

# Optimal Privacy-Preserving Energy Management for Smart Meters

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**Abstract**—Smart meters, designed for information collection and system monitoring in smart grid, report fine-grained power consumption to utility providers. With these highly accurate profiles of energy usage, however, it is possible to identify consumers’ specific activity or behavior patterns, thereby giving rise to serious privacy concerns. In this paper, this concern is addressed by using battery energy storage. Beyond privacy protection, batteries can also be used to cut down the electricity bill. From a holistic perspective, a dynamic optimization framework is designed for consumers to strike a tradeoff between the smart meter data privacy and the electricity bill. In general, a major challenge in solving dynamic optimization problems lies in the need of the knowledge of the future electricity consumption events. By exploring the underlying structure of the original problem, an equivalent problem is derived, which can be solved by using only the current observations. An online control algorithm is then developed to solve the equivalent problem based on the Lyapunov optimization technique. To overcome the difficulty of solving a mixed-integer nonlinear program involved in the online control algorithm, the problem is further decomposed into multiple cases and the closed-form solution to each case is derived accordingly. It is shown that the proposed online control algorithm can optimally control the battery operations to protect the smart meter data privacy and cut down the electricity bill, without the knowledge of the statistics of the time-varying load requirement and the electricity price processes. The efficacy of the proposed algorithm is demonstrated through extensive numerical evaluations using real data.

**Index Terms**—Smart Meter, Smart Grid, Data Privacy, Load Monitor, Cost Saving, Battery

## I. INTRODUCTION

### A. Motivation

Smart meters have been widely deployed in homes and businesses for information collection and system monitoring. The information collected by smart meters can be used to support both dynamic pricing and a two-way flow of electricity between homes (or micro grids) and power grids, thereby offering higher reliability, security and efficiency in power supply [1]–[4]. However, this information may be used by malicious users, giving rise to serious concerns of privacy, especially for residential consumers.

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The privacy of smart meter data has become an important topic of research. Recent work [5] provides an overview of the privacy implications of fine-grained power consumption monitoring. From the information collected by smart meters, complex usage patterns, such as residential occupancy and social activities, can be extracted without a priori knowledge of household activities [6]. Multiple methods [7]–[12] have been proposed to protect smart meter data privacy. One common approach is to introduce uncertainty in individual power consumption by perturbing the load measurements [7], [8]. However, this approach requires modification of the metering infrastructure, which may not be logistically and economically viable, with millions of smart meters already installed. Besides, the modification of usage data would result in inaccurate billing and grid controls, thereby undermining the grid management.

In this paper, we address the threats to electricity consumer privacy by using energy storage devices (such as a rechargeable battery). In particular, a battery can be used to flatten the usage patterns of electricity consumers. Ideally, all usage patterns can be perfectly masked by charging and discharging the battery to maintain a constant metered load, such that all load measurements equal to the average consumer’s load. However, it is challenging to control the battery charging/discharging operations to achieve this, without the knowledge of future electricity consumption events, not to mention the time-varying load requirements with possibly unknown statistics. Existing works develop heuristic algorithms to tackle this challenge. Kalogridis *et al.* [9] proposes a “best-effort” algorithm against power load changes by charging/discharging the battery to maintain the current load equal to the previous load. McLaughlin *et al.* [10] proposes a non-intrusive load leveling algorithm (NILL) to mask appliance features that are used by non-intrusive appliance load monitoring (NALM) algorithms (e.g., [13]–[15]) to detect appliance switch-on/off events. None of these works considered optimal control algorithms to protect smart meter data privacy. Besides, it is not cost-free to protect privacy by using the battery, due to the limited number of charging/discharging cycles for each battery.

Beyond privacy protection, the rechargeable battery can also be used to reduce the electricity bill, as the electricity price is time-varying. It is likely that consumers may tolerate some degree of information leakage to optimize the electricity bill by adapting their needs to the time-varying electricity price.

Recent work [11] proposes wallet-friendly privacy protection for smart meters by using stochastic dynamic programming. Along a different line, recent work [12] protects the smart meter data privacy by using energy harvesting and storage devices from an information theoretic perspective. Both works [11] and [12] require knowledge of the statistics of the load requirements, which may not be readily available, and do not take into consideration the battery operation cost. Moreover, the solution to dynamic programming requires solution of a value function that can be computationally difficult when the state space of the system is large and hence suffers from the curse of dimensionality.

### B. Summary of Major Contributions

Against this backdrop, the main thrust of this study is dedicated to developing an online control algorithm with low computational complexity that can jointly protect smart meter data privacy and reduce the electricity bill, while taking into account the cost of repeated charging and discharging on the battery’s lifetime. This is a challenging problem, due to the time-varying load requirements and electricity prices with possibly unknown statistics. Our main contributions are summarized as follows:

- We develop a dynamic optimization framework that can protect smart meter data privacy in a cost-effective manner, while taking into account the impact of repeated charging and discharging on the battery’s lifetime. One major challenge in solving this problem is the need for knowledge of future electricity consumption events. Moreover, due to the finite battery capacity, the control actions at all slots are coupled. It turns out that by relaxing the charging/discharging constraints, the average battery charging and discharging power can be evened out. By exploiting this structure, we recast the original problem as a decoupled optimization problem with a larger feasible set, such that the optimal control action at each slot can be solved by using only the current observations, and it is also optimal for the original problem, if it is feasible for the original problem.
- By carefully constructing the Lyapunov function, we develop an online control algorithm to solve the decoupled optimization problem based on the Lyapunov optimization technique [16], [17], such that the control action always lies in the feasible region of the original problem. The proposed online control algorithm requires solution of a mixed-integer nonlinear program, in order to consider the cost of repeated charging and discharging on the battery’s lifetime. By decomposing the problem into multiple cases, a closed-form solution to each case of the mixed-integer nonlinear program is derived.
- We show that the proposed online control algorithm can achieve an optimal solution to the original problem asymptotically. Moreover, we quantify the impact of the battery capacity on the performance of the proposed online control algorithm. Using real data, we demonstrate the efficacy of our online control algorithm through

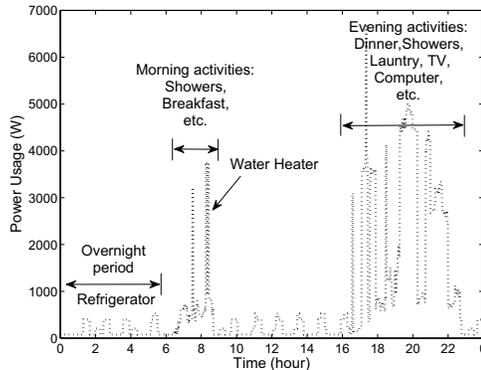


Fig. 1: A minute-level load profile of a typical day for a home [21].

extensive numerical exploration. The results corroborate that our algorithm can protect smart meter data privacy in a cost-effective manner.

