymbol{C}_{r,2}^{-2} - \frac{1}{\eta^2} \boldsymbol{U}_n \boldsymbol{U}_n^T \right) \boldsymbol{s}_1 \quad (75) \\ &= \left(\boldsymbol{C}_{r,3} + A_1^2 \boldsymbol{s}_1 \boldsymbol{s}_1^T \right)^{-2} \boldsymbol{s}_1 \\ &= \frac{\boldsymbol{C}_{r,3}^{-2} \boldsymbol{s}_1^T}{\left(1 + A_1^2 \boldsymbol{s}_1^T \boldsymbol{C}_{r,3}^{-1} \boldsymbol{s}_1\right)^2}. \quad (76)\end{aligned}$$

Combining (71), (72), and (76) yields

$$\frac{\tau}{\eta} = \frac{\boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} \boldsymbol{C}_{r,3}^{-2} \boldsymbol{s}_1}{\left(1 + A_1^2 \boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} \boldsymbol{s}_1\right) \left(1 + A_1^2 \boldsymbol{s}_1^T \boldsymbol{C}_{r,3}^{-1} \boldsymbol{s}_1\right)^2}. \quad (77)$$

Note that both $\boldsymbol{C}_{r,1}$ and $\boldsymbol{C}_{r,3}$ are independent of $\boldsymbol{s}_1$. Next we show that

$$\boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} \boldsymbol{C}_{r,3}^{-2} \boldsymbol{s}_1 - \frac{1}{N} \operatorname{tr} \left(\boldsymbol{C}_{r,1}^{-1} \boldsymbol{C}_{r,3}^{-2} \right) \xrightarrow{\text{a.s.}} 0. \quad (78)$$

To this end, observe that

$$\boldsymbol{C}_{r,3} = \boldsymbol{C}_{r,1} - \eta \boldsymbol{U}_s \boldsymbol{U}_s^T.$$

For any $\epsilon > 0$, replace $\boldsymbol{C}_{r,3}$ in (78) by $\boldsymbol{C}_{r,3} + \epsilon \boldsymbol{I}_N$. Letting $\epsilon \to 0$, we can show, via a continuity argument, that

$$\boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} \boldsymbol{C}_{r,3}^{-2} \boldsymbol{s}_1 = \lim_{\epsilon \to 0} \boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} (\boldsymbol{C}_{r,3} + \epsilon \boldsymbol{I}_N)^{-2} \boldsymbol{s}_1. \quad (79)$$

It is clear that the eigenvalues of $\boldsymbol{C}_{r,1}^{-1} (\boldsymbol{C}_{r,3} + \epsilon \boldsymbol{I}_N)^{-2}$ are bounded above by $\frac{1}{\eta \epsilon^2}$, which indicates that for a given ϵ, the spectral radius of $\boldsymbol{C}_{r,1}^{-1} (\boldsymbol{C}_{r,3} + \epsilon \boldsymbol{I}_N)^{-2}$ is bounded above. Using Theorem A.2 in Appendix A and the Borel–Cantelli lemma, it follows that

$$\boldsymbol{s}_1^T \boldsymbol{C}_{r,1}^{-1} (\boldsymbol{C}_{r,3} + \epsilon \boldsymbol{I}_N)^{-2} \boldsymbol{s}_1 - \frac{1}{N} \operatorname{tr} \left(\boldsymbol{C}_{r,1}^{-1} (\boldsymbol{C}_{r,3} + \epsilon \boldsymbol{I}_N)^{-2} \right) \xrightarrow{\text{a.s.}} 0. \quad (80)$$

Appealing to [17, Theorem 7.11], we obtain the result in (78). Alternatively, we can use [1, Theorem 2] to show that the spectral radius of $\boldsymbol{C}_{r,1}^{-1} \boldsymbol{C}_{r,3}^{-2}$ is bounded above with probability one as N gets large. Without loss of generality, let $\{\lambda_k, k = 1, \ldots, K\}$ denote the largest K eigenvalues of $\boldsymbol{S}\boldsymbol{A}^2\boldsymbol{S}^T$. Observe that

$$\boldsymbol{S}\boldsymbol{A}^2\boldsymbol{S}^T \succeq d_2^2 \boldsymbol{S}\boldsymbol{S}^T \quad (81)$$

and that the smallest nonzero eigenvalue of $\boldsymbol{S}\boldsymbol{S}^T$ is equal to the smallest eigenvalue of $\boldsymbol{R} = \boldsymbol{S}^T \boldsymbol{S}$. Furthermore, by [1, Theorem 2], the smallest eigenvalues of $\boldsymbol{R}$ converge to $(1 - \sqrt{\alpha_K})^2$. It then follows that the minimum among $\{\lambda_k, k = 1, \ldots, K\}$ is bounded below with probability one as $N \to \infty$. (This observation will be used to obtain e_1 and e_2 defined later.)

Note that by [21], the eigenvalues of $\boldsymbol{C}_r$ and $\boldsymbol{C}_{r,1}$ are asymptotically the same as $N \to \infty$. Then it follows that

$$\begin{aligned}&\frac{1}{N} \operatorname{tr} \left(\boldsymbol{C}_{r,1}^{-1} \boldsymbol{C}_{r,3}^{-2} \right) - \frac{1}{N} \operatorname{tr} \left(\boldsymbol{C}_r^{-1} \boldsymbol{C}_{r,2}^{-2} \right) \\ &\quad = \frac{1}{N} \operatorname{tr} \left(\boldsymbol{C}_{r,1}^{-1} \boldsymbol{C}_{r,3}^{-2} \right) - \frac{1}{N} \sum_{i=1}^{K} \frac{1}{\lambda_i + \eta} \frac{1}{\lambda_i^2} \\ &\quad - \frac{N-K}{N} \frac{1}{\eta^3} \xrightarrow{\text{a.s.}} 0. \quad (82)\end{aligned}$$

Also note that

$$\frac{1}{\lambda_i + \eta} \frac{1}{\lambda_i^2} = \frac{1}{\eta^2} \left(\frac{1}{\lambda_i + \eta} - \frac{1}{\lambda_i} + \frac{\eta}{\lambda_i^2} \right) \quad (83)$$

and $\lambda_k = 0$ for $k > K$. Thus, it follows that

$$\begin{aligned}\frac{1}{N} \sum_{i=1}^{K} \frac{1}{\lambda_i + \eta} &= \frac{1}{N} \sum_{i=1}^{N} \frac{1}{\lambda_i + \eta} - \frac{N-K}{N} \frac{1}{\eta} \\ &\xrightarrow{\text{a.s.}} \int_0^\infty \frac{1}{\lambda + \eta} \, \mathrm{d}G^*(\lambda) - \frac{1 - \alpha_K}{\eta} \quad (84) \\ &= \beta_0(\eta) - \frac{1 - \alpha_K}{\eta} \quad (85)\end{aligned}$$

where (84) follows from [21] and the bounded convergence theorem.

Recall that

$$e_1 = \lim_{\eta \to 0} \left(\beta_0(\eta) - \frac{1 - \alpha_K}{\eta} \right) \quad (86)$$

$$e_2 = \lim_{\eta \to 0} - \frac{\mathrm{d} \left(\beta_0(\eta) - \frac{1 - \alpha_K}{\eta} \right)}{\mathrm{d}\eta}. \quad (87)$$

(We note that as $\eta \to 0$, e_1 exists while $\beta_0(\eta) \to \infty$.) As shown above, the minimum among $\{\lambda_k, k = 1, \ldots, K\}$ is bounded below with probability one as $N \to \infty$, (hence the maximum among $\{1/\lambda_k, k = 1, \ldots, K\}$ is bounded above with probability one), therefore, $\frac{1}{N} \sum_{i=1}^{K} 1/\lambda_i$ converges with probability one. We use Lemma 5.1 to conclude that

$$\frac{1}{N} \sum_{i=1}^{K} \frac{1}{\lambda_i} \xrightarrow{\text{a.s.}} e_1. \quad (88)$$

