

# Distributed Opportunistic Scheduling for Wireless Networks Powered by Renewable Energy Sources

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**Abstract**—This paper considers an *ad hoc* network with multiple transmitter-receiver pairs, in which all transmitters are capable of harvesting renewable energy from the environment and compete for the same channel by random access. To quantify the roles of both the energy state information (ESI) and the channel state information (CSI), a distributed opportunistic scheduling (DOS) framework with a save-then-transmit scheme is proposed. First, in the channel probing stage, each transmitter probes the CSI via channel contention; next, in the data transmission stage, the successful transmitter decides to either give up the channel (if the expected reward calculated over the CSI and ESI is small) or hold and utilize the channel by optimally exploring the energy harvesting and data transmission tradeoff. With a constant energy arrival model, i.e., the energy harvesting rate keeps identical over the time of interest, the expected throughput maximization problem is formulated as an optimal stopping problem, whose solution is shown to exist and have a threshold-based structure, for both the homogeneous and heterogeneous cases. Furthermore, we prove that there exists a steady-state distribution for the stored energy level at each transmitter, and propose an efficient iterative algorithm for its computation. Finally, we show via numerical results that the proposed scheme can achieve a potential 175% throughput gain compared with the method of best-effort delivery.

## I. INTRODUCTION

Conventional wireless communication devices are usually powered by batteries that can provide stable energy supplies. However, the lifetime of the battery limits the operation time of such devices. Recently, energy harvesting (EH) technique has been proposed as a promising substitution for the conventional constant power supplies [1], [2], which is capable of converting the renewable energy from the environment into electrical energy. Compared with the conventional constant energy suppliers, the transmitter powered by energy harvesters is restricted by a new class of EH constraints, i.e., the consumed energy up to any time is bounded by the harvested energy until this point [3], [4]. Therefore, to meet certain performance requirements, such as throughput, stability, delay, etc., these EH constraints should be carefully taken into account in the design of the power allocation schemes for the EH-based communication systems.

### A. Related Work

Communication systems powered by energy harvesters have been investigated in recent years. For the point-to-point wireless systems, the authors in [3] [5] considered the throughput maximization problem over a finite horizon for both the cases

that the harvested energy information is non-causally and causally known to the transmitter, where the optimal solutions were obtained by the proposed one-dimension search algorithm and dynamic programming (DP) techniques, respectively. In [4], the authors extended the results to the classic three-node Gaussian relay channel with EH source and relay nodes, where the optimal power allocation algorithms were proposed. With a more practical circuit model by considering the half-duplex constraint of the battery, the authors in [6] proposed a save-then-transmit protocol, which divides each transmission frame into two parts: the first one for harvesting energy and the other for data transmission. For wireless networks with EH constraints, the authors in [7] investigated the performance of some standard medium access control protocols, e.g., TDMA, framed Aloha, and dynamic-framed Aloha.

In related work on *ad hoc* networking, opportunistic scheduling has been known as an effective method to utilize the wireless resource [8], [9]. In particular, a distributed opportunistic scheduling (DOS) scheme was introduced in [10], [11], where only local channel state information (CSI) is available to each transmitter. By applying optimal stopping theory [12], it has been shown in [10], [11] that the optimal solution for the expected throughput maximization problem has a threshold-based structure. When channel estimation is imperfect, the authors in [13] proposed a two-level channel probing framework that allows the accessing transmitter to perform one more round of channel estimation before data transmission to improve the quality of estimated CSI and possibly increase the system throughput. The optimal scheduling policy of the two-level probing framework was proved to be threshold-based as well by referring to the optimal stopping with two-level incomplete information [14].

### B. Summary of Main Contributions

Compared to the conventional networks powered by constant power supplies [8]–[11], [13], in our setup, both the CSI and the energy state information (ESI) play important roles on the system performance. Over various types of renewable energy sources, we consider the case when the EH rate can be approximated as a constant within the time duration of interest. For example, the power variation coherent time of wind and solar EH system is on the level of multiple seconds or higher [15], [16], while the duration of one communication block is about several milliseconds. Thus, over thousands of

communication blocks, the EH rate keeps almost identical<sup>1</sup>.

We investigate the DOS problem for the considered network powered by energy harvesters. All transmitters first adopt the random access scheme and do channel probing (CP), during which the successful link can obtain the CSI via channel contentions, similar to those in [10], [11], [13]. Then, based on the expected reward calculated, if the successful transmitter decides to transmit, it adopts a save-then-transmit protocol [6]: First, it possibly spends certain time to harvest more energy; and then transmits in the rest of the transmission block. Obviously, *when the total duration of the transmission block is fixed, spending more time on harvesting energy can increase the power level for transmission, while it decreases the portion of the time for transmission, which leads to a tradeoff to optimize*. In this paper, it is assumed that each transmitter consumes all the available energy at each transmission. Although such a power control scheme may be suboptimal, it is a simple while efficient method to fully utilize each chance contended for transmission, especially when the number of transmitters is large. The main contributions of this paper are summarized as follows.

- 1) We investigate the DOS framework for the ad hoc network with EH transmitters, which probes both the CSI and the ESI. The throughput maximization problem for the considered system is cast as a rate-of-return problem in optimal stopping theory. First, we compute the optimal saving ratio (EH duration vs. transmission duration) for the save-then-transmit scheme by maximizing the throughput with the given CSI and ESI, which can be transformed into a convex problem. Based on the obtained optimal saving ratio, we prove the existence of a optimal stopping rule for the optimal stopping problem in both the homogeneous and heterogenous cases.
- 2) Under the DOS framework with the save-then-transmit scheme, we prove the existence of the steady-state distribution for the stored energy level at each transmitter, by constructing a “super” Markov chain with its states being jointly determined by all transmitters. Moreover, we propose an efficient iterative algorithm to parallelly compute the steady-state distribution at each transmitter. When the network consists of  $I$  transmitters and each one is with  $B_{max}$  possible energy states, the computational complexity for one iteration of the proposed algorithm is on the order of  $O(I^4 B_{max}^5)$ , which is more efficient (when  $I$  and  $B_{max}$  are large) than that for directly solving the super Markov chain case, which is on the order of  $O(2B_{max}^{2I})$ .

The rest of this paper is organized as follows. Section II introduces the system model and formulates the throughput maximization problem. Section III derives the optimal saving ratio and the optimal stopping rule for the DOS framework. In Section IV, we propose an iterative algorithm to compute

<sup>1</sup>In this paper, we will not consider the optimization across different EH periods since it usually requires certain noncausal knowledge of the energy arrival process.

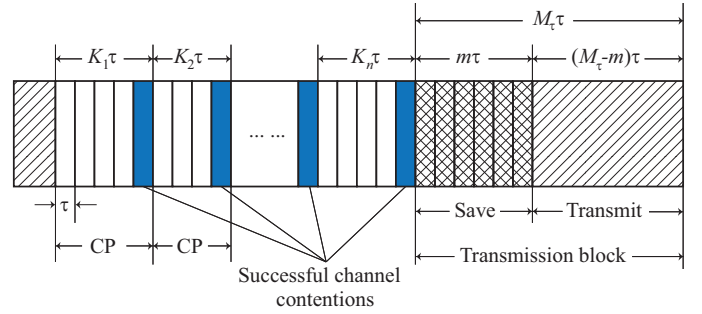


Fig. 1. One realization for the DOS system with the save-then-transmit scheme.

the steady-state distribution for each transmitter. In Section V, some numerical results are provided to show the influence of EH constraints on the throughput performance. Finally, Section VI concludes the paper.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

### A. System Model

We consider a single-hop *ad hoc* network, where all the  $I$  transmitter-receiver pairs contend for the channel by random access. For each link, the transmitter is powered by a renewable energy source and utilizes a rechargeable battery to temporally store the harvested energy. Here we assume that there is no loss for energy storage and retrieval at the battery, and the power consumed other than communication is negligible. In addition, all operations for the considered system, including the channel contentions and data transmissions, are time slotted, and the duration of each time slot is a constant, denoted as  $\tau > 0$ .

