A Social Group Utility Maximization Framework with Applications in Database Assisted Spectrum Access

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Abstract—In this paper, we develop a social group utility maximization (SGUM) framework for cooperative networking that takes into account both social relationships and physical coupling among users. Specifically, instead of maximizing its individual utility or the overall network utility, each user aims to maximize its social group utility that hinges heavily on its social ties with other users. We show that this framework provides rich modeling flexibility and spans the continuum space between non-cooperative game and network utility maximization (NUM) – two traditionally disjoint paradigms for network optimization. Based on this framework, we study an important application in database assisted spectrum access. We formulate the distributed spectrum access problem among white-space users with social ties as a SGUM game. We show that the game is a potential game and always admits a social-aware Nash equilibrium. We also design a distributed spectrum access algorithm that can achieve the social-aware Nash equilibrium of the game and quantify its performance gap. We evaluate the performance of the SGUM solution using real social data traces. Numerical results demonstrate that the performance gap between the SGUM solution and the NUM (social welfare optimal) solution is at most 15%.

I. INTRODUCTION

Network utility maximization (NUM) has been extensively studied for network optimization problems (see [1], [2] and the references therein). A key assumption underpinning network utility maximization is that the interests of all users are aligned, i.e., they act in an altruistic manner with the same social objective of maximizing the total network utility. Along a different line, game theory has found a wide variety of important applications for distributed resource allocation problems in various networking applications [3]. Many existing game-theoretic models assume that all users are selfish and rationally, aiming at maximizing its own benefit. In fact, the assumptions that users are altruistic and selfish by network utility maximization and non-cooperative game represent two extreme cases that users are fully social-aware (i.e., caring about the benefit of all the users) and socially oblivious (i.e., caring about their own benefits only), respectively. In many applications such as mobile social networking, however, such assumptions are not readily applicable since hand-held devices are carried by human beings who have diverse social relationships and care about their social neighbors at different levels [4]. Clearly, there is much room left to exploit diverse social
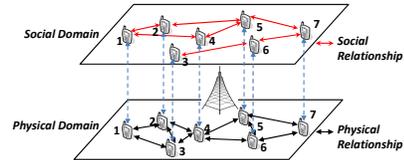
Fig. 1. An illustration of the social group utility maximization framework. In the physical domain, users have different physical coupling subject to physical relationships (e.g., interference). In the social domain, users have heterogeneous social coupling due to the social ties among users.

relationships among users for networking optimization, and it is of great interest to explore the continuum space between these two extreme cases. Indeed, with the explosive growth of online social networks such as Facebook and Twitter, more and more people are actively involved in online social interactions, and social relationships among people are hence extensively broadened and significantly enhanced [5]. This has opened up a new avenue to integrate the social interactions for cooperative network design.

With this motivation, we advocate a novel *social group utility maximization* (SGUM) framework that takes into account both the users' social relationships and physical coupling. As illustrated in Figure 1, a key observation is that users are coupled not only in the physical domain due to the physical relationship (e.g., interference), and but also in social domain due to the social ties among them. It would be a win-win case for users to help those users having social ties with them. With this insight, we view the network as an overlay/underlay system where a "virtual social network" overlays the physical communication network (the "social network" is virtual, in the sense that the social tie structure therein results from existing human relationship and online social networks). Then, the social tie structure is leveraged to facilitate cooperative networking. Specifically, we cast the distributed decision making problem for cooperative networking among users as a SGUM game, where each user maximizes its social group utility, defined as the sum of its own individual utility and the weighted sum of the utilities of other users having social tie with it. One primary objective of this study is to establish a general framework that bridges the gap between non-cooperative game and network utility maximization – two traditionally disjoint paradigms for network optimization. These two paradigms are captured under the proposed framework as two special cases

where no social tie exists among users (i.e., users are socially oblivious) and all users are connected by the strongest social ties (i.e., users are fully altruistic), respectively (as illustrated in Figure 2).

Under the SGUM framework, we study an important application in database assisted spectrum access. The very recent FCC ruling requires that white-space users (i.e., secondary TV spectrum users) must rely on a geo-location database to determine the spectrum availability [6]. Although the database-assisted approach obviates the need of spectrum sensing by individual users, it remains challenging to achieve reliable shared spectrum access, because different white-space users may choose to access the same vacant channel and thus incur severe interference to each other. To stimulate effective cooperation for channel allocation among whit-space users, we cast the database assisted distributed spectrum access problem among white-space users with social ties as a SGUM game. We also design a distributed spectrum access algorithm that can converge to a social-aware Nash equilibrium and quantify its performance gap from the social welfare optimal solution.

A. Summary of Main Contributions

The main contributions of this paper are as follows:

- *Social Group Utility Maximization Framework*: We propose a SGUM framework, which highlights the interplay between the physical coupling subject to users' physical relationships and the social coupling due to the social ties among users. The SGUM framework can provide rich flexibility for modeling network optimization problems and span the continuum space between non-cooperative game and network utility maximization – two extreme paradigms based on drastically different assumptions that users are selfish and altruistic, respectively.
- *Social Group Utility Maximization for Database Assisted Spectrum Access*: We apply the SGUM framework for database assisted spectrum access and formulate the distributed spectrum access problem among white-space users with social ties as a SGUM game. We prove that the game is a potential game and always admits a social-aware Nash equilibrium. Moreover, we show that the potential function of the game exhibits a nice structure that can be decomposed into two parts, capturing the impact of the physical coupling and social coupling in spectrum access, respectively.
- *Distributed Spectrum Access Algorithm for Achieving Social-aware Nash equilibrium*: We design a distributed spectrum access algorithm that can achieve the social-aware Nash equilibrium of the SGUM game for database assisted spectrum access. We also derive the upper-bound of the performance gap of the social-aware Nash equilibrium from the NUM solution. We further evaluate the performance of the SGUM solution using real social data traces. Numerical results demonstrate that the performance gap between the SGUM solution and the NUM solution is at most 15%.

B. Related Work

Although there exists a significant body of work on non-cooperative game and network utility maximization, surprisingly very little attention has been paid to the continuum space between these two extreme paradigms. Recent works [7], [8] have studied the impact of altruistic behavior in a routing game. [9] has recently investigated a random access game of two symmetrically altruistic players.

The social aspect is emerging as an important dimension for communication system design. A channel recommendation system based on cooperative social interactions is developed for dynamic spectrum access in [10]. Social structures, such as social community which are derived from the user contact patterns, have been exploited to design efficient data forwarding and dissemination algorithms in delay tolerant networks [11], [12]. In a recent study [13], we leveraged two key social social phenomena of social trust and social reciprocity to propose a coalitional game based mechanism for cooperative D2D communication. *We should emphasize that the SGUM framework in this paper is quite different from the coalitional game solution, since each user in the latter aims to maximize its individual benefit (although it is achieved by cooperating with other users in its coalition). Further, while a user in a coalitional game can only participate in one cooperative group (coalition), a user in the SGUM framework can be in multiple social groups associated with different users due to diverse social ties among them.*

II. SOCIAL GROUP UTILITY MAXIMIZATION FRAMEWORK

In this section we introduce the SGUM framework for cooperative networking. As illustrated in Figure 1, the framework can be projected onto two domains: the physical domain and the social domain. In the physical domain, different wireless users have different physical coupling due to their heterogeneous physical relationships (e.g., interference). In the social domain, different users have different social coupling due to their intrinsic social ties. We next discuss both physical and social domains in detail.