The rest of the paper is organized as follows. In Section II, we introduce the smart meter data privacy problem. In Section III, we describe the system model. In Section IV, we propose an optimal privacy-preserving energy management framework. In Section V, we propose an optimal online control algorithm. In Section VI, we evaluate the performance of the proposed online control algorithm using real data. The paper is concluded in Section VII.

## II. SMART METER DATA PRIVACY

In this section, we give a brief introduction to the data privacy of smart metering.

### A. Load Profiles

Load profiles are time series of electric demand that are delivered to the utility provider. Unlike conventional electromechanical watt-hour meters that do not record instantaneous demand, smart meters can generate load profiles in or near real time. According to [18], common low cost meters measure epochs at 15 minutes [19], while more sophisticated models can generate profiles at second-level or even finer granularity [20].

A minute-level load profile of a typical day for a home is shown in Fig. 1. From a privacy point of view, the information contained in the load profile comprises individual consumption events. The presence of appliances can be detected by observing the change in power consumption due to appliance switch-on/off events, which indicate the corresponding human activity. In Fig. 1, high variations correspond to human activities, in particular during periods 7:00 AM – 9:00 AM and 4:30 PM – 11:00 PM. For example, during 7:00 AM – 9:00 AM, the large spike matches the power consumption of the water heater, which indicates that the resident may take a shower during this period.

### B. Privacy Concerns with Smart Meters

As illustrated in Fig. 1, household smart meters inadvertently leak detailed information about household activities,

thereby giving rise to serious privacy concerns. With the load profile collected at a configured granularity, e.g., seconds or minutes, recent work [6] has proposed a load monitoring algorithm that can extract appliance profiles from load profiles, and thereby expose the human activities. As pointed out in [5], different use of such information implicates different kinds of privacy concerns, ranging from nefarious misuse to inadvertent sharing. For example, a relatively low level of power consumption and variation during 9:00 AM – 4:00 PM in Fig. 1 may indicate to intruders that no one is at home. Therefore, for the concern of home invaders, it is important to develop and implement technological protections and standards for protecting smart meter data privacy.

### C. Energy Storage: Privacy Protection and Power Cost Saving

It has been reported by existing works (e.g., [9], [10], and the references therein) that smart meter data privacy can be protected by using a rechargeable battery. The rechargeable battery can be used to mask appliance features in load profiles, thus eliminating exposure of the appliance-driven information used to compromise consumer privacy. However, existing works did not provide an optimal privacy protection algorithm to efficiently use the battery, as the optimal solution may require information about future usage. Furthermore, the cost of using the battery is often ignored. In this paper, we propose a control algorithm that can achieve an optimal solution asymptotically without the need of future information, while taking into account the cost of using the battery.

Beyond privacy protection, the rechargeable battery offers an opportunity to reduce the electricity bill. Recent work [11] considers both privacy protection and power cost saving simultaneously by using stochastic dynamic programming. However, the solution to dynamic programming requires the statistics of the electricity price and the load, which may not be available, and cannot necessarily adapt if these probabilities change, not to mention the computational difficulty that results from the curse of dimensionality. Moreover, the cost of using battery is not considered in [11]. In this paper, we approach this challenge by a Lyapunov optimization based approach that does not require any statistical information.

## III. SYSTEM MODEL

We consider a discrete-time system, in which the length of each time slot matches the typical sampling and operation time scale of the smart meter. An overview of the system model is given in Fig. 2. We assume that a smart home contains an energy storage device (battery) and a power controller that can control the combination of power drawn from the utility and the battery to satisfy the load requirements. In the following, the model of each component in Fig. 2 is described in detail, aiming to optimize the load profile to mask individual consumption events in a cost-effective manner.

### A. Battery Model

We denote by  $B^{\max}$  the battery capacity, by  $B(t)$  the energy level of the battery at slot  $t$ , and by  $P_B(t)$  the power charged to

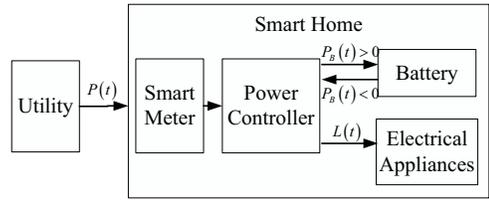


Fig. 2: System model.

(when  $P_B(t) > 0$ ) or discharged from (when  $P_B(t) < 0$ ) the battery during slot  $t$ . Assume that the battery energy leakage is negligible, which is a reasonable assumption, since the time scale that the loss takes place (e.g., about 3-20% a month for lead-acid batteries) is much larger than the minute-level time scale. Then, the dynamics of the battery energy level can be expressed as

$$B(t+1) = B(t) + P_B(t). \quad (1)$$

Since the charging and discharging rates of the battery are physically constrained, we denote by  $P_B^{\max}$  the maximum charging rate and by  $P_B^{\min}$  the maximum discharging rate.  $P_B^{\max}$  and  $P_B^{\min}$  are positive constants depending on the physical properties of the battery. Therefore, we have the following constraint on  $P_B(t)$ :

$$-P_B^{\min} \leq P_B(t) \leq P_B^{\max}. \quad (2)$$

Since the battery energy level should always be nonnegative and cannot exceed the battery capacity, we need to ensure that in each slot  $t$ ,

$$0 \leq B(t) \leq B^{\max}. \quad (3)$$

Based on constraints (1), (2) and (3), we have the following equivalent constraints in each slot  $t$  for  $P_B(t)$ :

$$P_B(t) \geq -\min\{P_B^{\min}, B(t)\}, \quad (4)$$

$$P_B(t) \leq \min\{P_B^{\max}, B^{\max} - B(t)\}. \quad (5)$$

As is standard, the number of charging/discharging cycles for each battery is limited. To model the cost of repeated charging and discharging on the battery's lifetime, we assume that an amortized cost  $C_B$  (in unit of dollars) is incurred with each charging or discharging operation. Therefore, at each slot, an operating cost of  $C_B$  is incurred whenever the battery is charging ( $P_B(t) > 0$ ) or discharging ( $P_B(t) < 0$ ).