Observe that

$$\begin{aligned}\frac{1}{N} \sum_{i=1}^{K} \frac{1}{\lambda_i^2} &= \lim_{\eta \to 0} \sum_{i=1}^{K} \frac{1}{(\lambda_i + \eta)^2} \\ &= - \lim_{\eta \to 0} \frac{\mathrm{d}}{\mathrm{d}\eta} \left(\sum_{i=1}^{K} \frac{1}{\lambda_i + \eta} \right) \\ &= - \lim_{\eta \to 0} \frac{\mathrm{d}}{\mathrm{d}\eta} \left(\sum_{i=1}^{N} \frac{1}{\lambda_i + \eta} - \frac{1 - \alpha_K}{\eta} \right). \quad (89)\end{aligned}$$

Using [17, Theorem 7.11] and Lemma 5.1, we conclude that

$$\frac{1}{N}\sum_{i=1}^{K}\frac{1}{\lambda_i^2} \xrightarrow{\text{a.s.}} e_2. \tag{90}$$

Combining (77), (78), (82), (83), (85), (88), and (90), we obtain that

$$\frac{\tau}{\eta} \xrightarrow{\text{a.s.}} \frac{\beta_0 - e_1 + \eta e_2}{\eta^2(1+A_1^2\beta_0)\left[1+A_1^2(e_1+(1-\alpha_K)/\eta)\right]^2}. \tag{91}$$

It then follows immediately that for the subspace blind MMSE receiver

$$\tau\eta \xrightarrow{\text{a.s.}} \frac{\beta_0 - e_1 + \eta e_2}{(1+A_1^2\beta_0)\left[1+A_1^2(e_1+(1-\alpha_K)/\eta)\right]^2} \tag{92}$$

which implies that

$$\mathcal{I}_3 \xrightarrow{\text{a.s.}} (1-\alpha_K)\frac{\beta_0 - e_1 + \eta e_2}{(1+A_1^2\beta_0)\left[1+A_1^2(e_1+(1-\alpha_K)/\eta)\right]^2}. \tag{93}$$

Combining (62), (63), (69), (70), and (93), we have that for the DMI MMSE receiver

$$\text{SINR}^{(N)}(\hat{\boldsymbol{w}}_1) \xrightarrow{\text{a.s.}} \frac{A_1^2\beta_0}{1+\frac{1}{\alpha_M}(1+A_1^2\beta_0)} \tag{94}$$

and for the subspace blind MMSE receiver

$$\text{SINR}^{(N)}(\hat{\boldsymbol{w}}_1) \xrightarrow{\text{a.s.}} \frac{A_1^2\beta_0}{1+\frac{1}{\alpha_M}\left[\alpha_K + c_1(1-\alpha_K)\right](1+A_1^2\beta_0)} \tag{95}$$

where

$$c_1 = \frac{\beta_0 - e_1 + \eta e_2}{\beta_0\left[1+A_1^2(e_1+(1-\alpha_K)/\eta)\right]^2}.$$

This concludes the proof of Theorem 3.1. □

C. *Proof of Theorem 3.2*

The proof of Theorem 3.2 again boils down to evaluating the interference terms. Define $\tilde{\boldsymbol{P}} = \text{diag}\{A_1^2, \ldots, A_{\tilde{K}}^2\}$. Note that by (27)

$$\boldsymbol{\Psi}_{11} = \tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}. \tag{96}$$

By (24), the group-blind linear hybrid receiver can also be written as

$$\boldsymbol{w}_1 = \boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{e}}_1. \tag{97}$$

It follows that

$$\begin{aligned}\mathcal{I}_0 &= \boldsymbol{w}_1^T\left(\sum_{k=1}^{K-\tilde{K}} A_{\tilde{K}+k}^2 s_{\tilde{K}+k}s_{\tilde{K}+k}^T + \eta\boldsymbol{I}_N\right)\boldsymbol{w}_1 \quad \text{[by (30)]}\\ &= \boldsymbol{w}_1^T\left(\boldsymbol{C}_r - \sum_{k=1}^{\tilde{K}}A_k^2\boldsymbol{s}_k\boldsymbol{s}_k^T\right)\boldsymbol{w}_1\\ &= \tilde{\boldsymbol{e}}_1^T\left[\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}\boldsymbol{C}_r^{-1}\left(\boldsymbol{C}_r - \tilde{\boldsymbol{S}}\tilde{\boldsymbol{P}}\tilde{\boldsymbol{S}}^T\right)\right.\\ &\qquad \left.\cdot\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\right]\tilde{\boldsymbol{e}}_1 \quad \text{[by (97)]}\\ &= \tilde{\boldsymbol{e}}_1^T\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{e}}_1 - \tilde{\boldsymbol{e}}_1^T\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\tilde{\boldsymbol{P}}\tilde{\boldsymbol{e}}_1\\ &= \left[\boldsymbol{\Psi}_{11}^{-1}\right]_{1,1} - A_1^2 \quad \text{[by (96)]}.\end{aligned} \tag{98}$$

A direct application of Lemma 5.2 allows us to conclude that

$$\mathcal{I}_0 \xrightarrow{\text{a.s.}} \frac{1+A_1^2\beta_0}{\beta_0} - A_1^2 = \frac{1}{\beta_0} \tag{99}$$

and by (31)

$$\mathcal{I}_1 \xrightarrow{\text{a.s.}} (\alpha_K - \alpha_{\tilde{K}})\frac{1+A_1^2\beta_0}{\beta_0}. \tag{100}$$

Using the same techniques as in proving Lemma 5.2, we can show that

$$\left[\boldsymbol{\Psi}_{22} - \boldsymbol{\Psi}_{12}^T\boldsymbol{\Psi}_{11}^{-1}\boldsymbol{\Psi}_{12}\right]_{k,k} \xrightarrow{\text{a.s.}} \frac{\beta_0}{1+A_K^2\beta_0} \le \frac{1}{A_K^2} \le \frac{1}{d_2}. \tag{101}$$

Furthermore, along the same line as in showing (65), (67), and (68), we have that

$$\frac{\sum_{k=1}^{K-\tilde{K}} A_{\tilde{K}+k}^4\left[\boldsymbol{\Psi}_{12}^T\boldsymbol{\Psi}_{11}^{-1}\right]_{k,1}^2}{N} \xrightarrow{\text{a.s.}} 0. \tag{102}$$

Substituting (101) and (102) into (32) yields

$$\mathcal{I}_2 \xrightarrow{\text{a.s.}} 0. \tag{103}$$

In what follows, we proceed to calculate the quantity $\mathcal{I}_3$. Denote

$$\boldsymbol{v}_1 = \tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{e}}_1. \tag{104}$$

After some algebra, we can show that [11]

$$\mathcal{I}_3 = \tau\eta(1-\alpha_K) \tag{105}$$

where

$$\begin{aligned}\tau &= \eta\boldsymbol{v}_1^T\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}(\boldsymbol{\Lambda}_s - \eta\boldsymbol{I}_K)^{-2}\boldsymbol{U}_s^T\boldsymbol{v}_1\\ &= \eta\tilde{\boldsymbol{e}}_1^T\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}^T\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T\boldsymbol{U}_s\\ &\qquad\cdot(\boldsymbol{\Lambda}_s - \eta\boldsymbol{I}_K)^{-2}\boldsymbol{U}_s^T\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{e}}_1.\end{aligned} \tag{106}$$

By (96)

$$\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}^T\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T = \boldsymbol{\Psi}_{11}^{-1}\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1} \tag{107}$$

and similar to (75), we have

$$\boldsymbol{U}_s(\boldsymbol{\Lambda}_s - \eta\boldsymbol{I}_K)^{-2}\boldsymbol{U}_s^T\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1} = \boldsymbol{C}_{r,2}^{-2}\tilde{\boldsymbol{S}}\boldsymbol{\Psi}_{11}^{-1}. \tag{108}$$

It then follows that

$$\tau = \eta\tilde{\boldsymbol{e}}_1^T\boldsymbol{\Psi}_{11}^{-1}\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\boldsymbol{C}_{r,2}^{-2}\tilde{\boldsymbol{S}}\boldsymbol{\Psi}_{11}^{-1}\tilde{\boldsymbol{e}}_1. \tag{109}$$