1) *Channel probing*: As illustrated in Fig. 1, the DOS procedure takes place in two stages: First, each transmitter probes the channel via random access; and then the successful transmitter may start the save-then-transmit process or directly give up the channel<sup>2</sup>. For the first stage, we define a successful channel contention as follows: All transmitters first independently contend for the channel until there is only one contending in one time slot. Furthermore, one round of CP is defined as the duration to achieve one successful channel contention. Denote the probability that transmitter  $i$  contends the channel as  $q_i$ ,  $1 \leq i \leq I$ , with  $0 \leq q_i \leq 1$ . As such, the probability that the  $i$ -th transmitter successfully occupies the channel is given by  $Q_i = q_i \prod_{j \neq i} (1 - q_j)$ . Then, the probability to achieve one successful channel contention at each time slot is given by  $Q = \sum_{i=1}^I Q_i$ , and it is easy to check that  $Q \leq 1$  [17]. Accordingly, for the  $n$ -th round of CP,  $n \geq 1$ , we use  $K_n$  to denote the number of time slots needed to achieve a successful channel contention, which is a random variable and satisfies the geometric distribution with parameter  $Q$  [10], [11], [13]. In this way, the expected duration of one round CP is given as  $\tau/Q$ . Suppose that the successful

<sup>2</sup>If the successful transmitter experiences a bad channel condition and a low energy level, it may directly skip the transmission.

transmitter  $i$  transmits signal  $x^i$ , and the received signal  $y^i$  is given by

$$y^i = h^i x^i + z^i, \quad (1)$$

where  $h^i$  is the channel gain and  $z^i$  is the circularly symmetric complex Gaussian (CSCG) noise with zero mean and variance  $\sigma^2$  at the receiver. Across different links,  $\{h^i\}$  are assumed to be independent with finite mean and variance, while not necessarily identically distributed. After one round of CP, the successful transmitter can perfectly estimate the corresponding channel gain via certain feedback mechanisms, and  $h^i$  is assumed to be a known constant during the whole transmission block. After CP, the successful transmitter chooses one of the following actions based on its local CSI and ESI:

(a) releases the channel (if the CSI and ESI indicate that the transmission rate is lower than a threshold) and let all links re-contend; or

(b) directly transmits during the next  $M_\tau$  time slots; or

(c) holds the channel, continues EH to save energy, and then transmits.

Note that action (b) can be treated as a special case of action (c) when there spends no more time for energy saving. From the whole system point of view, it may take  $n$  rounds of CPs before completing one data transmission as depicted in Fig. 1. In short, during the CPs, all transmitters keep harvesting energy, and after each round of CP, only the successful transmitter makes a choice among three actions as listed above.

2) *Data transmission*: When the successful transmitter decides not to take action (a) defined above, the system reaches its second stage, i.e., data transmission. For one transmission block, it contains  $M_\tau$  time slots, where  $M_\tau$  is a finite integer. During this stage, the transmitter may choose to continue EH over more time slots and then transmit over the rest of transmission block, i.e., to do action (c). When the transmitter starts transmission, it is required to exhaust all its available energy to fully take advantage of the opportunity for channel use [6]. Denote  $B_t^i \in \Delta$  as the energy level in the battery at the end of time slot  $t$  for transmitter  $i$ , where  $\Delta = \{0, \delta, 2\delta, \dots, B_{max}\delta\}$  denotes the set of all possible energy states, with  $\delta$  being the minimum energy unit and  $B_{max}\delta$  being the capacity of the battery. In general,  $\{B_t^i\}_{t \geq 1}$  is a random process with state distribution  $\{\Pi_t^i\}_{t \geq 1}$ . As noted in the previous section, we make the following assumption:

**Assumption A:** For many types of energy sources, including the wind or solar, the EH rate changes slowly after every thousands of communications blocks, and thus can be approximately the same over such a period.

As such, we assume that the EH rate  $E^i$  at transmitter  $i$  is a constant, and  $\{E^i\}$  can thus be learned and assumed known before transmissions. It will be shown in Section IV that  $\{B_t^i\}_{t \geq 1}$  turns out to be a non-homogeneous Markov chain, while there still exists a steady-state distribution as time goes to infinity (by Assumption A). We denote  $\Pi^i$  as the steady-state distribution for  $\{B_t^i\}_{t \geq 1}$  when  $t \rightarrow \infty$ .

## B. Problem Formulation

Now, we formulate the problem to maximize the expected throughput of the considered network. Since the energy level at transmitter  $i$ , i.e.,  $\{B_t^i\}_{t \geq 1}$ , is influenced by how the transmitter  $i$  takes its action after it successfully occupies the channel, we are interested in highlighting its value at the time slot after each round of CP. To do so, we slightly modify the notation  $B_t^i$  by reformatting the time index  $t$ : Suppose that transmitter  $i$  occupies the channel after the  $n$ -th round of CP, and we denote  $B_{n,0}^i$  as its energy level at this time before taking any action. If transmitter  $i$  decides to spend another  $m$  time slots on EH, its energy level becomes  $B_{n,m}^i$ , which is given by

$$B_{n,m}^i = \min \{B_{n,0}^i + m\tau E^i, B_{max}\delta\}, \quad (2)$$

where  $n \geq 1$ ,  $0 \leq m \leq M_\tau$ , and  $\min\{x, y\}$  denotes the smaller value between two real numbers  $x$  and  $y$ . For convenience, we omit the index  $i$  for either the CSI or the ESI in the sequel. Then, after the  $n$ -th round of CP and  $m$  additional time slots, the CSI and the ESI at the successful transmitter are given as  $\mathcal{F}_{n,m} = \{h_n, B_{n,m}\}$ . Note that the channel gain  $h_n$  is now indexed by  $n$ , since we assume that its value is determined at the end of the  $n$ -th round of CP and fixed during the following data transmission block. In particular,  $\mathcal{F}_{n,0} = \{h_n, B_{n,0}\}$  denotes the initial information after the  $n$ -th round of CP before any further EH and data transmission.

Given  $\mathcal{F}_{n,m}$  and the remaining transmission time  $(M_\tau - m)\tau$ , the average transmission rate over the  $M_\tau$  time slots is given by

$$R_{n,m} = \left(1 - \frac{m}{M_\tau}\right) \log \left(1 + |h_n|^2 \frac{B_{n,m}}{\sigma^2(M_\tau - m)\tau}\right). \quad (3)$$

When  $m = M_\tau$ , we define  $R_{n,m} = 0$  due to the fact that there is no transmission. Since  $h_n$  has finite mean and variance and the energy level  $B_{n,m}$  is also finite, it follows that  $\mathbb{E}[R_{n,m}] < \infty$  and  $\mathbb{E}[(R_{n,m})^2] < \infty$ . Recall that the channel gains and the battery states are independent across different transmitters at a given time slot; moreover, the probability that one transmitter occupies the channel in two consecutive channel contentions is relatively small. Therefore,  $\{R_{n,m}\}_{n \geq 1}$  are assumed to be independent random variables over  $n$ .