### B. Load Model

We denote by  $L(t)$  the residential load generated at slot  $t$ .  $L(t)$  is assumed to be independent and identical distributed (*i.i.d.*) over time slots with some unknown probability distribution. Assume that  $L(t)$  is deterministically bounded by a finite constant  $L^{\max}$ , so that

$$L(t) \leq L^{\max}, \quad \forall t. \quad (6)$$

$L^{\max}$  can be determined by the total power consumption of all electrical appliances at home.

With the battery, the total power used to serve the load is given by

$$L(t) = P(t) - P_B(t), \quad (7)$$

where  $P(t)$  denotes the power drawn from the grid at slot  $t$ . Assume that the maximum amount of power that can be drawn from the grid in any slot is upper bounded by  $P^{\max}$ , i.e.,

$$0 \leq P(t) \leq P^{\max}, \forall t. \quad (8)$$

Note that for the original scenario where the battery is not used,  $P^{\max}$  must be larger than  $L^{\max}$  in order to satisfy all residential loads. Therefore, we have  $P^{\max} \geq L^{\max}$ .

### C. Electricity Pricing Model

Since the electricity price is time-varying, e.g., time-of-use electricity pricing, the rechargeable battery offers an opportunity to lower the electricity bill, in addition to privacy protection. Let  $c(t)$  denote the cost per unit of power drawn from the grid at slot  $t$ . We assume that  $c(t)$  evolves according to an *i.i.d.* process with some unknown distribution and is deterministically bounded by a finite constant  $c^{\max}$ , so that  $c(t) \leq c^{\max}$ .

Given the system model, our goal is to design a control algorithm that jointly optimizes the cost of power and smart meter data privacy, while meeting all the constraints. For ease of exposition, the load process and the electricity price process are assumed to be *i.i.d.* over slots. The algorithm developed for this case can be applied to non *i.i.d.* scenarios, as shown in Section VI, where the numerical studies are carried out in non *i.i.d.* environments. Our results can be generalized for the non *i.i.d.* case by using the delayed Lyapunov drift and  $T$  slot drift techniques developed in [16] and [17].

## IV. OPTIMAL PRIVACY-PRESERVING ENERGY MANAGEMENT

Based on the system model above, we now study optimal privacy-preserving energy management.

### A. Control Objective

With the use of a battery, the original load profile  $L(t)$  becomes  $P(t) = L(t) + P_B(t)$ . Intuitively speaking, in order to mask energy usage profiles, the modified load profile  $P(t)$  needs to be as “flat” as possible. If  $P(t)$  is equal to a constant value representing the average residential load, then all individual consumption events would be perfectly masked. Let  $\bar{L} = \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t L(\tau)$  denote the average residential load. In real time, by charging/discharging the battery,  $P(t)$  needs to be controlled with as little deviation from  $\bar{L}$  as possible. Equivalently, the control objective is to minimize the “variance” of  $P(t)$ , i.e.,

$$\lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t \mathbb{E}\{(P(\tau) - \bar{L})^2\}. \quad (9)$$

It can be seen that minimizing (9) generally requires the knowledge of all future consumption events due to  $\bar{L}$ . Heuristic algorithms (e.g., [9] and [10]) have been devised to mitigate this issue, which inevitably sacrifices performance. As discussed later in Proposition 1, by exploring the problem structure, this issue can be solved by transforming (9) into an equivalent problem.

Beyond privacy protection, the rechargeable battery can also be used to lower the electricity bill, by utilizing the time-varying electricity price. In this paper, we jointly optimize the electricity bill and smart meter data privacy. The objective function can be expressed as

$$\lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t \mathbb{E}\left\{ \underbrace{c(\tau)P(\tau)}_{\text{Electricity cost}} + \underbrace{\mathbf{1}_B(\tau)C_B}_{\text{Battery operation cost}} + \underbrace{\beta(P(\tau) - \bar{L})^2}_{\text{Smart meter data privacy}} \right\}, \quad (10)$$

where the indicator function  $\mathbf{1}_B(t)$  corresponds to the battery operation, i.e.,  $\mathbf{1}_B(t) = 1$  if  $P_B(t) \neq 0$  (the battery is charging/discharging), otherwise  $\mathbf{1}_B(t) = 0$ .  $\beta \geq 0$  is a parameter chosen by the customer to trade off between reducing the electricity bill and protecting the smart meter data privacy. Assuming that the limit exists in (10), our goal is to design a control algorithm that can minimize this time average cost subject to the constraints described in the system model.

### B. Problem Formulation

The optimal privacy-preserving energy management problem can be formulated as the following stochastic dynamic optimization problem:

**P1:**

$$\begin{aligned} \min \quad & \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t \mathbb{E}\left\{ c(\tau)P(\tau) + \mathbf{1}_B(\tau)C_B + \beta(P(\tau) - \bar{L})^2 \right\} \\ \text{s.t.} \quad & \text{constraints (4), (5), (7), (8)}. \end{aligned} \quad (11)$$

One major challenge of solving **P1** is the lack of knowledge of future time-varying load requirements and electricity prices. Moreover, due to the *finite* battery capacity constraints (4) and (5), the current control action would impact the future control actions, making it more challenging to solve **P1**. Note that the traditional approach based on dynamic programming (e.g., the approach in [11]) is difficult to apply, because it requires the statistics of load requirements and electricity prices, and the computational complexity may suffer from the curse of dimensionality. As the statistics of load requirements and electricity prices may not be known, in this paper, we develop an online control algorithm to solve this problem by using only the current observations, while taking into account the *finite* battery capacity constraints.