Using the same techniques as in showing (78), (82), (92), and proving Lemma 5.2, we conclude that for $i = 1, \ldots, \tilde{K}$

$$\boldsymbol{s}_i^T\boldsymbol{C}_r^{-1}\boldsymbol{C}_{r,2}^{-2}\boldsymbol{s}_i \xrightarrow{\text{a.s.}} \frac{\beta_0 - e_1 + \eta e_2}{\eta^2(1+A_i^2\beta_0)\left[1+A_i^2(e_1+(1-\alpha_K)/\eta)\right]^2} \tag{110}$$

and $i \neq j$, $i, j \in \{1, \ldots, \tilde{K}\}$

$$\boldsymbol{s}_i^T \boldsymbol{C}_r^{-1} \boldsymbol{C}_{r,2}^{-2} \boldsymbol{s}_j \xrightarrow{\text{a.s.}} 0. \tag{111}$$

Then we have that

$$\tilde{\boldsymbol{S}}^T \boldsymbol{C}_r^{-1} \boldsymbol{C}_{r,2}^{-2} \tilde{\boldsymbol{S}} \xrightarrow{\text{a.s.}} \text{diag}\left(\frac{\beta_0 - e_1 + \eta e_2}{\eta^2 (1 + A_1^2 \beta_0)[1 + A_1^2 (e_1 + (1 - \alpha_K)/\eta)]^2}, \ldots, \frac{\beta_0 - e_1 + \eta e_2}{\eta^2 (1 + A_{\tilde{K}}^2 \beta_0) \left[1 + A_{\tilde{K}}^2 (e_1 + (1 - \alpha_K)/\eta)\right]^2} \right). \tag{112}$$

By using (112) and Lemma 5.2 in (109), we have

$$\tau\eta \xrightarrow{\text{a.s.}} \left(\frac{1 + A_1^2 \beta_0}{\beta_0} \right)^2 \cdot \frac{\beta_0 - e_1 + \eta e_2}{(1 + A_1^2 \beta_0)[1 + A_1^2 (e_1 + (1 - \alpha_K)/\eta)]^2}. \tag{113}$$

Combining (99), (100), (103), and (113), we conclude that the asymptotic output SINR achieved by the group-blind linear hybrid receiver is given by

$$\text{SINR}^{(N)}(\hat{\boldsymbol{w}}_1) \xrightarrow{\text{a.s.}} \frac{A_1^2 \beta_0}{1 + \frac{1}{\alpha_M} \left[(\alpha_K - \alpha_{\tilde{K}}) + c_1 (1 - \alpha_K) \right] (1 + A_1^2 \beta_0)} \tag{114}$$

where c_1 is given by (38). This concludes the proof of Theorem 3.2. □

D. Proof of Theorem 3.3

We prove Theorem 3.3 by the following three-step procedure.

1) For almost every realization of $\boldsymbol{S}$, $\mathbb{I}_0^{(N)}$ converges to a Gaussian distribution as $N \to \infty$.

2) For almost every realization of $\boldsymbol{S}$, $\mathbb{I}_1^{(N)}$ converges to a Gaussian distribution as $N \to \infty$.

3) For almost every realization of $\boldsymbol{S}$, the covariance of $\mathbb{I}_0^{(N)}$ and $\mathbb{I}_1^{(N)}$ vanishes as $N \to \infty$.

Step 1: A direct application of [37, Theorem 3.2] shows that $\mathbb{I}_0^{(N)}$ in (49) converges to a Gaussian distribution for almost every realization of $\boldsymbol{S}$.

Step 2: The main portion in the interference enhancement due to the estimation error $\mathbb{I}_1^{(N)}$ in (50) is $(\Delta \boldsymbol{w}_1^T \boldsymbol{SAb})$. In the following, we show that for almost realization of $\boldsymbol{S}$, $(\Delta \boldsymbol{w}_1^T \boldsymbol{SAb})$ has a limiting Gaussian distribution. Observe that for fixed $\boldsymbol{b}$ and $\boldsymbol{S}$, $(\Delta \boldsymbol{w}_1^T \boldsymbol{SAb})$ is Gaussian. Then, it suffices to show that for almost every $\boldsymbol{b}$ and $\boldsymbol{S}$, the variance of the corresponding $(\Delta \boldsymbol{w}_1^T \boldsymbol{SAb})$ is the same as $N \to \infty$. To this end, note that for fixed $\boldsymbol{b}$ and $\boldsymbol{S}$

$$\mathbb{E}\left\{ \left(\Delta \boldsymbol{w}_1^T \boldsymbol{SAb} \right)^2 \right\} = \boldsymbol{b}^T \boldsymbol{AS}^T \boldsymbol{C}_w \boldsymbol{SAb}. \tag{115}$$

We next show that the spectral radius of the matrix $(\boldsymbol{AS}^T \boldsymbol{C}_w \boldsymbol{SA})$ is bounded with probability one. Recall that the covariance of the estimation error $\Delta \boldsymbol{w}_1$ for the blind linear MMSE receiver is given by [11], [12]

$$\boldsymbol{C}_w = \frac{1}{M} \left[(\boldsymbol{w}_1^T \boldsymbol{s}_1) \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} \boldsymbol{U}_s^T + \boldsymbol{w}_1 \boldsymbol{w}_1^T - 2 \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} \boldsymbol{U}_s^T \boldsymbol{SDS}^T \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} \boldsymbol{U}_s^T + \tau \boldsymbol{U}_n \boldsymbol{U}_n^T \right] \tag{116}$$

where

$$\boldsymbol{D} \triangleq \text{diag}\left\{ A_1^4 \left(\boldsymbol{w}_1^T \boldsymbol{s}_1 \right)^2, A_2^4 \left(\boldsymbol{w}_1^T \boldsymbol{s}_2 \right)^2, \ldots, A_K^4 \left(\boldsymbol{w}_1^T \boldsymbol{s}_K \right)^2 \right\} \tag{117}$$

$$\tau \triangleq \begin{cases} \frac{1}{\eta} \boldsymbol{s}_1^T \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} \boldsymbol{U}_s^T \boldsymbol{s}_1, & \text{DMI} \\ \eta \boldsymbol{s}_1^T \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} (\boldsymbol{\Lambda}_s - \eta \boldsymbol{I}_K)^{-2} \boldsymbol{U}_s^T \boldsymbol{s}_1, & \text{subspace.} \end{cases} \tag{118}$$

It is straightforward to see that

$$\boldsymbol{S}^T \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} \boldsymbol{U}_s^T \boldsymbol{S} = \boldsymbol{R}(\boldsymbol{A}^2 \boldsymbol{R} + \eta \boldsymbol{I}_K)^{-1}. \tag{119}$$

Then it follows that

$$\boldsymbol{AS}^T \boldsymbol{C}_w \boldsymbol{SA} = \frac{1}{M} \left[(\boldsymbol{w}_1^T \boldsymbol{s}_1) \boldsymbol{AR}(\boldsymbol{A}^2 \boldsymbol{R} + \eta \boldsymbol{I}_K)^{-1} \boldsymbol{A} + \boldsymbol{AS}^T \boldsymbol{w}_1 \boldsymbol{w}_1^T \boldsymbol{SA} - 2 \boldsymbol{AR}(\boldsymbol{A}^2 \boldsymbol{R} + \eta \boldsymbol{I}_K)^{-1} \cdot \boldsymbol{DR}(\boldsymbol{A}^2 \boldsymbol{R} + \eta \boldsymbol{I}_K)^{-1} \boldsymbol{A} \right]. \tag{120}$$

By Condition 2, we use [9, p. 470] again and have that

$$\boldsymbol{AR}(\boldsymbol{A}^2 \boldsymbol{R} + \eta \boldsymbol{I}_K)^{-1} \boldsymbol{A} = \boldsymbol{A}(\boldsymbol{A}^2 + \eta \boldsymbol{R}^{-1})^{-1} \boldsymbol{A} \preceq \boldsymbol{A}(\boldsymbol{A}^2)^{-1} \boldsymbol{A} = \boldsymbol{I}_K. \tag{121}$$

Note that

$$\boldsymbol{w}_1^T \boldsymbol{s}_1 = \boldsymbol{s}_1^T \boldsymbol{C}_r^{-1} \boldsymbol{s}_1 \leq \frac{1}{\eta} \boldsymbol{s}_1^T \boldsymbol{s}_1 \leq \frac{1}{\eta}.$$