A. Physical Network Graph Model

We consider a set of wireless users $\mathcal{N} = \{1, 2, ..., N\}$ where N is the total number of users. We denote the set of feasible strategies for each user $n \in \mathcal{N}$ as $\mathcal{X}_n$. For instance, a strategy $x \in \mathcal{X}_n$ can be choosing either a channel or a power level for wireless transmissions. Subject to heterogeneous physical constraints, the strategy set $\mathcal{X}_n$ can be user-specific. For example, the strategy set $\mathcal{X}_n$ can be a set of feasible relay users that are in vicinity of user n for cooperative communication.

To capture the physical coupling among the users in the physical domain, we introduce the *physical graph* $\mathcal{G}^p = \{\mathcal{N}, \mathcal{E}^p\}$ (see Figure 1 for an example). Here the set of users $\mathcal{N}$ is the vertex set, and $\mathcal{E}^p \triangleq \{(n, m) : e_{nm}^p = 1, \forall n, m \in \mathcal{N}\}$ is the edge set where $e_{nm}^p = 1$ if and only if users n and m have physical coupling (e.g., cause interference to each other). We also denote the set of users that have physical coupling with user n as $\mathcal{N}_n^p \triangleq \{m \in \mathcal{N} : e_{nm}^p = 1\}$.

Let $\boldsymbol{x} = (x_1, ..., x_N) \in \prod_{n=1}^{N} \mathcal{X}_n$ be the strategy profile of all users. Given the strategy profile $\boldsymbol{x}$, the individual utility function of user n is denoted as $U_n(\boldsymbol{x})$, which represents the payoff of user n, accounting for the physical coupling among users. For example, $U_n(\boldsymbol{x})$ can be the achieved data rate or the satisfaction of quality of service (QoS) requirement of user n under the strategy profile $\boldsymbol{x}$. Note that in general the underlying physical graph plays a critical role in determining the individual utility $U_n(\boldsymbol{x})$. For example, users' achieved data rates are determined by the interference graph and channel quality.

B. Social Network Graph Model

We next introduce the social graph model to describe the social ties among users. The underlying rationale of considering social tie is that the hand-held devices are carried by human beings and the knowledge of human social ties can be utilized to enhance the performance of cooperative networking.

Specifically, we introduce the *social graph* $\mathcal{G}^s = \{\mathcal{N}, \mathcal{E}^s\}$ to model the social tie among the users. Here the vertex set is the same as the user set $\mathcal{N}$ and the edge set is given as $\mathcal{E}^s = \{(n, m) : e_{nm}^s = 1, \forall n, m \in \mathcal{N}\}$ where $e_{nm}^s = 1$ if and only if users n and m have social tie between each other, which can be kinship, friendship, or colleague relationship between two users. Furthermore, for a pair of users n and m who have a social edge between them on the social graph, we formalize the strength of social tie as $w_{nm} \in [0, 1]$, with a higher value of w_{nm} being a stronger social tie. We define user n's *social group* $\mathcal{N}_n^s$ as the set of users that have social ties with user n, i.e., $\mathcal{N}_n^s = \{m : e_{nm}^s = 1, \forall m \in \mathcal{N}\}$.

Based on the physical and social graph models above, users are coupled in the physical domain due to the physical relationships, and also coupled in social domain due to the social ties among them. It would be a win-win case for users to help those users having social ties with them. With this insight, we further define the *social group utility* of each user n as

$$S_n(\boldsymbol{x}) = U_n(\boldsymbol{x}) + \sum_{m \in \mathcal{N}_n^s} w_{nm} U_m(\boldsymbol{x}). \tag{1}$$

It follows that the social group utility of each user consists of two parts: 1) its own individual utility and 2) the weighted sum of the individual utilities of other users having social tie with it. In a nutshell, the social group utility function captures the feature that each user is social-aware and cares about the users having social tie with it.

C. Social Group Utility Maximization Game

We next consider the distributed decision making problem among the users for maximizing their social group utilities. Let $x_{-n} = (x_1, ..., x_{n-1}, x_{n+1}, ..., x_N)$ be the set of strategies chosen by all other users except user n. Given the other users' strategies x_{-n}, user n wants to choose a strategy $x_n \in \mathcal{X}_n$ to maximize its social group utility, i.e.,

$$\max_{x_n \in \mathcal{X}_n} S_n(x_n, x_{-n}), \forall n \in \mathcal{N}.$$

The distributed nature of the problem above naturally leads to a formulation based on game theory such that each user

Fig. 2. Different problem formulations for network optimization with different users' social awareness levels

aims to maximize its social group utility. We thus formulate the decision making problem among the users as a strategic game $\Gamma = (\mathcal{N}, \{\mathcal{X}_n\}_{n \in \mathcal{N}}, \{S_n\}_{n \in \mathcal{N}})$, where the set of users $\mathcal{N}$ is the set of players, $\mathcal{X}_n$ is the set of strategies for each user n, and the social group utility function S_n of each user n is the payoff function of player n. In the sequel, we call the game Γ as the SGUM game. We next introduce the concept of *social-aware Nash equilibrium* (SNE).

Definition 1. *A strategy profile $\boldsymbol{x}^* = (x_1^*, ..., x_N^*)$ is a social-aware Nash equilibrium of the SGUM game if no player can improve its social group utility by unilaterally changing its strategy, i.e.,*

$$x_n^* = \arg \max_{x_n \in \mathcal{X}_n} S_n(x_n, x_{-n}), \forall n \in \mathcal{N}.$$

It is worth noting that under different social graphs, the proposed SGUM game formulation can provide rich flexibility for modeling the network optimization problem (as illustrated in Figure 2). When the social graph consists of isolated nodes with $w_{nm} = 0$ for any $n, m \in \mathcal{N}$ (i.e., all users are selfish), the SGUM game degenerates to the non-cooperative game formulation. When the social graph is fully meshed with edge weight $w_{nm} = 1$ for any $n, m \in \mathcal{N}$ (i.e., each user is fully altruistic and cares enough about other users), the SGUM game becomes the network utility maximization problem, which aims to maximize the system-wide utility. The SGUM framework in this study is applicable to general social graphs and hence can bridge the gap between non-cooperative game and network utility maximization – two traditionally disjoint paradigms for network optimization.

The SGUM is a general framework that can be applied for many networking applications. To get a more concrete sense of the framework, in the following sections, we will focus on studying its application in database assisted spectrum access.

III. SOCIAL GROUP UTILITY MAXIMIZATION FOR DATABASE ASSISTED SPECTRUM ACCESS

In this section we apply the SGUM framework for the database assisted spectrum access.