### C. Problem Relaxation

To solve **P1**, we first consider a relaxed version of **P1**, by using (2) to relax the constraints (4) and (5). Define the average expected charging or discharging rate of the battery as

$$\bar{P}_B = \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t \mathbb{E}\{P_B(\tau)\}. \quad (12)$$

Since the battery energy evolves based on (1), summing over all  $t$  and taking expectation of both sides, we have

$$\mathbb{E}\{B(t+1)\} - B(1) = \sum_{\tau=1}^t \mathbb{E}\{P_B(\tau)\}, \quad (13)$$

where  $B(1)$  is the initial battery energy level. Since the battery capacity is finite, dividing both sides by  $t$ , and taking  $t \rightarrow \infty$  in (13) yields

$$\bar{P}_B = 0. \quad (14)$$

Thus, we have the following relaxed problem:

$$\begin{aligned} \mathbf{P2}: \\ \min \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t \mathbb{E} \left\{ c(\tau)P(\tau) + \mathbf{1}_B(\tau)C_B + \beta(P(\tau) - \bar{L})^2 \right\} \\ \text{s.t. constraints (2), (7), (8)} \\ \bar{P}_B = 0. \end{aligned} \quad (15)$$

One main challenge of solving **P2** is that it requires the knowledge of all future consumption events to compute  $\bar{L}$ . Now that  $\bar{P}_B = 0$ , the average charging power is equal to the average discharging power, indicating that the battery does not consume energy on average. By exploiting this structure, we show in the following proposition that the decision variable  $P(t)$  is independent of  $\bar{L}$ . Therefore, **P2** can be solved without using the information of  $\bar{L}$ .

**Proposition 1:** Consider the following stochastic optimization problem:

$$\begin{aligned} \mathbf{P3}: \min \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t \mathbb{E} \left\{ c(\tau)P(\tau) + \mathbf{1}_B(\tau)C_B + \beta P(\tau)^2 \right\} \\ \text{s.t. constraints (2), (7), (8)} \\ \bar{P}_B = 0. \end{aligned} \quad (16)$$

Any optimal solution to **P3** is also optimal for **P2** and vice versa.

*Proof:* Since any feasible solution to **P3** is also feasible for **P2** and vice versa, it is sufficient to show that the objective functions of **P3** and **P2** differ by at most a constant independent of  $P(t)$ . Equivalently, it is sufficient to show that

$$\lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t \mathbb{E} \{ \bar{L}^2 - 2\bar{L}P(\tau) \} \quad (17)$$

is a constant independent of the choice of  $P(t)$ . Since  $\bar{L}$  is a constant depending only on  $L(t)$ , it is sufficient to show that

$$\lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t \mathbb{E} \{ P(\tau) \} \quad (18)$$

is a constant independent of the choice of  $P(t)$ .

Since  $P(t) = L(t) + P_B(t)$ , summing over all  $t$ , taking expectation of both sides, dividing both sides by  $t$  and taking  $t \rightarrow \infty$ , we have

$$\begin{aligned} \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t \mathbb{E} \{ P(\tau) \} &= \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t L(\tau) \\ &+ \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t \mathbb{E} \{ P_B(\tau) \} \\ &= \bar{L} + \bar{P}_B. \end{aligned} \quad (19)$$

Since  $\bar{P}_B = 0$ , we have

$$\lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t \mathbb{E} \{ P(\tau) \} = \bar{L}, \quad (20)$$

thereby concluding the proof.  $\blacksquare$

From Proposition 1, any feasible solution to **P1** is also a feasible solution to **P3**, since **P2** is equivalent to **P3** and the feasible set of **P2** is larger than that of **P1**. Let  $\phi_{rel}$  and  $\phi_{opt}$  denote the optimal objective value of the relaxed problem **P3** and that of **P1**, respectively. Since **P3** is less constrained than **P1**, the optimal value of **P3** will not exceed that of **P1** by more than a constant  $\beta\bar{L}^2$  that can be computed based on

Proposition 1, i.e.,  $\phi_{rel} \leq \phi_{opt} - \beta\bar{L}^2$ .

Note that **P3** is a decoupled control problem, in the sense that the optimal control actions  $P(t)$  and  $P_B(t)$  at each slot can be determined purely by the current state  $L(t)$  and  $c(t)$ . Specifically, it can be shown that there exists a stationary, randomized policy that can achieve the optimal solution to **P3** as presented in the following lemma.

**Lemma 1:** If  $L(t)$  and  $c(t)$  are *i.i.d.* over time slots, then there exists a stationary, randomized policy that makes control decisions  $P^{stat}(t)$  and  $P_B^{stat}(t)$  at every slot purely as a function (possibly randomized) of the current state  $L(t)$  and  $c(t)$ , while satisfying the constraints of **P3** and providing the following guarantees:

$$\begin{aligned} \mathbb{E} \{ P_B^{stat}(t) \} &= 0, \\ \mathbb{E} \{ c(t)P^{stat}(t) + \mathbf{1}_{\{P^{stat}(t) \neq 0\}} C_B + \beta P^{stat}(t)^2 \} &= \phi_{rel}, \end{aligned}$$

where the expectations above are with respect to the stationary distributions of  $L(t)$  and  $c(t)$ , and the randomized control decisions.

The proof of Lemma 1 follows from the framework in [17] and is omitted for brevity. Note that the above stationary, randomized policy may not be feasible for the original problem **P1**, as the constraints (4) and (5) may be violated. However, the existence of such a policy can be used to design our online control policy that meets all constraints of **P1** and derive a performance guarantee for our algorithm as shown later in Proposition 2.