It follows that the spectral radius of the matrix

$$(\boldsymbol{w}_1^T \boldsymbol{s}_1) \boldsymbol{AR}(\boldsymbol{A}^2 \boldsymbol{R} + \eta \boldsymbol{I}_K)^{-1} \boldsymbol{A}$$

is bounded in N, with probability one. Because $|\boldsymbol{w}_1^T \boldsymbol{s}_k| \leq 1/\eta$, for $i = 1, \ldots, K$, it is clear that

$$\boldsymbol{D} \preceq \frac{1}{\eta^2} \text{diag}\left\{ A_1^4, \ldots, A_K^4 \right\} \preceq \frac{d_1^2}{\eta^2} \boldsymbol{A}^2 \tag{122}$$

where the last step follows from Condition 1. Then we have that

$$\boldsymbol{AR}(\boldsymbol{A}^2 \boldsymbol{R} + \eta \boldsymbol{I}_K)^{-1} \boldsymbol{DR}(\boldsymbol{A}^2 \boldsymbol{R} + \eta \boldsymbol{I}_K)^{-1} \boldsymbol{A} \preceq \frac{d_1^2}{\eta^2} \left[\boldsymbol{A}(\boldsymbol{A}^2 + \eta \boldsymbol{R}^{-1})^{-1} \boldsymbol{A} \right] \left[\boldsymbol{A}(\boldsymbol{A}^2 + \eta \boldsymbol{R}^{-1})^{-1} \boldsymbol{A} \right]^T. \tag{123}$$

As shown in (121), the spectral radius of the matrix

$$\boldsymbol{A}(\boldsymbol{A}^2 + \eta \boldsymbol{R}^{-1})^{-1} \boldsymbol{A}$$

is bounded above, it follows that the spectral radius of

$$\boldsymbol{AR}(\boldsymbol{A}^2 \boldsymbol{R} + \eta \boldsymbol{I}_K)^{-1} \boldsymbol{DR}(\boldsymbol{A}^2 \boldsymbol{R} + \eta \boldsymbol{I}_K)^{-1} \boldsymbol{A}$$

is also bounded above. Note that

$$\boldsymbol{AS}^T \boldsymbol{w}_1 \boldsymbol{w}_1^T \boldsymbol{SA}$$

is a rank-one nonnegative matrix, and

$$\text{tr}\left(\boldsymbol{AS}^T \boldsymbol{w}_1 \boldsymbol{w}_1^T \boldsymbol{SA} \right) = \boldsymbol{s}_1^T \boldsymbol{C}_r^{-1} \boldsymbol{s}_1 \leq \frac{1}{\eta}$$

which indicates that the spectral radius of $(\boldsymbol{AS}^T\boldsymbol{w}_1\boldsymbol{w}_1^T\boldsymbol{SA})$ is bounded above. Using Weyl's theorem [9, p. 181], we have that the spectral radius of the matrix $(\boldsymbol{AS}^T\boldsymbol{C}_w\boldsymbol{SA})$ is bounded with probability one. Therefore, by combining Theorem A.2 in Appendix A and the Borel–Cantelli lemma, we conclude that

$$\boldsymbol{b}^T\boldsymbol{AS}^T\boldsymbol{C}_w\boldsymbol{SAb} - \mathrm{tr}\left(\boldsymbol{AS}^T\boldsymbol{C}_w\boldsymbol{SA}\right) \xrightarrow{\text{a.s.}} 0. \tag{124}$$

Observe that

$$\begin{aligned}\mathrm{tr}\left(\boldsymbol{AS}^T\boldsymbol{C}_w\boldsymbol{SA}\right) &= \mathrm{tr}\left([\boldsymbol{C}_r - \eta\boldsymbol{I}_N]\boldsymbol{C}_w\right)\\ &= \frac{N}{M}\left(\mathcal{I}_1 + \mathcal{I}_2 + \mathcal{I}_3\right) - \eta\,\mathrm{tr}(\boldsymbol{C}_w).\end{aligned} \tag{125}$$

It is straightforward to see that $\mathrm{tr}\,(\boldsymbol{C}_w)$ converges to the same limit for almost every $\boldsymbol{S}$. Using the results on $\mathcal{I}_1$, $\mathcal{I}_2$, and $\mathcal{I}_3$, we conclude that $\mathrm{tr}\,(\boldsymbol{AS}^T\boldsymbol{C}_w\boldsymbol{SA})$ converges to the same limit for almost every $\boldsymbol{S}$ (the limits in the DMI and subspace cases are different). Thus,

$$\mathrm{var}(\Delta\boldsymbol{w}_1^T\boldsymbol{SAb}\,|\,\boldsymbol{S}) = \mathbb{E}\left\{\boldsymbol{b}^T\boldsymbol{AS}^T\boldsymbol{C}_w\boldsymbol{SAb}\,|\,\boldsymbol{S}\right\}$$

converges with probability one, where the expectation is taken with respect to $\boldsymbol{b}$.

Observe that $\Delta\boldsymbol{w}_1^T\boldsymbol{n}$ is the inner product of two Gaussian vectors. As the processing gain N gets large, by the central limit theorem, this term also approaches Gaussian distribution. Therefore, we conclude that for almost every $\boldsymbol{S}$, $\mathbb{I}_1^{(N)}$ converges to a Gaussian distribution as $N\to\infty$; for the DMI MMSE receiver, the variance of $\mathbb{I}_1^{(N)}$

$$\mathrm{var}\left(\mathbb{I}_1^{(N)}\right) = \mathrm{tr}\left(\boldsymbol{C}_r\boldsymbol{C}_w\right) \xrightarrow{\text{a.s.}} \frac{1}{\alpha_M}\,\frac{\beta_0}{1+A_1^2\beta_0} \tag{126}$$

and for the subspace blind MMSE receiver, the variance of $\mathbb{I}_1^{(N)}$

$$\begin{aligned}\mathrm{var}\left(\mathbb{I}_1^{(N)}\right) =&\ \mathrm{tr}\left(\boldsymbol{C}_r\boldsymbol{C}_w\right)\\ &\xrightarrow{\text{a.s.}} \frac{1}{\alpha_M}\left[\alpha_K + c_1(1-\alpha_K)\right]\frac{\beta_0}{1+A_1^2\beta_0}.\end{aligned} \tag{127}$$

Step 3: We proceed to show that the covariance of $\mathbb{I}_0^{(N)}$ and $\mathbb{I}_1^{(N)}$ vanishes as N gets large for almost every $\boldsymbol{S}$. The covariance of $\mathbb{I}_0^{(N)}$ and $\mathbb{I}_1^{(N)}$ is

$$\begin{aligned}&\mathbb{E}\left\{\mathbb{I}_0^{(N)}\mathbb{I}_1^{(N)}\middle|\,\boldsymbol{S}\right\}\\ &= \mathbb{E}\left\{\left(\sum_{k=2}^{K}\boldsymbol{w}_1^T\boldsymbol{s}_kA_kb_k + \boldsymbol{w}_1^T\boldsymbol{n}\right)\left(\Delta\boldsymbol{w}_1^T\boldsymbol{SAb} + \Delta\boldsymbol{w}_1^T\right)\middle|\,\boldsymbol{S}\right\}\\ &= \boldsymbol{w}_1^T\sum_{k=2}^{K}A_k^2\boldsymbol{s}_k\boldsymbol{s}_k^T\Delta\boldsymbol{w}_1^T + \eta\boldsymbol{w}_1^T\Delta\boldsymbol{w}_1\\ &= \boldsymbol{s}_1^T\boldsymbol{C}_r^{-1}\left(\boldsymbol{C}_r - A_1^2\boldsymbol{s}_1\boldsymbol{s}_1^T\right)\Delta\boldsymbol{w}_1\\ &= \left(1 - A_1^2\boldsymbol{s}_1^T\boldsymbol{C}_r^{-1}\boldsymbol{s}_1\right)\boldsymbol{s}_1^T\Delta\boldsymbol{w}_1.\end{aligned} \tag{128}$$

Then it suffices to show that $\boldsymbol{s}_1^T\Delta\boldsymbol{w}_1 \xrightarrow{\text{a.s.}} 0$, which follows directly from Proposition 3.1.