A. Social Group Utility Maximization Game Formulation

We consider a set of white-space users $\mathcal{N} = \{1, 2, ..., N\}$ where N is the total number of users. We denote the set of TV channels as $\mathcal{M} = \{1, 2, ..., M\}$. According to the recent ruling by FCC [6], to protect the incumbent primary TV users, each white-space user $n \in \mathcal{N}$ will first send a spectrum access request message containing its geo-location information to a Geo-location database (see Figure 3 for an illustration). The database then feeds back the set of vacant channels
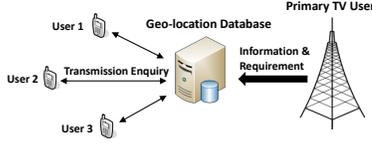
Fig. 3. An illustration of database assisted spectrum access

$\mathcal{M}_n \in \mathcal{M}$ and the allowable transmission power level P_n to user n. The ruling by FCC indicates that the allowable transmission power limit for personal/portable white-space devices (e.g., mobile phones) is 100 mW [6]. For ease of exposition, we hence assume that each user n accesses the white-space spectrum with the same power level. Each user n then chooses a feasible channel a_n from the vacant channel set $\mathcal{M}_n$ for data transmission. Although the database-assisted approach obviates the need of spectrum sensing by individual users, it remains challenging to achieve reliable distributed spectrum access, because many different white-space users may choose to access the same vacant channel and thus incur severe interference to each other [14], [15].

To stimulate effective cooperation among users for interference mitigation, we leverage the social ties among users and apply the SGUM approach. To capture the physical coupling, we construct the interference graph $\mathcal{G}^p = \{\mathcal{N}, \mathcal{E}^p\}$ based on the interference relationships among users. Here the set of white-space users $\mathcal{N}$ is the vertex set, and $\mathcal{E}^p \triangleq \{(n, m) : e_{nm}^p = 1, \forall n, m \in \mathcal{N}\}$ is the edge set where $e_{nm}^p = 1$ if and only if users n and m can generate significant interference and affect the data transmissions of each other. For example, we can construct the interference graph $\mathcal{G}^p$ based on spatial relationships of the users [16]. Let δ denote the transmission range of each user. We then have $e_{nm}^p = 1$ if and only if the distance d_{nm} between user n and m is not greater than the threshold δ, i.e., $d_{nm} \leq \delta$.

Let $\boldsymbol{a} = (a_1, ..., a_N) \in \prod_{n=1}^N \mathcal{M}_n$ be the channel selection profile of all users. Given the channel selection profile $\boldsymbol{a}$, the interference received by user n can be computed as

$$\gamma_n(\boldsymbol{a}) = \sum_{m \in \mathcal{N}_n^p} P_m d_{mn}^{-\alpha} I_{\{a_n = a_m\}} + \omega_{a_n}^n. \quad (2)$$

Here α is the path loss factor and $I_{\{A\}}$ is an indicator function with $I_{\{A\}} = 1$ if the event A is true and $I_{\{A\}} = 0$ otherwise. Furthermore, $\omega_{a_n}^n$ denotes the noisy power including the interference from primary TV users on the channel a_n. We then define the individual utility function $U_n(\boldsymbol{a})$ as

$$U_n(\boldsymbol{a}) = -\gamma_n(\boldsymbol{a}) = -\sum_{m \in \mathcal{N}_n^p} P_m d_{mn}^{-\alpha} I_{\{a_n = a_m\}} - \omega_{a_n}^n. \quad (3)$$

Here the negative sign comes from the convention that utility functions are typically the ones to be maximized. The individual utility of user n reflects the fact that each user n has interest to reduce its own received interference. To capture the social coupling in the social graph $\mathcal{G}^s$, we further introduce the social group utility of each white-space user n according to (1) as

$$S_n(\boldsymbol{a}) = U_n(\boldsymbol{a}) + \sum_{m \in \mathcal{N}_n^s} w_{nm} U_m(\boldsymbol{a}). \quad (4)$$

We then formulate the database assisted spectrum access problem as a SGUM game $\Gamma = (\mathcal{N}, \{\mathcal{M}_n\}_{n \in \mathcal{N}}, \{S_n\}_{n \in \mathcal{N}})$, where the set of white-space users $\mathcal{N}$ is the set of players, the set of vacant channels $\mathcal{M}_n$ is the set of strategies for each player n, and the social group utility function S_n of each user n is the payoff function of player n.

B. Properties of SGUM game

We next study the existence of SNE of the SGUM game for database assisted spectrum access. Here we resort to a useful tool of potential game [17].

Definition 2. *A game is called a potential game if it admits a potential function $\Phi(\boldsymbol{a})$ such that for every $n \in \mathcal{N}$ and $a_{-n} \in \prod_{i \neq n} \mathcal{M}_i$, for any $a_n, a_n' \in \mathcal{M}_n$,*

$$S_n(a_n', a_{-n}) - S_n(a_n, a_{-n}) = \Phi(a_n', a_{-n}) - \Phi(a_n, a_{-n}). \quad (5)$$

An appealing property of the potential game is that it always admits a Nash equilibrium, and any strategy profile that maximizes the potential function $\Phi(\boldsymbol{a})$ is a Nash equilibrium [17].

For the SGUM game Γ for database assisted spectrum access, we can show that it is a potential game. For ease of exposition, we first introduce the *physical-social graph* $\mathcal{G}^{sp} = \{\mathcal{N}, \mathcal{E}^{sp}\}$ to capture both physical coupling and social coupling simultaneously. Here the vertex set is the same as the user set $\mathcal{N}$ and the edge set is given as $\mathcal{E}^{sp} = \{(n, m) : e_{nm}^{sp} \triangleq e_{nm}^s \cdot e_{nm}^p = 1, \forall n, m \in \mathcal{N}\}$ where $e_{nm}^{sp} = 1$ if and only if users n and m have social tie between each other (i.e., $e_{nm}^s = 1$) and can also generate interference to each other (i.e., $e_{nm}^p = 1$). We denote the set of users that have social ties and can also generate interference to user n as $\mathcal{N}_n^{sp} = \{m : e_{nm}^{sp} = 1, \forall m \in \mathcal{N}\}$.

Based on the physical-social graph $\mathcal{G}^{sp}$, we show in Theorem 1 that the SGUM game Γ is a potential game with the following potential function

$$\Phi(\boldsymbol{a}) = \underbrace{-\frac{1}{2} \sum_{n=1}^N \sum_{m \in \mathcal{N}_n^p} P_m d_{mn}^{-\alpha} I_{\{a_n = a_m\}} - \sum_{n=1}^N \omega_{a_n}^n}_{\Phi_1(\boldsymbol{a}): \text{ due to Physical Coupling}}$$

$$\underbrace{-\frac{1}{2} \sum_{n=1}^N \sum_{m \in \mathcal{N}_n^{sp}} w_{nm} P_m d_{mn}^{-\alpha} I_{\{a_n = a_m\}}}_{\Phi_2(\boldsymbol{a}): \text{ due to Social Coupling}}. \quad (6)$$

The potential function in (6) can be decomposed into two parts: $\Phi_1(\boldsymbol{a})$ and $\Phi_2(\boldsymbol{a})$. The first part $\Phi_1(\boldsymbol{a})$ reflects the weighted system-wide interference level (including background noise) due to physical coupling in the physical domain and the second part $\Phi_2(\boldsymbol{a})$ captures the interdependence of user utilities due to social coupling in the social domain.