We denote  $T_n$  as the total time duration for completing one data transmission. It consists of the duration of  $n$  rounds of CP, which is given by  $\sum_{j=1}^n K_j \tau$  with  $K_j$  denoting the number of time slots for the  $j$ -th CP, and the data transmission block duration  $M_\tau \tau$ . Accordingly, we obtain

$$T_n = \sum_{j=1}^n K_j \tau + M_\tau \tau. \quad (4)$$

Here, the first thing to optimize is the value of  $n$ , which determines how long the system needs to wait for the successful transmitter to own both ‘‘good’’ CSI and ESI. We call this the wait vs. transmit tradeoff. In addition, as we discussed earlier, after the  $n$ -th round of CP, the transmitter may use  $m$  within

$M_\tau$  slots for EH and transmit with time  $(M_\tau - m)\tau$  afterwards. Apparently, the choice of  $m < M_\tau$  leads to an interesting save vs. transmit tradeoff for us to optimize.

Denote  $n = N \in \mathcal{N}$  as the stopping rule for CP with  $\mathcal{N}$  standing for the valid set (defined later) for  $N$ , and  $m = M \in [0, M_\tau]$  as the saving ratio for EH, which together tell the transmitter when to start the real data transmission. Note that although  $M$  indicates the number of time slots for EH, we still call it a ratio since it divides the whole transmission block into two parts. Then, with  $n = N$  in (4) and  $m = M$  in (3), the total time duration for completing one data transmission is given by  $T_N$  with a rate  $R_{N,M}$ . If such a process repeats  $L$  times with the total amount of transmitted bits  $R_{N_l, M_l} M_\tau \tau$  at each transmission,  $1 \leq l \leq L$ , we obtain the average throughput  $\lambda$  for the considered system as

$$\frac{\sum_{l=1}^L R_{N_l, M_l} M_\tau \tau}{\sum_{l=1}^L T_{N_l}} \rightarrow \lambda = \frac{\mathbb{E}[R_{N,M} M_\tau \tau]}{\mathbb{E}[T_N]}, \text{ a.s.}, \quad (5)$$

as  $L \rightarrow \infty^3$  by the renewal theory [18]. Note that the expectation of the numerator in (5) is taken over  $R_{N,M}$ , which incorporates  $h_N$ ,  $B_{N,0}$ ,  $M$ , and  $N$ . As it will be proved in Section IV, there exists a steady-state distribution for  $B_{N,0}$ . Thus,  $B_{N,0}$  is with the distribution given by this steady-state distribution, and this guarantees that (5) is valid. Define the maximum throughput  $\lambda^*$  as

$$\lambda^* \triangleq \sup_{N \in \mathcal{N}, 0 \leq M \leq M_\tau} \frac{\mathbb{E}[R_{N,M} M_\tau \tau]}{\mathbb{E}[T_N]}, \quad (6)$$

where  $\sup(\cdot)$  indicates the least upper bound for a set of real numbers, and

$$\mathcal{N} \triangleq \{N : N \geq 1, \mathbb{E}[T_N] < \infty, \text{ for } 0 \leq M \leq M_\tau\}. \quad (7)$$

From the definition of  $\lambda$ , we see that  $M$  only appears in the numerator, such that the optimal  $M$  to maximize  $\lambda$  is the same as the one to maximize  $\mathbb{E}[R_{N,M} M_\tau \tau]$ . In addition, maximizing  $\mathbb{E}[R_{N,M} M_\tau \tau]$  over  $M$  is equivalent to maximizing  $R_{N,M}$  for each given  $N$ . Therefore, to solve problem (6), we first find the optimal saving ratio  $M^*$  that maximizes  $R_{N,M}$ ; next, with  $M^*$ , we solve the optimal stopping problem (6) to find the optimal stopping rule  $N^*$ . The details are given in the next section.

### III. OPTIMAL SOLUTION

#### A. Optimal Saving Ratio $M^*$

After the  $N$ -th round of CP, the successful transmitter obtains the information  $\mathcal{F}_{N,0}$ . Then, the transmission rate  $R_{N,M}$  is deterministic over the transmission block. Over all possible transmission rates that the transmitter can achieve, define

$$V_N = \max_{0 \leq M \leq M_\tau} R_{N,M}, \quad (8)$$

and

$$M^* = \arg \max_{0 \leq M \leq M_\tau} R_{N,M}. \quad (9)$$

<sup>3</sup>By Assumption A,  $L \rightarrow \infty$  is valid under our constant energy arrival model.

Note that when  $M = M_\tau$ , it follows that  $R_{N,M} = 0$  according to our definition in Section II-A, which implies that  $M_\tau$  cannot be optimal, and thus, we take  $0 \leq M \leq M_\tau - 1$ . We first consider a continuous version of  $R_{N,M}$  (denoted as  $R_{N,\rho}$ ) by relaxing  $M/M_\tau$  as  $\rho$ ,  $0 \leq \rho < 1$ :

$$\max_{0 \leq \rho < 1} R_{N,\rho} = \max_{0 \leq \rho < 1} (1 - \rho) \cdot \log \left( 1 + |h_N|^2 \frac{\min\{B_{N,0} + \rho M_\tau \tau E, B_{max} \delta\}}{(1 - \rho) M_\tau \tau \sigma^2} \right). \quad (10)$$

After solving (10), we will show how to obtain the optimal solution of problem (8).

First, we establish some properties for the objective function of problem (10).

*Proposition 3.1:* For arbitrary  $a, b \geq 0$ , we have that

1) the function  $y(x) = (1 - x) \log \left( 1 + \frac{a+bx}{1-x} \right)$  is concave over  $[0, 1)$ , and  $\lim_{x \rightarrow 1^-} y'(x) < 0$ ;

2) the function  $g(x) = (1 - x) \log \left( 1 + \frac{a}{1-x} \right)$  is concave and non-increasing over  $[0, 1)$ .

Since both  $y(x)$  and  $g(x)$  are continuous and differentiable, we can prove the above properties by taking the first-order and second-order derivatives. The details of the proof can be found in [20].

Since  $\rho \in [0, 1)$ , when  $\frac{B_{max} \delta - B_{N,0}}{M_\tau \tau E} < 1$ ,  $R_{N,\rho}$  is concave over  $\left[0, \frac{B_{max} \delta - B_{N,0}}{M_\tau \tau E}\right]$  by part 1) of Proposition 3.1, and  $R_{N,\rho}$  is non-increasing on  $\left[\frac{B_{max} \delta - B_{N,0}}{M_\tau \tau E}, 1\right)$  by part 2) of Proposition 3.1. Thus,  $R_{N,\rho}$  cannot achieve its maximum on  $\left(\frac{B_{max} \delta - B_{N,0}}{M_\tau \tau E}, 1\right)$ . When  $\frac{B_{max} \delta - B_{N,0}}{M_\tau \tau E} \geq 1$ ,  $R_{N,\rho}$  is simply concave over  $\rho$  on  $[0, 1)$ . Therefore, we treat this fact as a new constraint over  $\rho$ , and rewrite problem (10) as

$$\begin{aligned} \max G_{N,\rho} &= \max (1 - \rho) \log \left( 1 + |h_N|^2 \frac{B_{N,0} + \rho M_\tau \tau E}{(1 - \rho) M_\tau \tau \sigma^2} \right) \\ \text{s.t. } & B_{N,0} + \rho M_\tau \tau E \leq B_{max} \delta, \quad 0 \leq \rho < 1. \end{aligned} \quad (11)$$

Next, we establish the following proposition to solve problem (11), where the obtained solution is optimal to problem (10) as well.

*Proposition 3.2:* The optimal solution  $\rho^*$  to problem (11) is given by:

$$\rho^* = \begin{cases} \min \left\{ \rho_0, \frac{B_{max} \delta - B_{N,0}}{M_\tau \tau E} \right\}, & \text{when } \frac{C+D}{1+C} \geq \log(1+C); \\ 0, & \text{otherwise,} \end{cases}$$

where  $C = \frac{|h_N|^2 B_{N,0}}{M_\tau \tau \sigma^2}$ ,  $D = \frac{|h_N|^2 E}{\sigma^2}$ , and  $\rho_0$  is the unique solution for the equation  $\log \left( 1 + \frac{C+D\rho}{1-\rho} \right) = \frac{C+D}{1-\rho+C+D\rho}$  when  $\frac{C+D}{1+C} \geq \log(1+C)$ .