## V. OPTIMAL ONLINE CONTROL ALGORITHM

In this section, we design an online control algorithm that can achieve the optimal solution to **P1** asymptotically. The proposed online control algorithm is designed based on **P3**, and uses a control parameter  $V > 0$  to quantify the impact of the battery capacity on the performance of the algorithm as discussed later. The key idea of our algorithm is to construct a Lyapunov function with a perturbed weight for determining the control actions. By carefully designing the weight, we can show that whenever the battery is charged or discharged, the battery energy level always lies in the feasible region of **P1**. From Proposition 1, if the optimal solution to **P3** can be found and lies in the feasible region of **P1**, it is also optimal for **P1**. To this end, we define a perturbed variable  $U(t)$  to track the battery energy level as

$$U(t) = B(t) - V(c^{\max} + 2\beta L^{\max}) - P_B^{\min}. \quad (21)$$

Note that  $B(t)$  is the actual battery energy level at slot  $t$  and  $U(t)$  is simply a shifted version of  $B(t)$  with the same dynamics:

$$U(t+1) = U(t) + P_B(t). \quad (22)$$

### A. Lyapunov Optimization

We define the following Lyapunov function:  $Z(U(t)) = \frac{1}{2}U(t)^2$ . The corresponding conditional Lyapunov drift can be defined as follows:

$$\Delta(U(t)) = \mathbb{E} \{ Z(U(t+1)) - Z(U(t)) | U(t) \}. \quad (23)$$

Following the drift-plus-penalty framework [17], our online control algorithm is designed to minimize an upper bound on

the following function, in order to achieve the optimal solution to **P1**:

$$\Delta(U(t)) + V\mathbb{E}\{c(t)P(t) + \mathbf{1}_B(t)C_B + \beta P(t)^2|U(t)\}. \quad (24)$$

The following lemma provides an upper bound for (24).

*Lemma 2:* (Drift Bound) For any control policy that satisfies the constraints of **P3**, we have

$$\begin{aligned} & \Delta(U(t)) + V\mathbb{E}\{c(t)P(t) + \mathbf{1}_B(t)C_B + \beta P(t)^2|U(t)\} \\ & \leq K + U(t)\mathbb{E}\{P_B(t)|U(t)\} \\ & \quad + V\mathbb{E}\{c(t)P(t) + \mathbf{1}_B(t)C_B + \beta P(t)^2|U(t)\}, \end{aligned} \quad (25)$$

where the constant  $K$  is defined as

$$K = \frac{1}{2} \max\{(P_B^{\min})^2, (P_B^{\max})^2\}. \quad (26)$$

*Proof:* Squaring both sides of (22), dividing by 2, and rearranging, we have

$$\frac{U(t+1)^2 - U(t)^2}{2} = \frac{1}{2}P_B(t)^2 + U(t)P_B(t).$$

Since  $-P_B^{\min} \leq P_B(t) \leq P_B^{\max}$ , we have

$$\frac{U(t+1)^2 - U(t)^2}{2} \leq \frac{1}{2} \max\{(P_B^{\min})^2, (P_B^{\max})^2\} + U(t)P_B(t).$$

Taking conditional expectations of the above and adding  $V\mathbb{E}\{c(t)P(t) + \mathbf{1}_B(t)C_B + \beta P(t)^2|U(t)\}$  to both sides, the proof is concluded. ■

### B. Online Control Algorithm

The design principle of our online control algorithm is to minimize the right-hand-side of the drift-plus-penalty bound (25) subject to the constraints of **P3** at each slot  $t$ , by observing the current state  $U(t)$ ,  $L(t)$ , and  $c(t)$ . The online control algorithm chooses the control actions  $P_B(t)$  and  $P(t)$  as the solution to the following optimization problem:

$$\begin{aligned} \mathbf{P4}: \min & U(t)P_B(t) + V(c(t)P(t) + \mathbf{1}_B(t)C_B + \beta P(t)^2) \\ \text{s.t.} & -P_B^{\min} \leq P_B(t) \leq P_B^{\max} \\ & 0 \leq P(t) \leq P^{\max} \\ & L(t) = P(t) - P_B(t). \end{aligned} \quad (27)$$

It can be observed that for each slot  $t$ , **P4** is a mixed-integer nonlinear programming. To solve it, we consider the optimal values of the objective in **P4** for two modes with and without charging/discharging operation respectively, and then choose the mode that yields the lowest value of the objective. The corresponding control actions of the mode can then be implemented in real time.

Let  $P_B^*(t)$  and  $P^*(t)$  denote the optimal solution to **P4**. For each case,  $P_B^*(t)$  and  $P^*(t)$  can be characterized by using the KKT conditions [22]. The optimal solution to **P4** is given as follows:

1) *The case without charging/discharging operation* ( $\mathbf{1}_B(t) = 0$ ): Let  $\theta_1(t)$  denote the optimal value of the objective in **P4** without charging/discharging operation. In this case, we have  $P_B^*(t) = 0$  and  $P^*(t) = L(t)$ .  $\theta_1(t)$  can be calculated as

$$\theta_1(t) = V(c(t)P^*(t) + \beta P^*(t)^2). \quad (28)$$

2) *The case with charging/discharging operation* ( $\mathbf{1}_B(t) = 1$ ): Let  $\theta_2(t)$  denote the optimal value of the objective in **P4** with charging/discharging operation. Letting  $UB =$

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### Algorithm 1 Online control algorithm

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**Initialization:** Given the initial battery charge level  $0 \leq B_{init} \leq B^{\max}$ , set  $U(1)$  based on (21).

**For each time slot  $t$**

- 1) Compute  $\theta_1(t)$  and  $\theta_2(t)$  based on (28) and (29).
  - 2) Choose the control decisions  $P_B^*(t)$  and  $P^*(t)$  associated with the lower value of  $\theta_1(t)$  and  $\theta_2(t)$ .
  - 3) Update  $U(t)$  according to the dynamics (22).
- 

$\min\{P_B^{\max}, P_B^{\max} + L(t)\}$  and  $LB = \max\{L(t) - P_B^{\min}, 0\}$ ,  $P_B^*(t)$  and  $P^*(t)$  can be calculated as follows:

- If  $-\frac{U(t)+Vc(t)}{2V\beta} > UB$ , then  $P^*(t) = UB$  and  $P_B^*(t) = P^*(t) - L(t)$ .
- If  $LB \leq -\frac{U(t)+Vc(t)}{2V\beta} \leq UB$ , then  $P^*(t) = -\frac{U(t)+Vc(t)}{2V\beta}$  and  $P_B^*(t) = P^*(t) - L(t)$ .
- If  $-\frac{U(t)+Vc(t)}{2V\beta} < LB$ , then  $P^*(t) = LB$  and  $P_B^*(t) = P^*(t) - L(t)$ .