Theorem 1. *The SGUM game Γ for database assisted spectrum access is a potential game with the potential function $\Phi(\boldsymbol{a})$ given in (6), and hence has a SNE.*

The proof can be found in Appendix A. Note that when

$w_{nm} = 0$ for any user $n, m \in \mathcal{N}$ (i.e., all users are selfish), the potential function $\Phi(\boldsymbol{a}) = \Phi_1(\boldsymbol{a})$, which does not involve the social coupling part $\Phi_2(\boldsymbol{a})$. In this case, the SGUM game Γ for database assisted spectrum access degenerates to the non-cooperative spectrum access game. When $w_{nm} = 1$ for any user $n, m \in \mathcal{N}$ (i.e., all users are fully altruistic), the potential function $\Phi(\boldsymbol{a}) = \sum_{n=1}^{N} U_n(\boldsymbol{a})$, which is the system-wide utility. In this case, the SGUM becomes the network utility maximization.

We next design a distributed spectrum access algorithm that can achieve the SNE of the SGUM game Γ for database assisted spectrum access.

IV. DISTRIBUTED SPECTRUM ACCESS ALGORITHM FOR SOCIAL GROUP UTILITY MAXIMIZATION

In this section we study the distributed spectrum access algorithm design.

A. Algorithm Design Principles

According to the property of potential game, any channel selection profile $\boldsymbol{a}$ that maximizes the potential function $\Phi(\boldsymbol{a})$ is a Nash equilibrium [17]. We hence design a distributed spectrum access algorithm that achieves the SNE of the SGUM Γ by maximizing the potential function $\Phi(\boldsymbol{a})$.

To proceed, first consider the problem that the users collectively compute the optimal channel selection profile such that the potential function is maximized, i.e.,

$$\max_{\boldsymbol{a} \in \Omega \triangleq \prod_{n=1}^{N} \mathcal{M}_n} \Phi(\boldsymbol{a}). \tag{7}$$

The problem (7) involves a combinatorial optimization over the discrete solution space Ω. In general, it is very challenging to solve such a problem exactly especially when the system size is large (i.e., the solution space Ω is large).

With this observation, it is plausible to search for approximate solutions to the potential function maximization problem. To this end, rewrite

We then consider to approach the potential function maximization solution approximately. To proceed, we first write the problem (7) as the following equivalent randomized problem:

$$\max_{(q_{\boldsymbol{a}} \geq 0 : \boldsymbol{a} \in \Omega)} \sum_{\boldsymbol{a} \in \Omega} q_{\boldsymbol{a}} \Phi(\boldsymbol{a}) \tag{8}$$

$$\text{s.t.} \sum_{\boldsymbol{a} \in \Omega} q_{\boldsymbol{a}} = 1,$$

where $q_{\boldsymbol{a}}$ is the probability that channel selection profile $\boldsymbol{a}$ is adopted. Obviously, the optimal solution to problem (8) is to choose the optimal channel selection profiles with probability one. It is known from the Markov approximation approach in [18] that problem (8) can be approximated by the following convex optimization problem:

$$\max_{(q_{\boldsymbol{a}} \geq 0 : \boldsymbol{a} \in \Omega)} \sum_{\boldsymbol{a} \in \Omega} q_{\boldsymbol{a}} \Phi(\boldsymbol{a}) - \frac{1}{\theta} \sum_{\boldsymbol{a} \in \Omega} q_{\boldsymbol{a}} \ln q_{\boldsymbol{a}} \tag{9}$$

$$\text{s.t.} \sum_{\boldsymbol{a} \in \Omega} q_{\boldsymbol{a}} = 1,$$

where θ is the parameter that controls the approximation ratio. Note that the approximation in (9) can guarantee the asymptotic optimality. This is because that when $\theta \to \infty$, the problem (9) boils down to exactly the same as problem (8). That is, when $\theta \to \infty$, the optimal solutions that maximize the potential function $\Phi(\boldsymbol{a})$ will be selected with probability one.

Algorithm 1 Distributed Spectrum Access Algorithm For Social Group Utility Maximization

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1: **initialization:**
2:     **set** the parameter θ and the channel update rate τ_n.
3:     **choose** a channel $a_n \in \mathcal{M}_n$ randomly for each user $n \in \mathcal{N}$.
4: **end initialization**

5: **loop** for each user $n \in \mathcal{N}$ in parallel:
6:     **compute** the social group utility $S_n(a_n, a_{-n})$ on the chosen channel a_n.
7:     **generate** a timer value following the exponential distribution with the mean equal to $\frac{1}{\tau_n}$.
8:     **count down** until the timer expires.
9:     **if** the timer expires **then**
10:       **choose** a new channel $a_n^{'} \in \mathcal{M}_n$ randomly.
11:       **compute** the social group utility $S_n(a_n^{'}, a_{-n})$ on the new channel $a_n^{'}$.
12:       **stay in** the new channel $a_n^{'}$ with probability $\frac{\exp\left(\theta S_n(a_n^{'}, a_{-n})\right)}{\max\{\exp\left(\theta S_n(a_n^{'}, a_{-n})\right), \exp(\theta S_n(a_n, a_{-n}))\}}$, Or **move back** to the original channel a_n with probability $1 - \frac{\exp\left(\theta S_n(a_n^{'}, a_{-n})\right)}{\max\{\exp\left(\theta S_n(a_n^{'}, a_{-n})\right), \exp(\theta S_n(a_n, a_{-n}))\}}$.
13:     **end if**
14: **end loop**

Moreover, the approximation in (9) enables us to obtain the close-form solution, which facilitates the distributed algorithm design later. More specifically, by the KKT conditions [19], the optimal solution to problem (9) is given as

$$q_{\boldsymbol{a}}^{*} = \frac{\exp(\theta \Phi(\boldsymbol{a}))}{\sum_{\hat{\boldsymbol{a}} \in \Omega} \exp(\theta \Phi(\hat{\boldsymbol{a}}))}. \tag{10}$$

Based on (10), we then design a self-organizing algorithm such that the asynchronous channel selection updates of the users form a Markov chain (with the system state as the channel selection profile $\boldsymbol{a}$ of all users). As long as the Markov chain converges to the stationary distribution as given in (10), we can approach the Nash equilibrium channel selection profile that maximizes the potential function by setting a large enough parameter θ.

B. Markov Chain Design for Distributed Spectrum Access

Motivated by the seminal work on the adaptive CSMA mechanism [2], we propose a distributed spectrum access algorithm in Algorithm 1 such that each user n updates its channel selection according to a timer value that follows the exponential distribution with a rate of τ_n. Note that the study in [2] focuses on the network utility maximization, while in this paper we consider the social group utility maximization, which results in significant differences in analysis.