*Proof:* See Appendix A. ■

Based on the optimal solution  $\rho^*$ , the optimal saving ratio  $M^*$  for  $R_{N,M}$  in (8) can be obtained easily: We only need to compare  $R_{N, \lfloor \rho^* M_\tau \rfloor}$  against  $R_{N, \lceil \rho^* M_\tau \rceil}$ , and  $M^*$  should attain the larger value. Specifically, we have the following result.

*Proposition 3.3:* The optimal saving ratio  $M^*$  to problem

(8) is given by

$$M^* = \begin{cases} \lfloor \rho^* M_\tau \rfloor, & \text{if } R_{N, \lfloor \rho^* M_\tau \rfloor} \geq R_{N, \lceil \rho^* M_\tau \rceil}; \\ \lceil \rho^* M_\tau \rceil, & \text{if } R_{N, \lceil \rho^* M_\tau \rceil} > R_{N, \lfloor \rho^* M_\tau \rfloor}; \\ 0, & \text{otherwise.} \end{cases} \quad (12)$$

where  $\rho^*$  is obtained by Proposition 3.2.

*Remark 3.1:* After each round of CP, the successful transmitter can compute  $M^*$  according to Proposition 3.3 based on the information  $\mathcal{F}_{N,0}$  before making any decisions. Then, it follows the optimal stopping rule  $N^*$  to decide whether to release the channel (when  $n < N^*$ ) or to start the save-then-transmit process (when  $n = N^*$ ), which will be discussed in the next subsection.

### B. Optimal Stopping Rule for Channel Probing

In previous subsection,  $V_N$  is treated as a real number for known  $\mathcal{F}_{N,0}$ . However, at each round of CP,  $\mathcal{F}_{N,0}$  is unknown before the successful CP and should be treated as a random variable, which implies that  $V_N$  should also be a random variable. With a little abuse of notation, we still use  $V_N$  to denote the maximum of transmission rate with  $\mathcal{F}_{N,0}$  being random. Thus, the optimal throughput  $\lambda^*$  in (6) can be rewritten as

$$\lambda^* = \sup_{N \in \mathcal{N}} \frac{\mathbb{E}[V_N M_\tau \tau]}{\mathbb{E}[T_N]}. \quad (13)$$

Then, we define the optimal stopping rule as

$$N^* \triangleq \arg \sup_{N \in \mathcal{N}} \frac{\mathbb{E}[V_N M_\tau \tau]}{\mathbb{E}[T_N]}. \quad (14)$$

This problem is related to the *rate of return* optimal stopping problem discussed in [12]. Note that in (13), the random sequence  $\{V_N\}_{N \geq 1}$  may not be identically distributed.

First, we consider the optimal stopping rule  $N^*$  for the homogeneous case: EH rates are the same, i.e.,  $E^1 = \dots = E^I$ ,  $\{h^1, \dots, h^I\}$  are i.i.d., and each transmitter contends for the channel with the same probability such that  $\{V_N\}_{N \geq 1}$  are i.i.d. Afterwards, we extend the results to the heterogenous case, i.e.,  $\{V_N\}_{N \geq 1}$  are independent but not identically distributed.

1) *Homogeneous case:* In order to solve problem (13), we introduce an auxiliary problem: Given some  $\lambda > 0$ , we define the maximum expected net reward as

$$r^*(\lambda) \triangleq \sup_{N \in \mathcal{N}} \mathbb{E}[r_N(\lambda)], \quad (15)$$

where  $r_N(\lambda) = V_N M_\tau \tau - \lambda T_N$  is the net reward if we transmit after the  $N$ -th round of CP, and the expectation in (15) is taken over  $V_N$  (which incorporates  $h_N$ ,  $B_{N,0}$  and  $M^*$ ),  $N$ , and  $T_N$  as given in (4). This problem is similar to the problem for selling an asset without recall [12]. Furthermore, we rewrite the net reward by  $r_N(\lambda) = (V_N - \lambda) M_\tau \tau - \lambda \tau \sum_{j=1}^N K_j$ . The term  $(V_N - \lambda) M_\tau \tau$  can be regarded as the gain from data transmission which is maximized by the optimal saving ratio  $M^*$  from Proposition 3.3, and the term  $\lambda \tau \sum_{j=1}^N K_j$  is treated as the accumulated cost for the total of  $N$  rounds of CP. Thus, similar to the problem for selling an asset without

recall [12], we consider the following sequence of rewards:

$$r_1(\lambda), r_2(\lambda), \dots$$

The following lemma shows that there exists an optimal stopping rule for problem (15).

*Lemma 3.1:* The optimal stopping rule  $N^*(\lambda)$  for (15) exists, such that

$$r^*(\lambda) = \mathbb{E}[r_{N^*(\lambda)}(\lambda)], \quad (16)$$

where  $r^*(\lambda)$  is the optimal value defined in (15). Moreover,  $r^*(\lambda)$  is decreasing over  $\lambda$ .

*Proof:* See Appendix B. ■

The next lemma connects problem (15) and problem (13), to prepare us for obtaining the optimal stopping rule in the original problem (13).

*Lemma 3.2:* (i) If there exists  $\lambda^*$  such that  $r^*(\lambda^*) = 0$ , this  $\lambda^*$  is the optimal throughput defined in (13). Moreover, if  $r^*(\lambda^*) = 0$  is attained at  $N^*(\lambda^*)$ , the stopping rule  $N^* = N^*(\lambda^*)$  is optimal for (13), which is defined in (14).

(ii) Conversely, if (13) and (14) are true, there is  $r^*(\lambda^*) = 0$ , which is attained at  $N^*$  given by (14).

*Proof:* The conclusion directly followings Theorem 1 in Chapter 6 of [12], which is skipped here. ■

By the second part of Lemma 3.1,  $r^*(\lambda)$  is decreasing over  $\lambda$  from a nonnegative value to  $-\infty$ . Then, there exists  $\lambda^*$  such that  $r^*(\lambda^*) = 0$ . Therefore, by the first part of Lemma 3.1 and Lemma 3.2, we know that the optimal stopping rule  $N^*(\lambda^*)$  for  $r^*(\lambda^*)$  exists, which is also optimal for (13), i.e.,  $N^* = N^*(\lambda^*)$ . The following proposition formally quantifies the optimal stopping rule  $N^*$  and the equation to compute the optimal throughput  $\lambda^*$ .

*Proposition 3.4:* The optimal stopping rule to solve problem (13) is given by

$$N^* = \min \{N \geq 1 : V_N \geq \lambda^*\}, \quad (17)$$

where  $V_N = R_{N, M^*}$  with  $M^*$  given in Proposition 3.3. Moreover,  $\lambda^*$  satisfies the following equation

$$\mathbb{E}[(V_N - \lambda^*)^+] = \frac{\lambda^*}{Q M_\tau}, \quad (18)$$

where the function  $(x)^+$  means  $\max\{x, 0\}$  for any real number  $x$ , and  $Q$  is the probability of a successful channel contention defined in Section II-A.

The proof can be found in [20].