Given  $P_B^*(t)$  and  $P^*(t)$ ,  $\theta_2(t)$  can be calculated as follows:  $\theta_2(t) = U(t)P_B^*(t) + V(C_B + c(t)P^*(t) + \beta P^*(t)^2)$ . (29)

After computing  $\theta_1(t)$  and  $\theta_2(t)$ , we choose the lower value of  $\theta_1(t)$  and  $\theta_2(t)$  and the corresponding control actions. A detailed description of the online control algorithm is given in Algorithm 1.

### C. Performance Analysis

In this section, we analyze the feasibility and performance of our online control algorithm. We define an upper bound  $V^{\max}$  on parameter  $V$  as follows:

$$V^{\max} = \frac{B^{\max} - P_B^{\max} - P_B^{\min}}{c^{\max} + 2\beta L^{\max}}. \quad (30)$$

**Proposition 2:** Suppose the initial battery charge level  $B_{init}$  satisfies  $0 \leq B_{init} \leq B^{\max}$ . Implementing the above online control algorithm with any fixed parameter  $0 < V \leq V^{\max}$  for all  $t$ , we have the following performance guarantees:

- 1) The battery energy level  $B(t)$  is always in the range  $0 \leq B(t) \leq B^{\max}$  for all  $t$ .
- 2) All control decisions are feasible for **P1**.
- 3) If  $L(t)$  and  $c(t)$  are *i.i.d.* over slots, then the time-average expected cost under the proposed online control algorithm is within  $K/V$  of the optimal value:

$$\begin{aligned} \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t \mathbb{E}\{c(\tau)P(\tau) + \mathbf{1}_B(\tau)C_B + \beta P(\tau)^2\} \\ \leq \phi_{rel} + \frac{K}{V}, \end{aligned} \quad (31)$$

where  $K$  is a constant given in (26).

*Proof:* The proof is given in Appendix A. ■

Proposition 2 shows that by choosing larger  $V$ , the time-average expected cost under the proposed online control algorithm can be pushed closer to the optimal solution to **P1**. Since  $V^{\max}$  is determined by the battery capacity, (31) quantifies the impact of the battery capacity on the performance of the proposed online control algorithm. The larger the battery capacity is, the better the proposed online control algorithm can optimize smart meter data privacy and the electricity bill.

**Remarks:** Note that Proposition 2 hold for all sample paths, including sample paths generated by  $L(t)$  and  $c(t)$  that are non *i.i.d.* over slots. The proposed online control algorithm can be applied to non *i.i.d.* scenarios, as shown in Section VI, where the numerical studies are carried out in non *i.i.d.* environments. Our results can be generalized for the non *i.i.d.* case by using the delayed Lyapunov drift and  $T$  slot drift techniques developed in [16] and [17].

## VI. PERFORMANCE EVALUATION

### A. Data and Simulation Setting

We evaluate the performance of the proposed algorithm by using the real-time measurements at a Georgian apartment [9] and the load profile constructed based on a domestic electricity demand model [21]. The time resolution of the load profile is one-minute. The real load profile given by the real-time measurements represents the case in which the power usage is high, while the constructed load profile based on the domestic electricity demand model [21] represents the case in which the power usage is low. The electricity prices are set according to SRP’s residential time-of-use price plan [23]. The on-peak price is 21.09 cent per kWh during 1:00 PM to 8:00 PM, while the off-peak price is 7.04 cent per kWh for the remaining time of a day. In the simulation, we fix the parameters  $P_B^{\min} = P_B^{\max} = 6$  kW,  $P^{\max} = 10$  kW and  $C_B = 0.1$  cent.

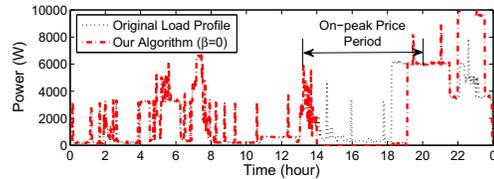
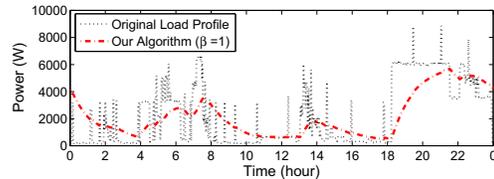
### B. Privacy Protection and Power Cost Saving

Based on the two data sets, Fig. 3 compares load profiles given by our algorithm under different values of  $\beta$ . As discussed in Section IV,  $\beta$  strikes a tradeoff between the electricity bill and smart meter data privacy. As illustrated in Fig. 3, when  $\beta = 1$ , the proposed algorithm smooths out the original load profile and the appliance switch-on/off events are masked accordingly. Fig. 4 takes a closer look at the load profile given by our algorithm during the period between 13:30 and 14:20, where the fluctuations in the original load profile due to the appliance switch-on/off events are smoothed out.

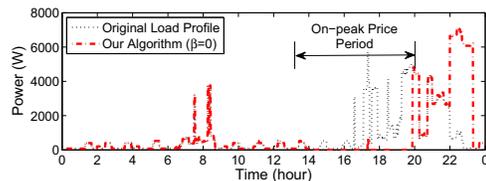
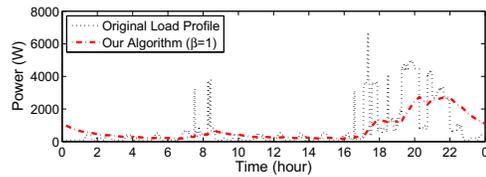
When  $\beta = 0$ , our algorithm minimizes only the electricity bill. As shown in Fig. 3, the proposed algorithm stores the energy in the battery in the off-peak price period and uses the energy in the battery as much as possible in the on-peak price period to reduce the electricity bill.