Appealing to the property of exponential distributions, we have that the probability that more than one users generate the same timer value and update their channels simultaneously equals zero. Since one user will activate for the channel selection update at a time, the direct transitions between two

system states $\boldsymbol{a}$ and $\boldsymbol{a}'$ are feasible if these two system states differ by one and only one user's channel selection. We also denote the set of system states that can be transited directly from the state $\boldsymbol{a}$ as $\Delta_{\boldsymbol{a}} = \{\boldsymbol{a}' \in \Omega : |\{\boldsymbol{a}' \cup \boldsymbol{a}\} \backslash \{\boldsymbol{a}' \cap \boldsymbol{a}\}| = 2\}$, where $|\cdot|$ denotes the size of a set. According to (4), a user n can compute the social group utility $S_n(\boldsymbol{a})$ by locally enquiring the users having social ties with it about their received interferences. Then user n will randomly choose a new channel $a_n' \in \mathcal{M}_n$ and stay in this channel with a probability of

$$\frac{\exp\left(\theta S_n(a_n', a_{-n})\right)}{\max\{\exp\left(\theta S_n(a_n', a_{-n})\right), \exp\left(\theta S_n(a_n, a_{-n})\right)\}}. \quad (11)$$

The underlying intuition behind (11) is as follows. When $S_n(a_n', a_{-n}) \geq S_n(a_n, a_{-n})$ (i.e., the new channel a_n' offers the better performance), user n will stay in the new channel a_n' with probability one. According to the property of potential game in (5), we know that choosing the new channel a_n' can help to increase both user n's social group utility $S_n(\boldsymbol{a})$ and the potential function $\Phi(\boldsymbol{a})$ of the SGUM game. When $S_n(a_n', a_{-n}) < S_n(a_n, a_{-n})$ (i.e., the original channel a_n offers the better performance), user n will switch back to the original channel a_n with a probability of $1 - \frac{\exp\left(\theta S_n(a_n', a_{-n})\right)}{\exp\left(\theta S_n(a_n, a_{-n})\right)}$. That is, the probability that user n will switch back to the original channel a_n increases with the performance gap $S_n(a_n, a_{-n}) - S_n(a_n', a_{-n})$. We would like to emphasize that such probabilistic channel selections are necessary such that all the users can explore the feasible channel selection space to prevent the algorithm from getting stuck at a local optimum.

Then from a system-wide perspective, the probability of transition from state (a_n, a_{-n}) to (a_n', a_{-n}) due to user n's channel selection update is given as

$$\frac{1}{|\mathcal{M}_n|} \frac{\exp\left(\theta S_n(a_n', a_{-n})\right)}{\max\{\exp\left(\theta S_n(a_n', a_{-n})\right), \exp\left(\theta S_n(a_n, a_{-n})\right)\}}. \quad (12)$$

Since each user n activates its channel selection update according to the countdown timer mechanism with a rate of τ_n, hence if $\boldsymbol{a}' \in \Delta_{\boldsymbol{a}}$, the transition rate from state $\boldsymbol{a}$ to state $\boldsymbol{a}'$ is given as

$$q_{\boldsymbol{a}, \boldsymbol{a}'} = \frac{\tau_n}{|\mathcal{M}_n|} \frac{\exp\left(\theta S_n(a_n', a_{-n})\right)}{\max\{\exp\left(\theta S_n(a_n', a_{-n})\right), \exp\left(\theta S_n(a_n, a_{-n})\right)\}}. \quad (13)$$

Otherwise, we have $q_{\boldsymbol{a}, \boldsymbol{a}'} = 0$. We show in Theorem 2 that the spectrum access Markov chain is time reversible. Time reversibility means that when tracing the Markov chain backwards, the stochastic behavior of the reverse Markov chain remains the same. A nice property of a time reversible Markov chain is that it always admits a unique stationary distribution, which is independent of the initial system state. This implies that given any initial channel selections the distributed spectrum access algorithm can drive the system converging to the stationary distribution given in (10).

Theorem 2. *The distributed spectrum access algorithm induces a time-reversible Markov chain with the unique stationary distribution as given in (10).*

Due to space limit, the proof is given in the online technical report [20]. One key idea of the proof is to show that the distribution in (10) satisfies the following detailed balance equations: $q_{\boldsymbol{a}}^* q_{\boldsymbol{a}, \boldsymbol{a}'} = q_{\boldsymbol{a}'}^* q_{\boldsymbol{a}', \boldsymbol{a}}, \forall \boldsymbol{a}, \boldsymbol{a}' \in \Omega$.

C. Performance Analysis

According to Theorem 2, we can achieve the SNE that maximizes the potential function $\Phi(\boldsymbol{a})$ of the SGUM game Γ by setting $\theta \to \infty$. However, in practice one has to choose only implement a finite value of θ. Let $\bar{\Phi} = \sum_{a \in \Omega} q_a^* \Phi(\boldsymbol{a})$ be the expected potential by Algorithm 1 and $\Phi^* = \max_{a \in \Omega} \Phi(\boldsymbol{a})$ be the maximum potential. We show in Theorem 3 that, when a large enough θ is adopted in practice, the gap between $\bar{\Phi}$ and Φ^* is very small.

Theorem 3. *For the distributed spectrum access algorithm, we have that $0 \leq \Phi^* - \bar{\Phi} \leq \frac{1}{\theta} \sum_{n=1}^{N} \ln |\mathcal{M}_n|$, where $|\mathcal{M}_n|$ denotes the number of vacant channels of user n.*

Proof. First of all, we must have that $\Phi^* \geq \bar{\Phi}$.

According to (8) and (9), we then have that

$$\max_{(q_{\boldsymbol{a}}: \boldsymbol{a} \in \Omega)} \sum_{a \in \Omega} q_a \Phi(\boldsymbol{a}) \leq \max_{(q_{\boldsymbol{a}}: \boldsymbol{a} \in \Omega)} \sum_{a \in \Omega} q_a \Phi(\boldsymbol{a}) - \frac{1}{\theta} \sum_{a \in \Omega} q_a \ln q_a, \quad (14)$$

which is due to the fact that $0 \leq -\frac{1}{\theta} \sum_{a \in \Omega} q_a \ln q_a \leq \frac{1}{\theta} \ln |\Omega|$. Since q_a^* is the optimal solution to (9) and $\Phi^* = \max_{(q_{\boldsymbol{a}}: \boldsymbol{a} \in \Omega)} \sum_{a \in \Omega} q_a \Phi(\boldsymbol{a})$, according to (14), we know that

$$\Phi^* \leq \sum_{a \in \Omega} q_a^* \Phi(\boldsymbol{a}) - \frac{1}{\theta} \sum_{a \in \Omega} q_a^* \ln q_a^*$$

$$\leq \sum_{a \in \Omega} q_a^* \Phi(\boldsymbol{a}) + \frac{1}{\theta} \ln |\Omega| \leq \bar{\Phi} + \frac{1}{\theta} \ln |\Omega|,$$

which completes the proof. $\square$

Due to space limitation, the proof is given in the online technical report [20]. We next discuss the efficiency of the SNE by the distributed spectrum access algorithm when θ is sufficiently large (i.e., $\theta \to \infty$). Let $V(\boldsymbol{a})$ be the total individual utility received by all the users under the channel selection profile $\boldsymbol{a}$, i.e., $V(\boldsymbol{a}) = \sum_{n=1}^{N} U_n(\boldsymbol{a})$. We denote $\bar{\boldsymbol{a}}$ as the NUM solution that maximizes the system-wide utility (i.e., $\bar{\boldsymbol{a}} = \arg\max_{a \in \Omega} V(\boldsymbol{a})$) and $\hat{\boldsymbol{a}}$ as the convergent SNE by the distributed spectrum access algorithm (i.e., $\hat{\boldsymbol{a}} = \arg\max_{a \in \Omega} \Phi(\boldsymbol{a})$). We then define the performance gap ρ as the difference between the total utility received at the NUM solution $\bar{\boldsymbol{a}}$ and that of the SNE $\hat{\boldsymbol{a}}$, i.e., $\rho = V(\bar{\boldsymbol{a}}) - V(\hat{\boldsymbol{a}})$. We can show the following result.