*Remark 3.2:* By solving (18), we can obtain the optimal throughput  $\lambda^*$ . Since  $V_N = R_{N, M^*}$  is a function of random variables  $h_N$  and  $B_{N,0}$ , we can calculate the expectation on the left-hand side of (18) for each given  $\lambda$ . Note that how to compute the distribution, i.e., the steady-state distribution  $\Pi$  with given  $\lambda$ , for  $B_{N,0}$  will be given in Section IV. It is worth noticing that for a given  $\lambda \geq 0$ , an upper bound of this expectation can be obtained by fixing  $\Pi = [0, \dots, 0, 1]$ . As  $\lambda$  increases from zero to infinity, this upper bound will decrease to zero at some  $\tilde{\lambda} < \infty$ . Since the right-hand side of (18) is strictly increasing over  $\lambda$  within the range  $[0, +\infty)$ , there at least exists one  $\lambda^*$  satisfying (18). Therefore, an

one-dimension search can be applied to obtain the optimal throughput over the range  $[0, \tilde{\lambda}]$ .

2) *Heterogeneous case*: For the case that  $\{V_N\}_{N \geq 1}$  are independent but not identically distributed (across both time  $N$  and different users), we use the similar idea in [11] to treat  $V_N$  as a compound random variable whose distribution is a composition of distributions over different transmitters, as shown below

$$\Pr\{V_N \leq \lambda\} = \sum_{i=1}^I \frac{Q_i}{Q} \Pr\{V_N^i \leq \lambda\}, \quad (19)$$

where  $\Pr\{V_N^i \leq \lambda\}$  is the conditional probability over  $i$ . Recall that  $Q$  is the probability of a successful channel contention, and  $Q_i$  is the probability that the  $i$ -th transmitter is the successful one for  $1 \leq i \leq I$ , which are both defined in Section II-A.

*Proposition 3.5*: The optimal stopping rule  $N^*$  has the same form as in (17) when  $\{V_N\}_{N \geq 1}$  are independent but not identically distributed. In addition,  $\lambda^*$  defined in (13) can be computed from the equation

$$\sum_{i=1}^I \frac{Q_i}{Q} \mathbb{E}[(V_N^i - \lambda^*)^+] = \frac{\lambda^*}{QM_\tau}. \quad (20)$$

*Remark 3.3*: Proposition 3.5 implies that when all transmitters have different statistics of the CSI and the ESI, each one still has the same threshold which is globally determined. The intuition is similar to that in [11]: In order to guarantee the overall system performance, the transmitter with a bad channel condition and a low energy level should “sacrifice” its own reward, while the one with good conditions should transmit more data.

The next theorem gives the overall optimal scheduling policy in the DOS with save-then-transmit scheme, for both homogeneous and heterogeneous cases.

*Theorem 3.1*: After the  $N$ -th round of CP, it is optimal for the successful transmitter to take one of the following two options:

1) transmit after  $M^*$  slots for EH if  $V_N \geq \lambda^*$ , where  $M^*$  is given by Proposition 3.3;

2) release the channel immediately if  $V_N < \lambda^*$ , and let all transmitters to perform the next round of CP.

*Proof*: The conclusion directly follows Propositions 3.3 and 3.5. ■

#### IV. BATTERY DYNAMICS

In this section, we show that the energy level stored at each transmitter forms a Markov chain over time, while the state transition probabilities for different transmitters are coupled together. However, we propose an iterative algorithm to compute the corresponding steady-state distribution, which is shown to converge the global optimal point. Note that the algorithm is valid under Assumption A that EH rates are identical over thousands of time slots, which guarantees the asymptotic analysis in this section sound.

Note that after CP, if the successful transmitter releases the channel, then the next round of CP will start. If the transmitter starts the transmission, its energy level will become zero at the end of the transmission block according to Section II-A. During this time, all other transmitters will keep harvesting energy within this period. Thus, the energy level transition over the transmission block can be determined. To simplify our analysis, the transmission block is treated as one time slot for the purpose of counting battery state transitions.

For transmitter  $i$  with EH rate  $E^i$ ,  $1 \leq i \leq I$ , the set of its energy states is given by  $\Delta_i = \{0, E^i\tau, 2E^i\tau, \dots, \lfloor \frac{B_{max}\delta}{E^i} \rfloor E^i, B_{max}\delta\}$ , and the energy level  $B_t^i \in \Delta_i$ . In addition, we denote  $\Pi_t^i = [\pi_{t,0}^i \cdots \pi_{t,B_{max}}^i]$  as distribution of the energy level for the  $i$ -th transmitter at time  $t$ .

Suppose that transmitter  $i$  is at energy level  $u_i \in \Delta_i$ , there are three events that may happen at time slot  $t$ :

(i) It occupies the channel and transmits. According to Section II-A, transmitter  $i$  consumes all the energy for the transmission, and transfers to the energy level 0 after the transmission. Thus, the transition probability is given by

$$p_{u_i,0}^i = Q_i p_{tr}^i(u_i), \quad (21)$$

where  $Q_i$  is the probability that the  $i$ -th transmitter occupies the channel, and  $p_{tr}^i(u_i)$  is the probability that it successfully transmits at the energy level  $u_i$ . Furthermore, according to (17),  $p_{tr}^i(u_i)$  can be computed as

$$\begin{aligned} p_{tr}^i(u_i) &= \Pr\{V^i \geq \lambda^*\} \\ &= \Pr\left\{\log\left(1 + |h|^2 \frac{u_i + M^* \tau E^i}{(M_\tau - M^*) \tau \sigma^2}\right) \geq \frac{\lambda^*}{1 - \frac{M^*}{M_\tau}}\right\}, \end{aligned} \quad (22)$$

where  $M^*$  is the optimal saving ratio according to Proposition 3.3. Note that in (22),  $|h|^2$  is the only random variable while its distribution is known.

(ii) Other transmitters occupy the channel and transmit. If anyone among the other  $I - 1$  transmitters sends data, transmitter  $i$  will harvest  $M_\tau \tau E^i$  units of energy during this period, and then attain level  $v_i = \min\{u_i + E^i M_\tau \tau, B_{max}\delta\}$ . Suppose the  $j$ -th transmitter transmits. Similar to the first case, the probability of transmission performed by the  $j$ -th transmitter is given by  $Q_j \sum_{b=0}^{B_{max}} \pi_{t,b}^j p_{tr}^j(bE^j\tau)$ , where  $bE^j\tau \in \Delta_j$  and thus  $b \in \{0, 1, 2, \dots, \lfloor \frac{B_{max}\delta}{E^j\tau} \rfloor, B_{max}\}$ . Since there are in total  $I - 1$  transmitters, the transition probability for the transmitter  $i$  from level  $u_i$  to  $v_i$  is given by

$$p_{u_i,v_i}^i = \sum_{j \neq i} Q_j \sum_{b=0}^{B_{max}} \pi_{t,b}^j p_{tr}^j(bE^j\tau). \quad (23)$$

(iii) No transmission happens. In this case, transmitter  $i$  just harvests  $E^i\tau$  units of the energy and goes into  $w_i = \min\{u_i + E^i\tau, B_{max}\delta\}$ . The probability of this case happening can be directly obtained as

$$p_{u_i,w_i}^i = 1 - p_{u_i,0}^i - p_{u_i,v_i}^i. \quad (24)$$

Note that when  $\tilde{u}_i = v_i = w_i$ , the transition probability is just

given by

$$p_{u_i, \tilde{u}_i}^i = p_{u_i, v_i}^i + p_{u_i, w_i}^i = 1 - p_{u_i, 0}^i. \quad (25)$$