Built on Steps 1–3, a simple application of Lemma B.2 in Appendix B completes the proof of Theorem 3.3. □

E. Proof of Theorem 3.4

Recall that the exact group-blind linear hybrid receiver is

$$\boldsymbol{w}_1 = \boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{e}}_1. \tag{129}$$

By definition, $\boldsymbol{w}_1^T\boldsymbol{s}_k = 0$ for $k = 2, \ldots, \tilde{K}$. Then, the output residual interference of the estimated group-blind receiver is given by $\mathbb{I}_0^{(N)} + \mathbb{I}_1^{(N)}$, with

$$\begin{aligned}\mathbb{I}_0^{(N)} &= \sum_{k=2}^{K}(\boldsymbol{w}_1^T\boldsymbol{s}_k)A_kb_k + \boldsymbol{w}_1^T\boldsymbol{n}\\ &= \sum_{k=1}^{K-\tilde{K}}(\boldsymbol{w}_1^T\boldsymbol{s}_{k+\tilde{K}})A_{k+\tilde{K}}b_{k+\tilde{K}} + \boldsymbol{w}_1^T\boldsymbol{n}\end{aligned} \tag{130}$$

$$\mathbb{I}_1^{(N)} = \Delta\boldsymbol{w}_1^T\boldsymbol{SAb} + \Delta\boldsymbol{w}_1^T\boldsymbol{n}. \tag{131}$$

We now prove Theorem 3.4, using the same three-step approach as in the proof of Theorem 3.3.

Step 1: The proof of Step 1 hinges on the use of dependent central limit theorem for martingale difference arrays [15], and follows essentially the same line as that of [37, Theorem 3.2]. We outline the necessary details as follows. Define

$$t_k^{(N)} \triangleq (\boldsymbol{w}_1^T\boldsymbol{s}_{k+\tilde{K}})A_{k+\tilde{K}}b_{k+\tilde{K}}, \qquad k = 1, \ldots, K-\tilde{K}. \tag{132}$$

It is clear that the array $\{t_k^{(N)}, k = 1, \ldots, K-\tilde{K}\}$ is a martingale difference array with respect to the σ-algebra $\sigma(t_1^{(N)}, \ldots, t_k^{(N)})$ [15]. Using the central limit theorem in [15], it suffices to show that for almost every realization of $\boldsymbol{S}$, the following three conditions hold:

a) $\max_{1\le k\le K-\tilde{K}}|t_k^{(N)}|$ is bounded in L_2 norm;

b) $\max_{1\le k\le K-\tilde{K}}|t_k^{(N)}|$ converges to 0 in probability as $N\to\infty$;

c) $\sum_{k=1}^{K-\tilde{K}}(t_k^{(N)})^2$ converges to the same limit in probability as $N\to\infty$.

To this end, it is straightforward to see that condition a) holds because $\mathbb{E}\{(\boldsymbol{w}_1^T\boldsymbol{s}_{k+\tilde{K}})^2\}$ is bounded. We proceed to establish condition b). Using [37, Lemma 4.3], we have that

$$\mathbb{E}\left[\left(\boldsymbol{s}_{k+\tilde{K}}^T\boldsymbol{C}_r^{-1}\boldsymbol{s}_j\right)^6\right] \le \frac{225}{N^3\eta^6},$$
$$k = 1, \ldots, K-\tilde{K},\ j = 1, \ldots, \tilde{K}. \tag{133}$$

Along the lines of proving [37, Lemma 5.1], we combine Markov's inequality and the Borel–Cantelli lemma to conclude that

$$\max_{1\le k\le K-\tilde{K}}\left|\boldsymbol{s}_{k+\tilde{K}}^T\boldsymbol{C}_r^{-1}\boldsymbol{s}_j\right| \xrightarrow{\text{a.s.}} 0, \qquad j = 1, \ldots, \tilde{K}. \tag{134}$$

It follows that

$$\max_{1\le k\le K-\tilde{K}}\left\|\boldsymbol{s}_{k+\tilde{K}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right\| \xrightarrow{\text{a.s.}} \boldsymbol{0}. \tag{135}$$

Therefore, we have that

$$\max_{1\le k\le K-\tilde{K}}\left|t_k^{(N)}\right| \xrightarrow{\text{a.s.}} 0 \tag{136}$$

and condition b) follows. Now, observe that

$$\sum_{k=1}^{K-\tilde{K}} \left(t_k^{(N)}\right)^2 = \mathcal{I}_0 - \eta\|\boldsymbol{w}_1\|^2. \tag{137}$$

By Lemma 5.2

$$\|\boldsymbol{w}_1\|^2 = \tilde{\boldsymbol{e}}_1^T \left(\tilde{\boldsymbol{S}}^T \boldsymbol{C}_r^{-1} \tilde{\boldsymbol{S}}\right)^{-1} \tilde{\boldsymbol{e}}_1 \xrightarrow{\text{a.s.}} \frac{1 + A_1^2\beta_0}{\beta_0}. \tag{138}$$

Combining the above result with (99) yields condition c), thereby completing the proof of Step 1.

Step 2: We establish in what follows that for almost every realization of $\boldsymbol{S}$, $\Delta\boldsymbol{w}_1^T\boldsymbol{SAb}$ has a limiting Gaussian distribution. To this end, we show that for almost every $\boldsymbol{b}$ and $\boldsymbol{S}$, the variance of the corresponding $\Delta\boldsymbol{w}_1^T\boldsymbol{SAb}$ is the same as $N \to \infty$. Again, note that for fixed $\boldsymbol{b}$ and $\boldsymbol{S}$

$$\mathbb{E}\left\{\left(\Delta\boldsymbol{w}_1^T\boldsymbol{SAb}\right)^2\right\} = \boldsymbol{b}^T\boldsymbol{AS}^T\boldsymbol{C}_w\boldsymbol{SAb}$$

and recall [11], [12] that the covariance of the estimation error $\Delta\boldsymbol{w}_1$ of the group-blind hybrid receiver is

$$\boldsymbol{C}_w = \frac{1}{M}\left\{\boldsymbol{Q}\left[\left(\boldsymbol{w}_1^T\boldsymbol{v}_1\right)\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T - 2\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T \cdot \boldsymbol{SDS}^T\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T\right]\boldsymbol{Q}^T + \tau\boldsymbol{U}_n\boldsymbol{U}_n^T\right\} \tag{139}$$

where

$$\boldsymbol{v}_1 \triangleq \tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{e}}_1 \tag{140}$$

$$\boldsymbol{D} \triangleq \text{diag}\left\{A_1^4\left(\boldsymbol{w}_1^T\boldsymbol{s}_1\right)^2, \ldots, A_K^4\left(\boldsymbol{w}_1^T\boldsymbol{s}_K\right)^2\right\} \tag{141}$$

$$\tau = \eta\boldsymbol{v}_1^T\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\left(\boldsymbol{\Lambda}_s - \eta\boldsymbol{I}_K\right)^{-2}\boldsymbol{U}_s^T\boldsymbol{v}_1 \tag{142}$$

$$\boldsymbol{Q} \triangleq \boldsymbol{I}_N - \boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}^T. \tag{143}$$

Plugging (139) into $\boldsymbol{S}^T\boldsymbol{C}_w\boldsymbol{S}$ yields that

$$\boldsymbol{S}^T\boldsymbol{C}_w\boldsymbol{S} = \boldsymbol{B}_1 + \boldsymbol{B}_2 + \boldsymbol{B}_3 + \boldsymbol{B}_4 + \boldsymbol{B}_5 + \boldsymbol{B}_6 + \boldsymbol{B}_7 + \boldsymbol{B}_8 \tag{144}$$

where

$$\begin{aligned}
\boldsymbol{B}_1 &= (\boldsymbol{w}_1^T\boldsymbol{v}_1)\boldsymbol{S}^T\boldsymbol{C}_r^{-1}\boldsymbol{S} \\
\boldsymbol{B}_2 &= -(\boldsymbol{w}_1^T\boldsymbol{v}_1)\boldsymbol{S}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\boldsymbol{S} \\
\boldsymbol{B}_3 &= -2\boldsymbol{S}^T\boldsymbol{C}_r^{-1}\boldsymbol{SDS}^T\boldsymbol{C}_r^{-1}\boldsymbol{S} \\
\boldsymbol{B}_4 &= 2\boldsymbol{S}^T\boldsymbol{C}_r^{-1}\boldsymbol{SDS}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\boldsymbol{S} \\
\boldsymbol{B}_5 &= -(\boldsymbol{w}_1^T\boldsymbol{v}_1)\boldsymbol{S}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\boldsymbol{S} \\
\boldsymbol{B}_6 &= (\boldsymbol{w}_1^T\boldsymbol{v}_1)\boldsymbol{S}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\boldsymbol{S} \\
\boldsymbol{B}_7 &= (\boldsymbol{w}_1^T\boldsymbol{v}_1)\boldsymbol{S}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\boldsymbol{SDS}^T\boldsymbol{C}_r^{-1}\boldsymbol{S} \\
\boldsymbol{B}_8 &= -\boldsymbol{S}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1} \\
&\quad \cdot \boldsymbol{SDS}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\boldsymbol{S}.
\end{aligned}$$