Theorem 4. *The performance gap ρ of the SNE by the distributed spectrum access algorithm is at most*

$$\frac{1}{2} \sum_{n=1}^{N} \sum_{m \in \mathcal{N}_n^{sp}} (1 - w_{nm}) P_m d_{mn}^{-\alpha} + \frac{1}{2} \sum_{n=1}^{N} \sum_{m \in \mathcal{N}_n^{p} \backslash \mathcal{N}_n^{sp}} P_m d_{mn}^{-\alpha}.$$

Proof. According to (2) and (6), we have that

$$V(\boldsymbol{a}) = \sum_{n=1}^{N} U_n(\boldsymbol{a}) = -\sum_{n=1}^{N} \sum_{m \in \mathcal{N}_n^{p}} P_m d_{mn}^{-\alpha} I_{\{a_n = a_m\}} - \sum_{n=1}^{N} \omega_{a_n}^{n}$$

$$=\Phi(\boldsymbol{a}) - \frac{1}{2}\sum_{n=1}^{N}\sum_{m\in\mathcal{N}_n^{sp}}(1-w_{nm})P_m d_{mn}^{-\alpha}I_{\{a_n=a_m\}}$$

$$-\frac{1}{2}\sum_{n=1}^{N}\sum_{m\in\mathcal{N}_n^{p}\backslash\mathcal{N}_n^{sp}}P_m d_{mn}^{-\alpha}I_{\{a_n=a_m\}}.$$

We then have that

$$\rho = V(\overline{\boldsymbol{a}}) - V(\hat{\boldsymbol{a}}) = \Phi(\overline{\boldsymbol{a}}) - \Phi(\hat{\boldsymbol{a}})$$

$$-\frac{1}{2}\sum_{n=1}^{N}\sum_{m\in\mathcal{N}_n^{sp}}(1-w_{nm})P_m d_{mn}^{-\alpha}\left(I_{\{\overline{a}_n=\overline{a}_m\}} - I_{\{\hat{a}_n=\hat{a}_m\}}\right)$$

$$-\frac{1}{2}\sum_{n=1}^{N}\sum_{m\in\mathcal{N}_n^{p}\backslash\mathcal{N}_n^{sp}}P_m d_{mn}^{-\alpha}\left(I_{\{\overline{a}_n=\overline{a}_m\}} - I_{\{\hat{a}_n=\hat{a}_m\}}\right). \tag{15}$$

Since $\Phi(\hat{\boldsymbol{a}}) = \max_{a\in\Omega}\Phi(\boldsymbol{a}) \geq \Phi(\overline{\boldsymbol{a}})$ and $V(\overline{\boldsymbol{a}}) = \max_{a\in\Omega}V(\boldsymbol{a}) \geq V(\hat{\boldsymbol{a}})$, we know from (15) that

$$\rho \leq \frac{1}{2}\sum_{n=1}^{N}\sum_{m\in\mathcal{N}_n^{sp}}(1-w_{nm})P_m d_{mn}^{-\alpha}\left(I_{\{\hat{a}_n=\hat{a}_m\}} - I_{\{\overline{a}_n=\overline{a}_m\}}\right)$$

$$+\frac{1}{2}\sum_{n=1}^{N}\sum_{m\in\mathcal{N}_n^{p}\backslash\mathcal{N}_n^{sp}}P_m d_{mn}^{-\alpha}\left(I_{\{\hat{a}_n=\hat{a}_m\}} - I_{\{\overline{a}_n=\overline{a}_m\}}\right)$$

$$\leq \frac{1}{2}\sum_{n=1}^{N}\sum_{m\in\mathcal{N}_n^{sp}}(1-w_{nm})P_m d_{mn}^{-\alpha} + \frac{1}{2}\sum_{n=1}^{N}\sum_{m\in\mathcal{N}_n^{p}\backslash\mathcal{N}_n^{sp}}P_m d_{mn}^{-\alpha}.$$

$\square$

Theorem 4 indicates that the upper-bound of the performance gap ρ decreases as the strength of social tie w_{nm} among users increases. When $w_{nm} = 0$ for any user $n, m \in \mathcal{N}$ (i.e., all users are selfish), the social group utility maximization game Γ degenerates to the non-cooperative spectrum access game and the upper-bound of the performance gap ρ reaches the maximum of $\frac{1}{2}\sum_{n=1}^{N}\sum_{m\in\mathcal{N}_n^{p}}P_m d_{mn}^{-\alpha}$. When $w_{nm} = 1$ for any user $n, m \in \mathcal{N}$ (i.e., all users are fully altruistic), the SGUM becomes the NUM and the performance gap $\rho = 0$. In Section V, we also evaluate the performance of the SGUM solution by real social data traces. Numerical results demonstrate that the performance gap between the SGUM solution and the NUM solution is at most 15%.

V. Numerical Results

In this section, we evaluate the SGUM solution for database assisted spectrum access by numerical studies based on both Erdos-Renyi social graphs and real trace based social graphs.

A. Social Graph with 8 White-Space Users

We first consider a database assisted spectrum access network consisting of $M = 5$ channels and $N = 8$ white-space users, which are scattered across a square area of a length of 500 m (see Figure 4). The transmission power of each user is $P_n = 100$ mW [6], the path loss factor $\alpha = 4$, and the background interference power ω_m^n for each channel m and user n is randomly assigned in the interval of $[-100, -90]$ dBm. Each user n has a different set of vacant channels by consulting the geo-location database. For example, the vacant

channels for user 1 are $\{2, 3, 4\}$. For the interference graph $\mathcal{G}^p$, we define that the user's transmission range $\delta = 1000$ m and two users can generate inference to each other if their distance is not greater than δ. The social graph $\mathcal{G}^s$ is given in Figure 4 where two users have social tie if there is an edge between them and the numerical value associated with each edge represents the strength of social tie.

We implement the proposed distributed spectrum access algorithm for the SGUM game with different parameters θ in Figure 5. We see that the convergent potential function value Φ of the SGUM game increases as the parameter θ increases. When the parameter θ is large enough (e.g., $\theta \geq 10^6$), the algorithm can approach the maximum potential function value $\Phi^* = \max_{\boldsymbol{a}}\Phi(\boldsymbol{a})$. Figure 6 shows the dynamics of user's time average interference $\gamma_n(\boldsymbol{a})$. It demonstrates that the distributed spectrum access algorithm can drive users' time average interference decreasing and converging to an equilibrium such that each user only receives a small interference level. To verify that the algorithm can approach the SNE of the SGUM game, we show the dynamics of the potential value $\Phi(\boldsymbol{a})$ in Figure 7. We see that the distributed spectrum access algorithm can drive the potential value Φ increasing and approach the maximum potential value Φ^*. According to the property of potential game, the algorithm hence can approach the SNE of the SGUM game.