In this way, we can compute all  $\{p_{u_i, \tilde{u}_i}^i\}$  for  $1 \leq i \leq I$ , where  $u_i \in \Delta_i$  and  $\tilde{u}_i \in \{0, v_i, w_i, B_{max}\delta\}$ . The transition probability matrix is nothing but  $\mathbf{P}_t^i = \{p_{u_i, \tilde{u}_i}^i\}$  with dimension  $(\lceil \frac{B_{max}\delta}{E^i\tau} \rceil + 1) \times (\lceil \frac{B_{max}\delta}{E^i\tau} \rceil + 1)$ . Obviously,  $\mathbf{P}_t^i$  is a stochastic matrix, i.e., a square matrix in which all elements are nonnegative and the row sum is 1. However,  $\mathbf{P}_t^i$  depends on  $t$  since  $p_{u_i, v_i}^i$  depends on the state distribution  $\Pi_t^j$  for all  $j \neq i$ . Therefore,  $\{\mathbf{P}_t^i\}_{t \geq 0}$  is a non-homogeneous Markov chain, whose state evolution is given by

$$\Pi_{t+1}^i = \Pi_t^i \mathbf{P}_t^i, \quad t \geq 0. \quad (26)$$

We propose Algorithm I, which is summarized in Table I, to compute the steady-state distribution for all transmitters. Here, the infinity norm is applied, which is defined as  $\|\mathbf{a}\|_\infty = \max_{1 \leq i \leq n} |a_i|$  for  $\mathbf{a} = [a_1 \ \dots \ a_n]$ .

TABLE I

ALGORITHM I: COMPUTE THE STEADY-STATE DISTRIBUTION FOR ALL TRANSMITTERS.

- 
- Initialize  $\Pi_0^i$  for  $1 \leq i \leq I$ ,  $\varepsilon$ , and compute  $p_{u_i, 0}^i$  by (21) for all  $u_i \in \Delta_i$  and  $1 \leq i \leq I$ ;
  - Set  $t = 0$ , compute  $\mathbf{P}_0^i$  by (23)–(25) for all  $1 \leq i \leq I$ , and compute  $\Pi_1^i$  by (26) for all  $1 \leq i \leq I$ . Then:
    - While  $\max_{1 \leq i \leq I} \|\Pi_{t+1}^i - \Pi_t^i\|_\infty > \varepsilon$ , repeat:
      - 1)  $t = t + 1$ ;
      - 2) Update  $\mathbf{P}_t^i$  by (23)–(25) for all  $1 \leq i \leq I$ ;
      - 3) Compute  $\Pi_{t+1}^i$  by (26) for all  $1 \leq i \leq I$ ;
    - end.
  - Algorithm ends.
- 

*Proposition 4.1:* For any given initial state distribution  $\Pi_0^i$ ,  $\Pi_t^i = [\pi_{t,0}^i \ \dots \ \pi_{t, B_{max}\delta}^i]$ , generated by Algorithm I, converges to a unique steady-state distribution  $\Pi^i$  for all  $1 \leq i \leq I$ .

*Proof:* See Appendix C. ■

*Remark 4.1:* The steady-state distribution for all transmitters can be obtained by the iterative computation  $\Pi_{t+1} = \Pi_t \mathbf{P}$  over the “super” Markov system as well, which is constructed in Appendix C. However, this is not as efficient as Algorithm I. From the computational complexity point of view, suppose that each transmitter has  $B_{max}$  energy levels, and there are  $I$  transmitters in total. The number of the states in the “super” Markov chain is  $B_{max}^I$ . If there is only one processor, one iteration of the state distribution for the “super” Markov chain requires approximately  $O(2B_{max}^{2I})$  floating-point calculations. On the contrary, by using Algorithm I, (23) requires  $O(I^2 B_{max}^2)$  calculations, and updating  $\{\mathbf{P}_t^i\}$  requires  $2IB_{max}$  calculations according to (24). In addition,  $\{\Pi_t^i \mathbf{P}_t^i\}$  requires  $2IB_{max}^2$  calculations. Overall, one iteration for all transmitters requires approximately  $O(I^4 B_{max}^5)$  floating-point calculations, which is more efficient than the case

for the “super” Markov chain especially when  $B_{max}$  and  $I$  are large. Moreover, our algorithm can also be operated in a parallel way, i.e., computing  $\Pi_{t+1}^i = \Pi_t^i \mathbf{P}_t^i$  for  $1 \leq i \leq I$  at the same time over different cores.

## V. NUMERICAL RESULTS

In this section, we show the impact of different parameters on the throughput performance. The baseline is a best-effort delivery where the data is transmitted whenever the channel contention is successful. Note that such method can be realized by fixing  $M = 0$  and setting the threshold of CP in (17) as zero. Denote  $\lambda_0$  as throughput obtained by such a best-effort delivery, which can be calculated as

$$\lambda_0 = \frac{\sum_{i=1}^I \frac{Q_i}{Q} \mathbb{E} \left[ M_\tau \log \left( 1 + |h_n^i|^2 \frac{B_{n,0}^i}{M_\tau \tau \sigma^2} \right) \right]}{\frac{1}{Q} + M_\tau}. \quad (27)$$

Denote  $\lambda^*$  as the throughput obtained by the proposed DOS with the optimal saving ratio  $M^*$ .

In general, a typical button cell has the capacity of 150 mAh with the end-point voltage of 0.9 V, which is equal to 150 mAh  $\times$  3600 s/h  $\times$  0.9 V = 486 J. A thin-film rechargeable battery can offer 50  $\mu$ Ah with 3.3 V, which is equal to 0.594 J. Since a typical transmission time interval is on the time scale of milliseconds, we let the energy unit be  $\delta = 10^{-3}$  J in the simulation. Accordingly, we set the capacity of the battery  $B_{max}\delta \in [100\delta, 20000\delta]$ , which is smaller than the capacity volume of a button cell battery. Also, the current commercial solar panel can provide power from 1 W to more than 400 W, which is equivalent to  $1\delta$ /ms  $\sim$   $400\delta$ /ms. According to this fact, in our simulation, we let the EH rate vary within the range  $[1\delta, 16\delta]$ . In addition, the channel gains are i.i.d for different links and the squares of the channel gains follow exponential distribution with mean 5. The variance of the noise is set to be 100 mW. The length of one time slot is also unified with  $\tau = 1$  ms, and the number of slots for data transmission is  $M_\tau = 300$ .

In Fig. 2(a), we draw the throughput as the function of the EH rates with fixed battery capacity  $B_{max}\delta = 20000\delta$ . We consider a homogeneous *ad hoc* network with 5 transmitter-receiver pairs in total. We observe that the proposed DOS with the save-then-transmit scheme outperforms the best-effort delivery and the gain increases as the EH rate increases. When  $E = 2\delta$ ,  $\lambda^*$  is about 2.75 times of  $\lambda_0$ . In Fig. 2(b), we plot the throughput as the function of the battery capacity  $B_{max}\delta$  over a fixed EH rate at  $E = 6\delta$ . The intuition is similar as the case of various EH rates. However, when the battery capacity is more than  $12000\delta$ ,  $\lambda^*$  is relatively stable as  $B_{max}\delta$  increases. It implies that if the battery capacity is large enough, it will not influence the throughput performance of the DOS with the save-then-transmit scheme. Intuition is that since the EH rate is fixed, the extra storage for the energy is rarely used.