We next show that

$$\boldsymbol{b}^T\boldsymbol{AB}_i\boldsymbol{SAb} \xrightarrow{\text{a.s.}} 0$$

for $i = 1, \ldots, 8$. Recall that

$$\boldsymbol{w}_1^T\boldsymbol{v}_1 = \left[\boldsymbol{\Psi}_{11}^{-1}\right]_{11} \xrightarrow{\text{a.s.}} \frac{\beta_0}{1 + A_1^2\beta_0}.$$

For convenience, define

$$\boldsymbol{E}_1 \triangleq \boldsymbol{S}^T\boldsymbol{C}_r^{-1}\boldsymbol{S} \tag{145}$$

$$\boldsymbol{E}_2 \triangleq \boldsymbol{S}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\boldsymbol{S}. \tag{146}$$

Also, note that

$$\boldsymbol{S}^T\boldsymbol{C}_r^{-1}\boldsymbol{SDS}^T\boldsymbol{C}_r^{-1}\boldsymbol{S} = (\boldsymbol{E}_1\boldsymbol{D}_1)(\boldsymbol{E}_1\boldsymbol{D}_1)^T \tag{147}$$

where $\boldsymbol{D}_1 = \boldsymbol{D}^{\frac{1}{2}}$ and

$$\begin{aligned}
&\boldsymbol{S}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1} \\
&\quad \cdot \boldsymbol{SDS}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\boldsymbol{S} \\
&= (\boldsymbol{E}_2\boldsymbol{D}_1)(\boldsymbol{E}_2\boldsymbol{D}_1)^T.
\end{aligned} \tag{148}$$

Then, based on Theorem A.2 in Appendix A and the Borel–Cantelli lemma, we have that it suffices to show that the spectral radii of the following three matrices are bounded above: $\boldsymbol{E}_1, \boldsymbol{E}_2$, and $\boldsymbol{E}_1\boldsymbol{DE}_2$.

To this end, we need the following results on the spectral radius of square matrices [9].

Lemma 5.3: If $\|\cdot\|$ is any matrix norm, then the spectral radius of any square matrix $\boldsymbol{C}$ is bounded above by $\|\boldsymbol{C}\|$.

The next result shows that the spectral radius of any square matrix $\boldsymbol{C}$ is the greatest lower bound for the values of all matrix norms of $\boldsymbol{C}$.

Lemma 5.4: Denote the spectral radius of $\boldsymbol{C}$ as $\rho(\boldsymbol{C})$. Given any $\epsilon > 0$, there is a matrix norm $\|\cdot\|$ such that

$$\rho(\boldsymbol{C}) \le \|\boldsymbol{C}\| \le \rho(\boldsymbol{C}) + \epsilon.$$

We have shown in the proof of Theorem 3.3 that the spectral radius of $\boldsymbol{E}_1$ is bounded above. Define

$$\boldsymbol{H} = \boldsymbol{C}_r^{-1/2}\tilde{\boldsymbol{S}}. \tag{149}$$

Then

$$\begin{aligned}
\boldsymbol{E}_2 &= \boldsymbol{S}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\boldsymbol{S} \\
&= \boldsymbol{S}^T\boldsymbol{C}_r^{-\frac{1}{2}}\boldsymbol{H}(\boldsymbol{H}^T\boldsymbol{H})^{-1}\boldsymbol{H}^T\boldsymbol{C}_r^{-\frac{1}{2}}\boldsymbol{S}.
\end{aligned} \tag{150}$$

Observe that $\boldsymbol{H}(\boldsymbol{H}^T\boldsymbol{H})^{-1}\boldsymbol{H}^T$ is idempotent, and can be written as [18, p. 48]

$$\boldsymbol{H}(\boldsymbol{H}^T\boldsymbol{H})^{-1}\boldsymbol{H}^T = \boldsymbol{O}\begin{bmatrix}\boldsymbol{I} & 0\\ 0 & 0\end{bmatrix}\boldsymbol{O}^T \tag{151}$$

where $\boldsymbol{O}$ is some unitary matrix. It follows that

$$\begin{aligned}
\boldsymbol{E}_2 &= \boldsymbol{S}^T\boldsymbol{C}_r^{-1/2}\boldsymbol{O}\begin{bmatrix}\boldsymbol{I} & 0\\ 0 & 0\end{bmatrix}\boldsymbol{O}^T\boldsymbol{C}_r^{-1/2}\boldsymbol{S} \\
&\preceq \boldsymbol{S}^T\boldsymbol{C}_r^{-1/2}\boldsymbol{OO}^T\boldsymbol{C}_r^{-1/2}\boldsymbol{S} \\
&\preceq \boldsymbol{S}^T\boldsymbol{C}_r^{-1}\boldsymbol{S} = \boldsymbol{E}_1.
\end{aligned} \tag{152}$$

Because $\boldsymbol{E}_1$ has bounded spectral radius, and both $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ are nonnegative definite, we have that the spectral radius of $\boldsymbol{E}_2$ is bounded above. Built on the boundedness of the spectral radii of $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$, we use Lemmas 5.3 and 5.4 to conclude that the spectral radius of $\boldsymbol{E}_1\boldsymbol{D}\boldsymbol{E}_2$ is bounded above. It then follows that

$$\begin{aligned}\boldsymbol{b}^T\boldsymbol{A}\boldsymbol{S}^T\boldsymbol{C}_w\boldsymbol{S}\boldsymbol{A}\boldsymbol{b} - \frac{1}{K}\,\mathrm{tr}\left(\boldsymbol{A}\boldsymbol{S}^T\boldsymbol{C}_w\boldsymbol{S}\boldsymbol{A}\right) \\ = \boldsymbol{b}^T\boldsymbol{A}\boldsymbol{S}^T\boldsymbol{C}_w\boldsymbol{S}\boldsymbol{A}\boldsymbol{b} - \frac{1}{K}\,\mathrm{tr}\left[\boldsymbol{A}\left(\sum_{i=1}^{8}\boldsymbol{B}_i\right)\boldsymbol{A}\right] \xrightarrow{\text{a.s.}} 0. \qquad (153)\end{aligned}$$

Along the same line as in proving Theorems 3.1 and 3.2, we can obtain that $\mathrm{tr}\,(\boldsymbol{A}\boldsymbol{S}^T\boldsymbol{C}_w\boldsymbol{S}\boldsymbol{A})$ converges to the same limit for almost every $\boldsymbol{S}$. It then follows that $\mathbb{E}\{\boldsymbol{b}^T\boldsymbol{A}\boldsymbol{S}^T\boldsymbol{C}_w\boldsymbol{S}\boldsymbol{A}\boldsymbol{b}\,|\,\boldsymbol{S}\}$ converges with probability one, where the expectation is taken with respect to $\boldsymbol{b}$. We conclude that for almost every $\boldsymbol{S}$, $\mathbb{I}_1^{(N)}$ converges to a Gaussian distribution with zero mean and variance

$$v^2 = \frac{1}{\alpha_M}\left[(\alpha_K - \alpha_{\tilde{K}}) + c_1(1-\alpha_K)\right]\frac{\beta_0}{1+A_1^2\beta_0}. \qquad (154)$$

Step 3: It is straightforward to use Proposition 3.1 to prove Step 3, thereby completing the proof of Theorem 3.4. □

APPENDIX A
TWO RANDOM MATRIX RESULTS

The first result is on the empirical distribution function of eigenvalues of large random matrices [21]. Denote the eigenvalues of the random matrix $\boldsymbol{S}\boldsymbol{A}^2\boldsymbol{S}^T$ by $\lambda_1, \ldots, \lambda_N$ (they are random, depending on the realization of $\boldsymbol{S}$), and the empirical distribution of the eigenvalues by G_N. We restate [21, Theorem 1.1] in the following.