B. Erdos-Renyi Social Graph

We then consider $N = 100$ users that randomly scattered across a square area of a length of 2000 m. We evaluate the SGUM game solution by the distributed spectrum access algorithm with the social graph represented by the Erdos-Renyi (ER) graph model [21], where a social link exists between any two users with a probability of P_L. We set the strength of social tie $w_{nm} = 1$ for each social link. To evaluate the impact of social link density of the social graph, we implement the simulations with different social link probabilities $P_L = 0, 0.1, ..., 1.0$, respectively. For each given P_L, we average over 100 runs. To benchmark the SGUM solution, we also implement the the following two solutions:

(1) Non-cooperative spectrum access: we implement the non-cooperative game based solution such that each user aims to maximize its individual utility, i.e., we set $S_n(\boldsymbol{a}) = U_n(\boldsymbol{a})$ in the distributed spectrum access algorithm.

(2) Network utility maximization: we implement the social optimal solution such that the system-wide utility is maximized, i.e., we set $S_n(\boldsymbol{a}) = \sum_{n=1}^{N}U_n(\boldsymbol{a})$ in the distributed spectrum access algorithm.

Similar to the price of anarchy in non-cooperative game [22], we normalize the system-wide interference of these solutions with respect to that of the social optimal solution (i.e., network utility maximum solution). The results are given in Figure 8. We see that the performance of the SGUM solution always dominates that of the non-cooperative spectrum access. This is non-trivial since non-cooperative game promotes the competition among users to increase the system-wide utility and has been widely adopted to devise efficient
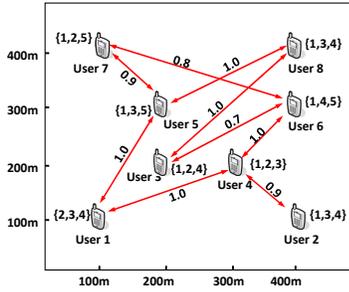
Fig. 4. A square area of a length of 500 m with 8 scattered white-space users
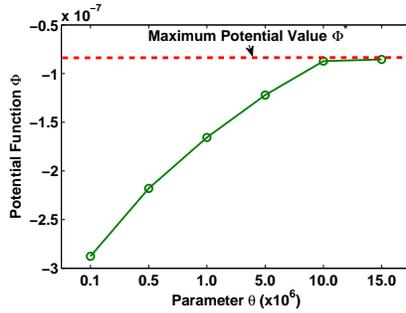

Fig. 5. The convergent potential value Φ with different parameters θ
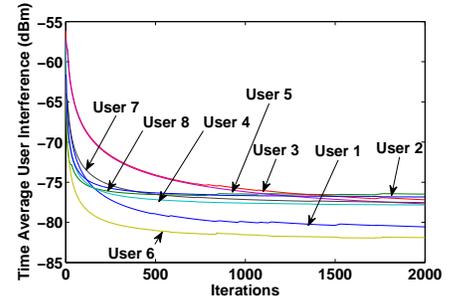

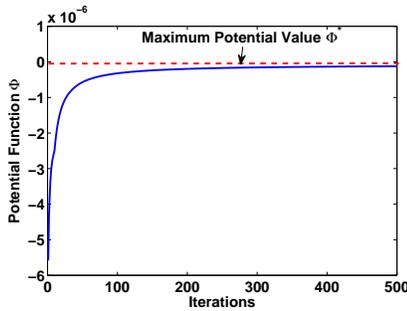Fig. 6. Dynamics of users' time average interference when $\theta = 10^6$

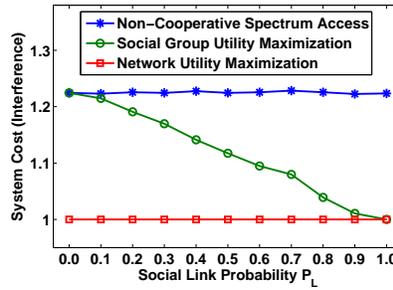Fig. 7. Dynamics of potential value Φ when $\theta = 10^6$

Fig. 8. Normalized system-wide interference with number of nodes $N = 100$ and different social network density
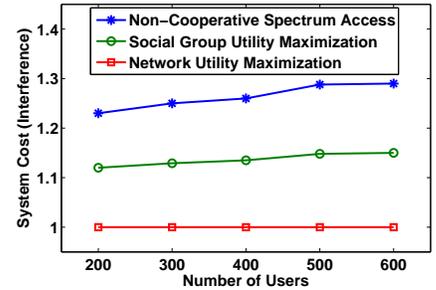

Fig. 9. Normalized system-wide interference with different number of users

distributed resource allocation mechanisms in wireless networks [3]. Moreover, we observe that the performance gain of the SGUM solution increases as the social link probability P_L increases. When the social link probability $P_L = 1$, the SGUM solution achieves the same performance of the network utility maximization and can reduce 23% system-wide interference over the non-cooperative spectrum access. This also demonstrates that the proposed SGUM framework spans the continuum space between non-cooperative game and network utility maximization – two extreme paradigms based on drastically different assumptions that users are selfish and altruistic, respectively.

C. Real Trace Based Social Graph

We next evaluate the SGUM solution by the distributed spectrum access algorithm based on the social graph represented by the friendship network of the real data trace Brightkite [23]. We implement experiments with the number of users $N = 200, 300, ..., 600$, respectively. As the benchmark, we also implement the solutions of non-cooperative spectrum access and network utility maximization.

The results are shown in Figure 9. We see that the non-cooperative spectrum access solution will increase the system-wide interference up-to 29% over the network utility maximization solution. Upon comparison, the system-wide interference by the the SGUM solution will increase at most 15%, compared with the network utility maximization solution. This verifies the effectiveness of leveraging social tie to stimulate

user cooperation for achieving efficient distributed spectrum access in practices.

VI. CONCLUSION

In this paper, we have developed a general SGUM framework that highlights the interplay between the physical coupling and the social coupling among users. We showed that the SGUM framework can provide rich modeling flexibility and span the continuum space between non-cooperative game and network utility maximization. In particular, we have studied the application in database assisted spectrum access under this framework. We showed that the SGUM game for database assisted spectrum access problem is a potential game and always admits a social-aware Nash equilibrium. We also designed a distributed spectrum access algorithm that can achieve the social-aware Nash equilibrium. We further evaluated the performance of the algorithm based on and real trace based social graphs. Numerical results demonstrate that the performance gap between the SGUM solution and the NUM solution is at most 15%.

We are currently studying the SGUM to build a thorough understanding of the impact of social behavior on networking performance. Beyond the potential game approach in this paper, one can resort to other powerful tools such as *supermodular game* to characterize social-aware Nash equilibrium and design distributed algorithm accordingly. We believe that this framework will open a new door for future networking system design by exploiting social interactions.