### C. Cost vs. Battery Capacity

We now study the impact of  $V$  on the performance of the proposed algorithm. Based on (30), if the electricity price is given,  $V^{\max}$  is determined by the battery capacity. For different values of the battery capacity, the system cost is evaluated by using the real load profile. The results are shown in Fig. 5, where the results are normalized by the system cost without using the battery. As illustrated in Fig. 5, the system cost decreases with the battery capacity, since the larger the battery capacity is, the better the proposed online control algorithm can optimize smart meter data privacy and the electricity bill.



(a) The real load profile.



(b) The constructed load profile.

Fig. 3: Comparison of load profiles given by our algorithm under different values of  $\beta$ , where  $B^{\max} = 12$  kWh, and  $V = V^{\max}$ .

### D. Privacy Protection vs. Battery Capacity

From the discussion in Section II, the privacy information is contained in appliance switch-on/off events, i.e., the differences between successive power measurements. Let  $dP(t) = P(t) - P(t-1)$  represent the difference between successive power measurements. The privacy information contained in  $dP(t)$  can be measured using entropy [8]–[12]. Given the original load profile  $L(t)$ , we evaluate the performance of the proposed algorithm on privacy protection based on the relative entropy or Kullback Leibler divergence. Let  $f_P(x)$  and  $f_L(x)$  denote the probability density functions of  $dP(t)$  and  $dL(t)$ , respectively. The Kullback Leibler divergence  $D(P||L)$  is defined as

$$D(P||L) = \int_{-\infty}^{\infty} f_P(x) \log \frac{f_P(x)}{f_L(x)} dx. \quad (32)$$

Fig. 6 illustrates the value of  $D(P||L)$  obtained based on the load profile given by our algorithm under different battery capacities by using the real load profile. As shown in Fig. 6, the relative entropy increases with the battery capacity in general, i.e., for a battery with larger capacity, our algorithm can provide better protection for smart meter data privacy.

### E. Our Algorithm vs. The “Best-Effort” Algorithm

Next, we compare our algorithm with the “best-effort” algorithm proposed in [9], which tries to maintain the current

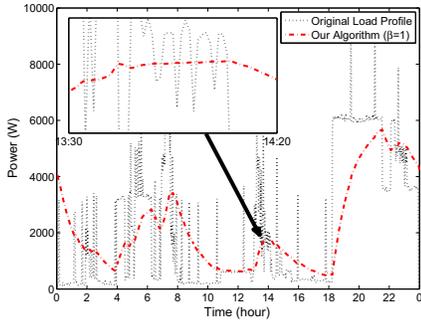


Fig. 4: Load profile given by our algorithm for the real load profile, where  $\beta = 1$ ,  $B^{\max} = 12$  kWh, and  $V = V^{\max}$ .

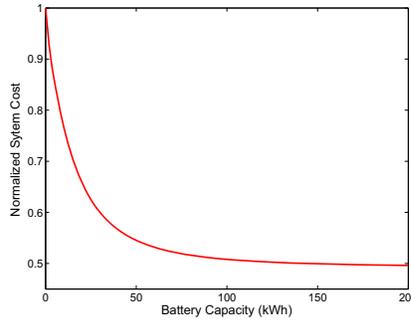


Fig. 5: System cost under different values of the battery capacity, where  $\beta = 1$  and  $V = V^{\max}$  for each value of the battery capacity.

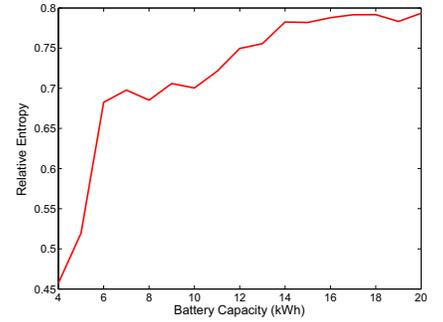


Fig. 6: Relative entropies of load profiles given by our algorithm under different battery capacities, where  $\beta = 1$  and  $V = V^{\max}$  for each value of the battery capacity.

load to be the same as the previous load. Since our algorithm can optimize both the cost of power and smart meter data privacy with asymptotically optimal performance, we compare the power cost and the relative entropy given by our algorithm and the “best-effort” algorithm. The results are shown in Fig. 7. For different battery capacities, Fig. 7(a) compares the power cost given by our algorithm with that given by the “best-effort” algorithm, where the results are normalized by the power cost computed based on the original load profile. As shown in Fig. 7(a), our algorithm can reduce the power cost (the results given by our algorithm are all less than the power cost computed based on the original load profile). Fig. 7(b) compares the relative entropy given by our algorithm with that given by the “best-effort” algorithm. As shown in Fig. 7(b), our algorithm can provide a load profile with a higher relative entropy to protect smart meter data privacy.

## VII. CONCLUSION

This paper has studied cost-effective smart meter data privacy protection by using batteries. A dynamic optimization framework has been designed for consumers to jointly protect smart meter data privacy and reduce the cost of the electricity. By exploring the underlying structure of the original problem, an equivalent problem has been derived, which can be solved by using only the current observations. Then an online control algorithm has been developed to solve the equivalent problem based on the Lyapunov optimization technique. It has been shown that the proposed online control algorithm can achieve the optimal solution asymptotically, without requiring any knowledge of the statistics of the load requirements and electricity prices. Using real data, numerical results have corroborated that compared with the existing algorithm, our algorithm can not only provide better protection for smart meter data privacy, but also reduce the cost of electricity.

For future work, it is of great interest to integrate demand response management into the current framework. For example, the response to fluctuations in renewable energy generation can be considered.