Theorem A.1: Under Condition 1, G_N converges weakly to a distribution function G^* with probability one, and the Stieltjes transform $m(z)$ of G^* is the solution of the following functional equation:

$$m(z) = \frac{1}{-z + \alpha\int\frac{\mu}{1+\mu m(z)}\,\mathrm{d}F_\mu(\mu)} \qquad (155)$$

where the Stieltjes transform $m(z)$ of any distribution G is defined as

$$m_G(z) \triangleq \int\frac{1}{\lambda - z}\,\mathrm{d}G(\lambda)$$

for $z \in \mathbb{C}^+ \equiv \{z \in \mathbb{C}, \mathrm{Im}\{z\} > 0\}$.

We also use the following result repeatedly, which follows directly from [21, Lemma 3.1]:

Theorem A.2: Let $\boldsymbol{C} = (c_{ij})$, $c_{ij} \in \mathbb{C}$, be an $N \times N$ matrix with spectral radius bounded in N, and $\boldsymbol{s} = \frac{1}{\sqrt{N}}[s_1, \ldots, s_N]^T$, where the s_i's are i.i.d. with $P\{s_i = 1\} = P\{s_i = -1\} = \frac{1}{2}$. Then

$$\mathbb{E}\left\{\left|\boldsymbol{s}^T\boldsymbol{C}\boldsymbol{s} - \frac{1}{N}\,\mathrm{tr}\,(\boldsymbol{C})\right|^6\right\} \le \frac{c}{N^3}$$

where the constant c does not depend on N and A.

APPENDIX B
PROOFS OF TECHNICAL LEMMAS

Lemma B.1: Suppose $X_n \xrightarrow{\text{a.s.}} a$ and $Y_n \xrightarrow{\text{a.s.}} b$ as $n \to \infty$, where both a and b are finite constants. Then

$$\lim_{n\to\infty} X_n Y_n \xrightarrow{\text{a.s.}} ab.$$

It is straightforward to establish the above lemma via standard real analysis techniques.

Lemma B.2: Suppose both $\{X_n\}$ and $\{Y_n\}$ have limiting Gaussian distributions $n \to \infty$, and $\{X_n\}$ and $\{Y_n\}$ are asymptotically uncorrelated. Then, $\{X_n + Y_n\}$ has a limiting Gaussian distribution.

The proof of Lemma B.2 follows by using an argument via characteristic functions.

A. Proof of Lemma 5.1

Outline of the proof: Let

$$\begin{aligned}\eta_m &= \frac{1}{m}\\ E_m &= \{\omega\colon X_n(\omega, \eta_m) \to X(\omega, \eta_m)\}, \qquad m = 1, 2, \ldots.\end{aligned}$$

Let $E = \bigcap_{m=1}^{\infty} E_m$. It is clear that $P[E] = 1$. Observe that for any $\omega \in E$, as $n \to \infty$

$$\begin{aligned}\left|\lim_{m\to\infty} X_n(\omega, \eta_m) - \lim_{m\to\infty} X(\omega, \eta_m)\right| \\ \le \limsup_{m\to\infty}\left|X_n(\omega, \eta_m) - X(\omega, \eta_m)\right| \to 0. \qquad (156)\end{aligned}$$

It then follows immediately that for any $\omega \in E$

$$\lim_{\eta\to 0} X_n(\eta) \xrightarrow{\text{a.s.}} \lim_{\eta\to 0} X(\eta) \qquad (157)$$

thereby completing the proof. □

B. Proof of Lemma 5.2

We know that for $i = 1, \ldots, \tilde{K}$

$$\boldsymbol{s}_i^T\boldsymbol{C}_r^{-1}\boldsymbol{s}_i \xrightarrow{\text{a.s.}} \frac{\beta_0}{1+A_1^2\beta_0}. \qquad (158)$$

Next we show that for $i \ne j$

$$\boldsymbol{s}_i^T\boldsymbol{C}_r^{-1}\boldsymbol{s}_j \xrightarrow{\text{a.s.}} 0. \qquad (159)$$

To this end, it suffices to show that

$$(\boldsymbol{s}_i^T\boldsymbol{C}_r^{-1}\boldsymbol{s}_j)^2 \xrightarrow{\text{a.s.}} 0.$$

For convenience, define

$$\begin{aligned}\boldsymbol{C}_{r,i} &\triangleq \boldsymbol{C}_r - A_i^2\boldsymbol{s}_i\boldsymbol{s}_i^T\\ \boldsymbol{C}_{r,i,j} &\triangleq \boldsymbol{C}_r - A_i^2\boldsymbol{s}_i\boldsymbol{s}_i^T - A_j^2\boldsymbol{s}_j\boldsymbol{s}_j^T.\end{aligned}$$

Note that $\boldsymbol{C}_{r,i}$ is independent from $\boldsymbol{s}_i$, and $\boldsymbol{C}_{r,i,j}$ is independent from both $\boldsymbol{s}_i$ and $\boldsymbol{s}_j$. Using the matrix inverse lemma, it can be shown that

$$\boldsymbol{s}_i^T\boldsymbol{C}_r^{-1}\boldsymbol{s}_j = \frac{1}{1+A_i^2\boldsymbol{s}_i^T\boldsymbol{C}_{r,i}^{-1}\boldsymbol{s}_i}\,\boldsymbol{s}_i^T\boldsymbol{C}_{r,i}^{-1}\boldsymbol{s}_j. \qquad (160)$$

It follows that

$$\left(\boldsymbol{s}_i^T\boldsymbol{C}_r^{-1}\boldsymbol{s}_j\right)^2 \le \left(\boldsymbol{s}_i^T\boldsymbol{C}_{r,i}^{-1}\boldsymbol{s}_j\right)^2. \qquad (161)$$

Since $\boldsymbol{C}_{r,i}^{-1}\boldsymbol{s}_j\boldsymbol{s}_j^T\boldsymbol{C}_{r,i}^{-1}$ is a rank one matrix, and

$$\operatorname{tr}\left(\boldsymbol{C}_{r,i}^{-1}\boldsymbol{s}_j\boldsymbol{s}_j^T\boldsymbol{C}_{r,i}^{-1}\right) = \boldsymbol{s}_j^T\boldsymbol{C}_{r,i}^{-2}\boldsymbol{s}_j \le \frac{1}{\eta^2};$$

the spectral radius of $\boldsymbol{C}_{r,i}^{-1}\boldsymbol{s}_j\boldsymbol{s}_j^T\boldsymbol{C}_{r,i}^{-1}$ is bounded above. By Lemma A.2

$$\left(\boldsymbol{s}_i^T\boldsymbol{C}_{r,i}^{-1}\boldsymbol{s}_j\right)^2 - \frac{1}{N}\operatorname{tr}\left(\boldsymbol{C}_{r,i}^{-1}\boldsymbol{s}_j\boldsymbol{s}_j^T\boldsymbol{C}_{r,i}^{-1}\right) \xrightarrow{\text{a.s.}} 0. \qquad (162)$$

Combining (161) and (162) yields that

$$\boldsymbol{s}_i^T\boldsymbol{C}_r^{-1}\boldsymbol{s}_j \xrightarrow{\text{a.s.}} 0.$$

Let $E_{i,j}$ denote the set on which $\boldsymbol{s}_i^T\boldsymbol{C}_r^{-1}\boldsymbol{s}_j$ converges. It is straightforward to see that

$$P\left\{\bigcap_{i,j=1}^{\tilde{K}} E_{i,j}\right\} = 1.$$

That is,

$$\boldsymbol{\Psi}_{11} \xrightarrow{\text{a.s.}} \operatorname{diag}\left(\frac{\beta_0}{1+A_1^2\beta_0}, \ldots, \frac{\beta_0}{1+A_{\tilde{K}}^2\beta_0}\right). \qquad (163)$$

Then, it can be shown that

$$\left[\boldsymbol{\Psi}_{11}^{-1}\right]_{1,1} = \left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)_{1,1}^{-1} \xrightarrow{\text{a.s.}} \frac{1+A_1^2\beta_0}{\beta_0}. \qquad (164)$$

Along the same lines, we can obtain the other elements in $\boldsymbol{\Psi}_{11}^{-1}$. The proof is completed. □