Finally, in Fig. 3, we show the steady-state distribution of the stored energy at each transmitter. In this setup, there are 6 transmitter-receiver pairs in the network; and each transmitter has a unique EH rate ( $E = \delta, 2\delta, \dots, 6\delta$ ) and contends for the

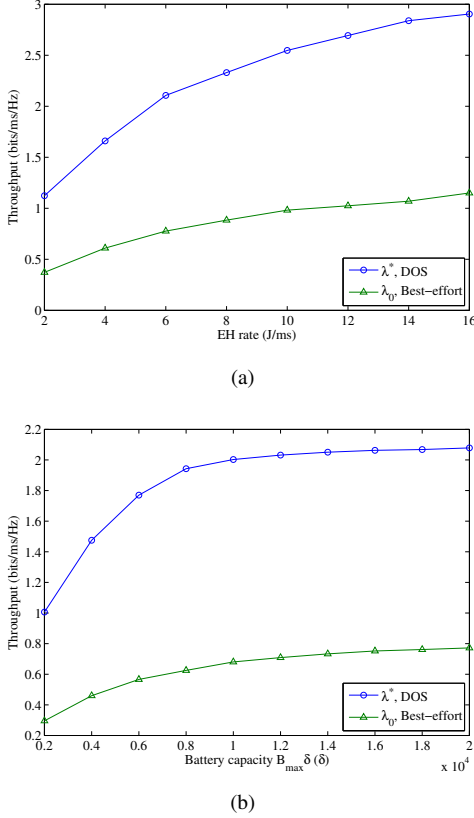


Fig. 2. Throughput comparison ( $\lambda^*$ - DOS with the same-then-transmit scheme,  $\lambda_0$ - Best-effort delivery (a) Throughput v.s. EH rate; (b) Throughput v.s. battery capacity.

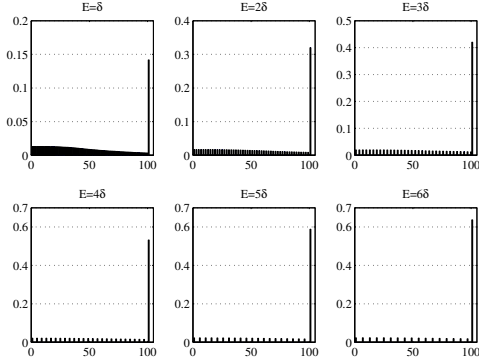


Fig. 3. Steady-state distribution in a heterogeneous network.

channel with the same probability  $1/6$ . Each bar in the figure indicates the probability of staying at the corresponding energy level. We have  $B_{max}\delta = 100\delta$  and  $M_\tau = 5$  in this case. We observe that the transmitter with a higher EH rate is more likely to stay in the state with full energy.

## VI. CONCLUSION

In this paper, we investigated the DOS for a single-hop *ad hoc* network in which each transmitter is powered by a renewable energy source and accesses the channel randomly.

Our DOS framework includes two successive processes: All transmitters first probe the channel via random access, and then the successful transmitter decides whether to give up the channel or to start the save-then-transit process. Given the CSI and the ESI, the optimal saving ratio was first obtained by maximizing the average transmission rate over the transmission block. Then, based on this result, the expected throughput maximization problem was solved to obtain the optimal stopping rule, which was formulated as a rate-of-return optimal stopping problem. The optimal stopping rule was proved to be a threshold policy for both the homogeneous and heterogeneous cases. Furthermore, the stored energy level at each transmitter was shown to be a non-homogeneous Markov chain, which was proven to own a steady-state distribution as time goes to infinity, where we proposed an efficient iterative algorithm for its computation.

## ACKNOWLEDGMENT

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## APPENDICES

### A. Proof of Proposition 3.2

By part 1) of Proposition 3.1, we obtain that  $G_{N,\rho}$  is concave over  $\rho \in [0, 1)$ , which means that  $G'_{N,\rho} = \frac{dG_{N,\rho}}{d\rho}$  is decreasing over  $[0, 1)$  and attains its maximum at  $\rho = 0$ . Then, finding the maximum of  $G_{N,\rho}$  boils down to two cases:

- 1)  $G'_{N,\rho}|_{\rho=0} < 0$ : It follows that  $G_{N,\rho}$  is decreasing over  $[0, 1)$ , and  $\rho^* = 0$  is the optimum.
- 2)  $G'_{N,\rho}|_{\rho=0} \geq 0$ : The point  $\rho_0$ , satisfying  $G'_{N,\rho}|_{\rho=\rho_0} = 0$ , lies on the right-hand side of  $\rho = 0$ . By part 1) of Proposition 3.1,  $G'_{N,\rho} < 0$  as  $\rho \rightarrow 1^-$ , which implies that  $\rho_0 \in [0, 1)$ . Since the optimal point  $\rho^* \leq \frac{B_{max}\delta - B_{N,0}}{M_\tau\tau E}$  due to (11), it follows that  $\rho^* = \min\left\{\rho_0, \frac{B_{max}\delta - B_{N,0}}{M_\tau\tau E}\right\}$ .

Note that  $G'_{N,\rho}|_{\rho=0} \geq 0$  is equivalent to  $\frac{C+D}{1+C} \geq \log(1+C)$ , where  $C = \frac{|h_N|^2 B_{N,0}}{M_\tau\tau\sigma^2} \geq 0$ ,  $D = \frac{|h_N|^2 E}{\sigma^2} \geq 0$ , and  $G'_{N,\rho}|_{\rho=\rho_0} = 0$  is equivalent to

$$\log\left(1 + \frac{C + D\rho_0}{1 - \rho_0}\right) = \frac{C + D}{1 - \rho_0 + C + D\rho_0}. \quad (28)$$

The proof that (28) has a unique solution when  $\frac{C+D}{1+C} \geq \log(1+C)$  can be found in [20]. In short, it is shown in [20] that the left-hand side of (28) has only one intersection with the right-hand side for  $\rho \in [0, 1)$ , and the intersection point is attained at  $\rho_0$ . Since  $\rho_0$  is unique in (28),  $\rho_0$  can be found just by adopting bisection search. In conclusion, the proposition is proved.

### B. Proof of Lemma 3.1

For the first part of lemma, it follows by Theorem 1 in Chapter 3 of [12] that  $N^*(\lambda)$  exists and  $r^*(\lambda)$  is attained by this  $N^*(\lambda)$  if the following two conditions are satisfied:

$$(C1) \quad \limsup_{N \rightarrow \infty} r_N(\lambda) \leq \infty, \text{ a.s.};$$



$$(C2) \quad \mathbb{E} [\sup_N r_N(\lambda)] \leq \infty.$$

For (C1), since  $\mathbb{E} [R_{N,M}] < \infty$  and  $\mathbb{E} [(R_{N,M})^2] < \infty$ , both  $\mathbb{E}[V_N]$  and  $\mathbb{E}[V_N^2]$  are finite as well. Note that  $\lambda\tau \sum_{j=1}^N K_j \rightarrow \infty$  as  $N \rightarrow \infty$ . Thus, by Theorem 1 in chapter 4 of [12], we obtain  $\limsup_{N \rightarrow \infty} r_N(\lambda) \rightarrow -\infty$  a.s., which proves that (C1) holds.

For (C2), it can be shown that

$$\begin{aligned} \mathbb{E} \left[ \sup_N r_N(\lambda) \right] &= \mathbb{E} \left[ \sup_N \left( (V_N - \lambda)M_{\tau\tau} - \lambda\tau \sum_{j=1}^N K_j \right) \right] \\ &\leq \mathbb{E} \left[ \sup_N ((V_N - \lambda)M_{\tau\tau} - \lambda\tau N) \right], \end{aligned} \quad (29)$$

due to the fact that  $K_j \geq 1$  for  $1 \leq j \leq N$ . Since  $\mathbb{E} [V_N^2] < \infty$  and  $\{V_N\}_{N \geq 1}$  are i.i.d., it follows that the right-hand side of (29) is finite by Theorem 1 in Chapter 4 of [12], which implies that (C2) also holds.

For the second part, it directly follows Lemma 1 in Chapter 6 of [12] that  $r^*(\lambda)$  is decreasing over  $\lambda$ .

In conclusion, the lemma is proved.