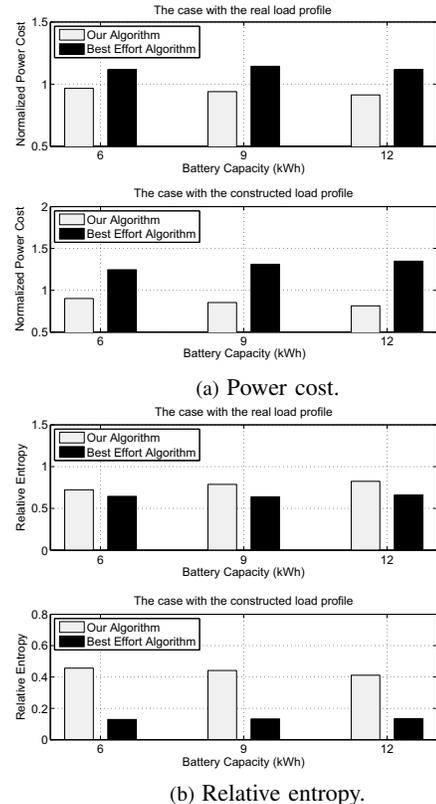


Fig. 7: Comparison of power cost and relative entropy given by our algorithm and the best effort algorithm, where  $\beta = 1$  and  $V = V^{\max}$  for each value of the battery capacity.

## ACKNOWLEDGEMENT

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## APPENDIX

### A. Proof of Proposition 2

1) *Proof of Part 1:* To show  $0 \leq B(t) \leq B^{\max}$  for all  $t$ , it suffices to show that  $-V(c^{\max} + 2\beta L^{\max}) - P_B^{\min} \leq U(t) \leq$

$B^{\max} - V(c^{\max} + 2\beta L^{\max}) - P_B^{\min}$  for all  $t$ , according to (22). As  $0 \leq B_{init} \leq B^{\max}$ , it is easy to verify  $-V(c^{\max} + 2\beta L^{\max}) - P_B^{\min} \leq U(1) \leq B^{\max} - V(c^{\max} + 2\beta L^{\max}) - P_B^{\min}$ .

To ease the proof, we first show that the optimal solution to **P4** has the following properties:

*Lemma 3:* The optimal solution to **P4** has certain properties:

- If  $U(t) \geq 0$ , then  $P_B^*(t) \leq 0$ .
- If  $U(t) \leq -V(c^{\max} + 2\beta L^{\max})$ , then  $P_B^*(t) \geq 0$ .

Lemma 3 can be verified based on the solution to **P4** derived in Section V-B.

Then, we show  $-V(c^{\max} + 2\beta L^{\max}) - P_B^{\min} \leq U(t) \leq B^{\max} - V(c^{\max} + 2\beta L^{\max}) - P_B^{\min}$  by induction. Suppose that  $-V(c^{\max} + 2\beta L^{\max}) - P_B^{\min} \leq U(t) \leq B^{\max} - V(c^{\max} + 2\beta L^{\max}) - P_B^{\min}$  holds for slot  $t$ . We need to show that it also holds for slot  $t + 1$ .

First, if  $0 \leq U(t) \leq B^{\max} - V(c^{\max} + 2\beta L^{\max}) - P_B^{\min}$ , then  $P_B^*(t) \leq 0$  based on Lemma 3. Therefore, using (22), we have  $U(t+1) \leq U(t) \leq B^{\max} - V(c^{\max} + 2\beta L^{\max}) - P_B^{\min}$ . If  $U(t) \leq 0$ , then  $U(t+1) \leq P_B^{\max}$  based on (22), since the maximum charging rate is  $P_B^{\max}$ . Using (30), for any  $0 < V \leq V^{\max}$  we have  $B^{\max} - P_B^{\min} - V(c^{\max} + 2\beta L^{\max}) \geq B^{\max} - P_B^{\min} - V^{\max}(c^{\max} + 2\beta L^{\max}) = P_B^{\max}$ . Therefore, we have  $U(t+1) \leq B^{\max} - P_B^{\min} - V(c^{\max} + 2\beta L^{\max})$ .

Second, if  $-V(c^{\max} + 2\beta L^{\max}) - P_B^{\min} \leq U(t) \leq -V(c^{\max} + 2\beta L^{\max})$ , then  $P_B^*(t) \geq 0$  based on Lemma 3. Then, using (22), we have  $U(t+1) \geq U(t) \geq -V(c^{\max} + 2\beta L^{\max}) - P_B^{\min}$ . If  $U(t) \geq -V(c^{\max} + 2\beta L^{\max})$ ,  $U(t+1) \geq -V(c^{\max} + 2\beta L^{\max}) - P_B^{\min}$  based on (22), since the maximum discharging rate is  $P_B^{\min}$ . Therefore, we have  $-V(c^{\max} + 2\beta L^{\max}) - P_B^{\min} \leq U(t+1)$ , which concludes the proof.

2) *Proof of Part 2:* From the proof above, the constraint on  $B(t)$  is satisfied for all  $t$ . Since the control decisions we make satisfy all the constraints in **P3**, with  $0 \leq B(t) \leq B^{\max}$  for all  $t$ , all the constraints in **P1** are also satisfied. Therefore, our control decisions are feasible to **P1**.

3) *Proof of Part 3:* The proposed online control algorithm is designed to minimize the right-hand-side of (25) over all possible feasible control policies, including the optimal, stationary policy given in Lemma 1. Therefore, we have the following:

$$\Delta(U(t)) + V\mathbb{E}\{c(t)P(t) + \mathbf{1}_{B(t)}C_B + \beta P(t)^2|U(t)\} \leq K + V\mathbb{E}\{c(t)P^{stat}(t) + \mathbf{1}_{P^{stat}(t) \neq 0}(t)C_B + \beta P^{stat}(t)^2|U(t)\}.$$

Taking the expectation of both sides, using the law of iterative expectation, and summing over all  $t$ , we have

$$V \sum_{\tau=1}^t \mathbb{E}\{c(\tau)P(\tau) + \mathbf{1}_{B(\tau)}C_B + \beta P(\tau)^2\} \leq Kt + Vt\phi_{rel} - \mathbb{E}\{Z(U(t))\} + \mathbb{E}\{Z(U(1))\}.$$

Dividing both sides by  $Vt$ , taking the limit as  $t \rightarrow \infty$  and using the facts that  $\mathbb{E}\{Z(U(t))\}$  is finite and  $\mathbb{E}\{Z(U(1))\}$  is nonnegative, we have

$$\lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t \mathbb{E}\{c(\tau)P(\tau) + \mathbf{1}_{B(\tau)}C_B + \beta P(\tau)^2\} \leq \phi_{rel} + \frac{K}{V}.$$

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