C. Proof of Proposition 3.1

In the following, we show that for the blind linear MMSE receiver case, $\boldsymbol{s}_k^T\Delta\boldsymbol{w}_1 \xrightarrow{\text{a.s.}} 0$, which is equivalent to showing that for almost every realization of signatures, $(\boldsymbol{s}_k^T\Delta\boldsymbol{w}_1)^2 \to 0$. To this end, it suffices to show that

$$\mathbb{E}\left\{\left(\boldsymbol{s}_k^T\Delta\boldsymbol{w}_1\right)^2 \middle| \boldsymbol{S}\right\} \xrightarrow{\text{a.s.}} 0. \qquad (165)$$

Observe that for the blind linear MMSE receiver (DMI or subspace), by (116), we have

$$\begin{aligned}
&\mathbb{E}\left\{\left(\boldsymbol{s}_k^T\Delta\boldsymbol{w}_1\right)^2 \middle| \boldsymbol{S}\right\} \\
&\quad= \boldsymbol{s}_k^T\boldsymbol{C}_w\boldsymbol{s}_k \\
&\quad= \frac{1}{M}\Big[\left(\boldsymbol{w}_1^T\boldsymbol{s}_1\right)\boldsymbol{s}_k^T\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T\boldsymbol{s}_k + \boldsymbol{s}_k^T\boldsymbol{w}_1\boldsymbol{w}_1^T\boldsymbol{s}_k \\
&\qquad - 2\boldsymbol{s}_k^T\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T\boldsymbol{S}\boldsymbol{D}\boldsymbol{S}^T\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T\boldsymbol{s}_k\Big] \\
&\quad= \frac{1}{M}\left[\left(\boldsymbol{w}_1^T\boldsymbol{s}_1\right)\boldsymbol{w}_k^T\boldsymbol{s}_k + \left(\boldsymbol{w}_1^T\boldsymbol{s}_k\right)^2 - 2\boldsymbol{w}_1^T\boldsymbol{S}\boldsymbol{D}\boldsymbol{S}^T\boldsymbol{w}_1\right]. \qquad (166)
\end{aligned}$$

Because $\boldsymbol{w}_1^T\boldsymbol{s}_1$, $\boldsymbol{w}_k^T\boldsymbol{s}_k$, $\boldsymbol{w}_1^T\boldsymbol{s}_k$, and $\boldsymbol{w}_1^T\boldsymbol{S}\boldsymbol{D}\boldsymbol{S}^T\boldsymbol{w}_1$ converge to their finite limits with probability one, we conclude that

$$\mathbb{E}\left\{\left(\boldsymbol{s}_1^T\Delta\boldsymbol{w}_1\right)^2 \middle| \boldsymbol{S}\right\} \xrightarrow{\text{a.s.}} 0. \qquad (167)$$

Next we also show that $\boldsymbol{s}_k^T\Delta\boldsymbol{w}_1 \xrightarrow{\text{a.s.}} 0$, where $\Delta\boldsymbol{w}_1$ is the estimation error of the group-blind hybrid receiver. Since by (139)

$$\begin{aligned}
&\mathbb{E}\left\{\left(\boldsymbol{s}_k^T\Delta\boldsymbol{w}_1\right)^2 \middle| \boldsymbol{S}\right\} \\
&\quad= \boldsymbol{s}_k^T\boldsymbol{C}_w\boldsymbol{s}_k \\
&\quad= \frac{1}{M}\left[\boldsymbol{s}_k^T - \boldsymbol{s}_k^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}^T\right] \\
&\qquad \cdot\Big[(\boldsymbol{w}_1^T\boldsymbol{v}_1)\boldsymbol{C}_r^{-1}\boldsymbol{s}_k - (\boldsymbol{w}_1^T\boldsymbol{v}_1)\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1} \\
&\qquad\quad \cdot\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\boldsymbol{s}_k - 2\boldsymbol{C}_r^{-1}\boldsymbol{S}\boldsymbol{D}\boldsymbol{S}^T\boldsymbol{C}_r^{-1}\boldsymbol{s}_k \\
&\qquad\quad + 2\boldsymbol{C}_r^{-1}\boldsymbol{S}\boldsymbol{D}\boldsymbol{S}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\left(\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}^T\boldsymbol{C}_r^{-1}\boldsymbol{s}_k\Big]
\end{aligned} \qquad (168)$$

using the findings on the boundedness of the spectral radii of the matrices in $\boldsymbol{E}_1$, $\boldsymbol{E}_2$, and $\boldsymbol{E}_1\boldsymbol{D}\boldsymbol{E}_2$ (cf. Section V-E, Step 2), we apply Theorem A.2 in Appendix A to conclude that for the group-blind hybrid receiver

$$\mathbb{E}\left\{\left(\boldsymbol{s}_k^T\Delta\boldsymbol{w}_1\right)^2 \middle| \boldsymbol{S}\right\} \xrightarrow{\text{a.s.}} 0. \qquad (169)$$

It then follows that $\boldsymbol{s}_k^T\Delta\boldsymbol{w}_1 \xrightarrow{\text{a.s.}} 0$, completing the proof. □

APPENDIX C
DERIVATIONS OF (54) AND (55)

We first derive e_1 in (54). Substituting (53) into (39), we have

$$\begin{aligned}
e_1 &= \lim_{\eta\to 0}\left(\beta_0(\eta) - \frac{1-\alpha_K}{\eta}\right) \\
&= \lim_{\eta\to 0}\left[\sqrt{\frac{(1-\alpha_K)^2}{4\eta^2} + \frac{1+\alpha_K}{2A^2\eta} + \frac{1}{4A^4}} - \left(\frac{1-\alpha_K}{2\eta} + \frac{1}{2A^2}\right)\right] \\
&= \lim_{\eta\to 0}\frac{\frac{\alpha_K}{A^2\eta}}{\sqrt{\frac{(1-\alpha_K)^2}{4\eta^2} + \frac{1+\alpha_K}{2A^2\eta} + \frac{1}{4A^4}} + \frac{1-\alpha_K}{2\eta} + \frac{1}{2A^2}} \\
&= \lim_{\eta\to 0}\frac{\alpha_K}{\sqrt{\frac{(1-\alpha_K)^2A^4}{4} + \frac{(1+\alpha_K)A^2\eta}{2} + \frac{\eta^2}{4}} + \frac{(1-\alpha_K)A^2}{2} + \frac{\eta}{2}} \\
&= \frac{\alpha_K}{A^2(1-\alpha_K)}.
\end{aligned} \qquad (170)$$

We next derive e_2 in (55). Using (53), we have

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\beta_0(\eta) - \frac{1-\alpha_K}{\eta}\right) \\
&\quad= \frac{1}{2\eta^2}\left[(1-\alpha_K) - \frac{\frac{(1-\alpha_K)^2}{2} + \frac{(1+\alpha_K)\eta}{A^2}}{\sqrt{\underbrace{\frac{(1-\alpha_K)^2}{4} + \frac{(1+\alpha_K)\eta}{2A^2} + \frac{\eta^2}{4A^4}}_{g(\eta)}}}\right]
\end{aligned}$$

$$= \frac{\overbrace{(1-\alpha_K)\sqrt{g(\eta)} - \left[\frac{(1-\alpha_K)^2}{2} + \frac{1+\alpha_K}{A^2}\right]}^{f_1(\eta)}}{\underbrace{2\eta^2\sqrt{g(\eta)}}_{f_2(\eta)}}. \quad (171)$$

After some manipulations, we have

$$f_1''(\eta) = -\frac{1-\alpha_K}{4}[g'(\eta)]^2[g(\eta)]^{-\frac{3}{2}} + \frac{1-\alpha_K}{4A^4}[g(\eta)]^{-\frac{1}{2}} \quad (172)$$

$$f_2''(\eta) = 4[g(\eta)]^{\frac{1}{2}} + 2\eta[g(\eta)]^{-\frac{1}{2}}g'(\eta) + o(\eta) \quad (173)$$

where

$$g'(\eta) = \frac{1+\alpha_K}{2A^2} + \frac{\eta}{2A^4}. \quad (174)$$

Using (40) we obtain

$$\begin{aligned} e_2 &= -\lim_{\eta\to 0}\frac{f_1(\eta)}{f_2(\eta)} = -\lim_{\eta\to 0}\frac{f_1''(\eta)}{f_2''(\eta)} \\ &= \frac{\frac{1}{2A^4}\left[\frac{(1+\alpha_K)^2}{(1-\alpha_K)^2} - 1\right]}{2(1-\alpha_K)} \\ &= \frac{\alpha_K}{A^6(1-\alpha_K)^3}. \end{aligned} \quad (175)$$

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