### C. Proof of Proposition 4.1

To prove this proposition, we construct a ‘‘super’’ Markov chain in which each state is a ‘‘super’’ vector of aggregated energy levels across the whole network, whose the transition probability matrix does not change over time  $t$ . Afterwards, we prove that such a ‘‘super’’ Markov chain has a unique steady-state distribution. Then, we show that for any time  $t$  in the original Markov chain, one iteration to update  $\Pi_t^i$  for  $1 \leq i \leq I$  in Algorithm I is equivalent to the evolution of the state distribution in the ‘‘super’’ Markov chain, which proves the convergence of Algorithm I.

To construct such a ‘‘super’’ Markov chain, we need to jointly consider the states of energy levels across all transmitters. Denote  $\Sigma$  as the set of all possible battery states of the whole system, i.e.,

$$\Sigma = \{ \mathbf{u} = (u_1 \cdots u_I) : u_1 \in \Delta_1, \dots, u_I \in \Delta_I \}. \quad (30)$$

Furthermore, we denote  $\mathbf{B}_t$  as the battery state of the system at time  $t$ , where  $\mathbf{B}_t \in \Sigma$ . Note that the number of elements in  $\Sigma$  is  $(\lceil \frac{B_{max}\delta}{E^1\tau} \rceil + 1) \times \cdots \times (\lceil \frac{B_{max}\delta}{E^I\tau} \rceil + 1)$ .

Suppose that  $\mathbf{B}_t = \mathbf{u}$ . There are  $I + 1$  possible events at time  $t$ : a transmission is performed by transmitter  $i$ , where  $1 \leq i \leq I$ , and no transmission happens. Then, we can compute the transition probability between any two states, and the transition probability matrix  $\mathbf{P}$  for  $\{\mathbf{B}_t\}_{t \geq 0}$  can be obtained. The details for the computation can be found in [20]. It can be shown that  $\mathbf{P}$  is a stochastic matrix and is invariant over time. Therefore, there exists a unique probability vector  $\Pi$  such that the following equation holds [21]:

$$\Pi = \Pi \mathbf{P}. \quad (31)$$

In fact,  $\Pi$  is the steady-state distribution of  $\{\mathbf{B}_t\}_{t \geq 0}$ .

So far, we have constructed a ‘‘super’’ Markov chain  $\{\mathbf{B}_t\}_{t \geq 0}$  for the whole system of which the steady-state

distribution exists and is unique. Therefore, by the iteration  $\Pi_{t+1} = \Pi_t \mathbf{P}$ , it follows that  $\lim_{t \rightarrow \infty} \Pi_t = \Pi$  [21]. Thus, we only need to show that

$$\Pi_{t+1} = \Pi_t \mathbf{P} \Leftrightarrow \Pi_{t+1}^i = \Pi_t^i \mathbf{P}_t^i, \quad 1 \leq i \leq I, \quad t \geq 0. \quad (32)$$

If (32) is true, the state distribution of each transmitter converges to the unique steady-state distribution. The proof for (32) can be found in [20].

In conclusion, the convergence of Algorithm I is proved.

### REFERENCES

- [1] S. Sudevalayam and P. Kulkarni, ‘‘Energy harvesting sensor nodes: survey and implications,’’ *IEEE Communications Surveys & Tutorials*, vol. 13, no. 3, pp. 443-461, Third Quarter 2011.
- [2] B. Medepally, N. B. Mehta, and C. R. Murthy, ‘‘Implications of energy profile and storage on energy harvesting sensor link performance,’’ in *Proc. GLOBECOM*, Hawaii, U. S., Nov. 2009.
- [3] C. K. Ho and R. Zhang, ‘‘Optimal energy allocation for wireless communications with energy harvesting constraints,’’ *IEEE Trans. Signal Processing*, vol. 60, no. 9, pp. 4808-4818, Sept. 2012.
- [4] C. Huang, R. Zhang, and S. Cui, ‘‘Throughput maximization for the Gaussian relay channel with energy harvesting constraints,’’ *IEEE J. Sel. Areas Commun.*, vol. 31, no. 8, pp. 1469-1479, Aug. 2013.
- [5] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, ‘‘Transmission with energy harvesting nodes in fading wireless channels: optimal policies,’’ *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1732-1743, Sept. 2011.
- [6] S. Luo, R. Zhang, and T. J. Lim, ‘‘Optimal save-then-transmit protocol for energy harvesting wireless transmitters,’’ *IEEE Trans. Wireless Commun.*, vol. 12, no. 3, pp. 1196-1207, Mar. 2013.
- [7] F. Iannello, O. Simeone, and U. Spagnolini, ‘‘Medium access control protocols for wireless sensor networks with energy harvesting,’’ *IEEE Trans. Commun.*, vol. 60, no. 5, pp. 1381-1389, May 2012.
- [8] M. Andrews, K. Kumaran, K. Ramanan, A. Stolyar, P. Whiting, and R. Vijayakumar, ‘‘Providing quality of service over a shared wireless link,’’ *IEEE Commun. Mag.*, vol. 39, no. 2, pp. 150-154, Feb. 2001.
- [9] P. Viswanath, D. N. Tse, and R. Laroia, ‘‘Opportunistic beamforming using dumb antennas,’’ *IEEE Trans. Information Theory*, vol. 48, no. 6, pp. 1277-1294, Jun. 2002.
- [10] D. Zheng, W. Ge, and J. Zhang, ‘‘Distributed opportunistic scheduling for ad hoc networks with random access: An optimal stopping approach,’’ *IEEE Trans. Information Theory*, vol. 55, no. 1, pp. 205-222, Jan. 2009.
- [11] D. Zheng, ‘‘Physical-layer aware control and optimization in stochastic wireless networks,’’ Ph.D. thesis, Arizona State Univ., Aug. 2007.
- [12] T. S. Ferguson, ‘‘Optimal stopping and applications,’’ available at: <http://www.math.ucla.edu/~tom/Stopping/Contents.html>, 2006.
- [13] C. Thejaswi P. S., J. Zhang, M.-O. Pun, and H. V. Poor, ‘‘Distributed opportunistic scheduling with two-level channel probing,’’ *IEEE INFOCOM*, Rio de Janeiro, Brazil, pp. 1683-1691, Apr. 2009.
- [14] W. Stadje, ‘‘An optimal stopping problem with two levels of incomplete information,’’ *Mathematical Methods of Operations Research*, vol. 45, no. 1, pp. 119-131, Feb. 1997.
- [15] M. Beaudin, H. Zareipour, A. Schellenberglobe, and W. Rosehart, ‘‘Energy storage for mitigating the variability of renewable electricity sources: an updated review,’’ *Energy for Sustainable Development*, vol. 14, no. 4, pp. 302-314, Dec. 2010.
- [16] S. Combs, *The energy report*, Chapter 10, available at: <http://www.window.state.tx.us/specialrpt/energy/>, May, 2008.
- [17] N. Abramson, ‘‘The throughput of packet broadcasting channels,’’ *IEEE Trans. Commun.*, vol. 25, no. 1, pp. 117-128, Jan. 1977.
- [18] P. Billingsley, *Probability and Measure*, 3rd ed., New York: John Wiley & Sons, Inc., 1995.
- [19] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge, U.K.: Cambridge Univ. Press, 2004.
- [20] H. Li, C. Huang, J. Zhang, and S. Cui, ‘‘Distributed opportunistic scheduling for wireless networks powered by renewable energy sources,’’ *Technical Report, Department of ECE, TAMU*, Texas, available at: [http://www.public.asu.edu/~chuang78/tech\\_report.pdf](http://www.public.asu.edu/~chuang78/tech_report.pdf), 2013.
- [21] R. Gallager, *Discrete Stochastic Processes*, Boston: Kluwer Academic Publishers, 1996.