APPENDIX

A. Proof of Theorem 1

Suppose that a user k changes its channel a_k to $a_k^{'}$ such that the channel selection profile changes from $\boldsymbol{a}$ to $\boldsymbol{a}^{'}$. We have that

$$\Phi(\boldsymbol{a}^{'}) - \Phi(\boldsymbol{a}) = \Phi_1(\boldsymbol{a}^{'}) - \Phi_1(\boldsymbol{a}) + \Phi_2(\boldsymbol{a}^{'}) - \Phi_2(\boldsymbol{a}). \quad (16)$$

For the part Φ_1, we have that

$$\Phi_1(\boldsymbol{a}^{'}) - \Phi_1(\boldsymbol{a})$$

$$= -\frac{1}{2} \sum_{m \in \mathcal{N}_k^p} P_m d_{mk}^{-\alpha} I_{\{a_k^{'}=a_m\}} - \frac{1}{2} \sum_{n \neq k} \sum_{k \in \mathcal{N}_n^p} P_k d_{kn}^{-\alpha} I_{\{a_n = a_k^{'}\}}$$

$$- \omega_{a_k^{'}}^k + \frac{1}{2} \sum_{m \in \mathcal{N}_k^p} P_m d_{mk}^{-\alpha} I_{\{a_k = a_m\}}$$

$$+ \frac{1}{2} \sum_{n \neq k} \sum_{k \in \mathcal{N}_n^p} P_k d_{kn}^{-\alpha} I_{\{a_n = a_k\}} + \omega_{a_k}^k. \quad (17)$$

Since users access the spectrum with the same power level and <mark>the interference relationship and distance measurement are symmetry, we know that</mark>

$$\sum_{n \neq k} \sum_{k \in \mathcal{N}_n^p} P_k d_{kn}^{-\alpha} = \sum_{n \in \mathcal{N}_k^p} P_k d_{kn}^{-\alpha} = \sum_{n \in \mathcal{N}_k^p} P_n d_{nk}^{-\alpha}. \quad (18)$$

Combining (17) and (18), we have that

$$\Phi_1(\boldsymbol{a}^{'}) - \Phi_1(\boldsymbol{a})$$

$$= -\frac{1}{2} \sum_{m \in \mathcal{N}_k^p} P_m d_{mk}^{-\alpha} I_{\{a_k^{'}=a_m\}} - \frac{1}{2} \sum_{n \in \mathcal{N}_k^p} P_n d_{nk}^{-\alpha} I_{\{a_n = a_k^{'}\}} - \omega_{a_k^{'}}^k$$

$$+ \frac{1}{2} \sum_{m \in \mathcal{N}_k^p} P_m d_{mk}^{-\alpha} I_{\{a_k = a_m\}} + \frac{1}{2} \sum_{n \in \mathcal{N}_k^p} P_n d_{nk}^{-\alpha} I_{\{a_n = a_k\}} + \omega_{a_k}^k$$

$$= - \sum_{m \in \mathcal{N}_k^p} P_m d_{mk}^{-\alpha} I_{\{a_k^{'}=a_m\}} - \omega_{a_k^{'}}^k$$

$$+ \sum_{m \in \mathcal{N}_k^p} P_m d_{mk}^{-\alpha} I_{\{a_k = a_m\}} + \omega_{a_k}^k$$

$$= - \gamma_k(\boldsymbol{a}^{'}) + \gamma_k(\boldsymbol{a}) = U_k(\boldsymbol{a}^{'}) - U_k(\boldsymbol{a}). \quad (19)$$

Similarly, for the part Φ_2, we have that

$$\Phi_2(\boldsymbol{a}^{'}) - \Phi_2(\boldsymbol{a})$$

$$= \sum_{n \in \mathcal{N}_k^{sp}} w_{kn} \left(-P_k d_{kn}^{-\alpha} I_{\{a_n = a_k^{'}\}} + P_k d_{kn}^{-\alpha} I_{\{a_n = a_k\}} \right)$$

$$= \sum_{n \in \mathcal{N}_k^{sp}} w_{kn} \times$$

$$\left(-P_k d_{kn}^{-\alpha} I_{\{a_n = a_k^{'}\}} - \sum_{m \neq k} \sum_{m \in \mathcal{N}_n^p} P_m d_{mn}^{-\alpha} I_{\{a_n = a_m\}} - \omega_{a_n}^k \right.$$

$$\left. + P_k d_{kn}^{-\alpha} I_{\{a_n = a_k\}} + \sum_{m \neq k} \sum_{m \in \mathcal{N}_n^p} P_m d_{mn}^{-\alpha} I_{\{a_n = a_m\}} + \omega_{a_n}^k \right)$$

$$= \sum_{n \in \mathcal{N}_k^{sp}} w_{kn} \left(-\gamma_n(\boldsymbol{a}^{'}) + \gamma_n(\boldsymbol{a}) \right)$$

$$= \sum_{n \in \mathcal{N}_k^{sp}} w_{kn} \left(U_n(\boldsymbol{a}^{'}) - U_n(\boldsymbol{a}) \right). \quad (20)$$

Finally, substituting (19) and (20) into (16), we obtain that

$$\Phi(\boldsymbol{a}^{'}) - \Phi(\boldsymbol{a})$$

$$= U_k(\boldsymbol{a}^{'}) - U_k(\boldsymbol{a}) + \sum_{n \in \mathcal{N}_k^{sp}} w_{kn} \left(U_n(\boldsymbol{a}^{'}) - U_n(\boldsymbol{a}) \right). \quad (21)$$

<mark>Since user k can not generate interference to any user $n \in \mathcal{N}_k^s \backslash \mathcal{N}_k^{sp}$, we have that</mark>

$$U_n(\boldsymbol{a}^{'}) = U_n(\boldsymbol{a}), \forall n \in \mathcal{N}_k^s \backslash \mathcal{N}_k^{sp}.$$

This implies that

$$\Phi(\boldsymbol{a}^{'}) - \Phi(\boldsymbol{a})$$

$$= U_k(\boldsymbol{a}^{'}) - U_k(\boldsymbol{a}) + \sum_{n \in \mathcal{N}_k^{sp}} w_{kn} \left(U_n(\boldsymbol{a}^{'}) - U_n(\boldsymbol{a}) \right)$$

$$+ \sum_{n \in \mathcal{N}_k^s \backslash \mathcal{N}_k^{sp}} w_{kn} \left(U_n(\boldsymbol{a}^{'}) - U_n(\boldsymbol{a}) \right)$$

$$= U_k(\boldsymbol{a}^{'}) - U_k(\boldsymbol{a}) + \sum_{n \in \mathcal{N}_k^s} w_{kn} \left(U_n(\boldsymbol{a}^{'}) - U_n(\boldsymbol{a}) \right),$$

which completes the proof. $\square$

REFERENCES

[1] M. Chiang, S. H. Low, A. R. Calderbank, and J. C. Doyle, "Layering as optimization decomposition: A mathematical theory of network architectures," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 255–312, 2007.

[2] L. Jiang and J. Walrand, "A distributed csma algorithm for throughput and utility maximization in wireless networks," *IEEE/ACM TON*, vol. 18, no. 3, pp. 960–972, 2010.

[3] Z. Han, D. Niyato, W. Saad, T. Basar, and A. Hjorungnes, *Game theory in wireless and communication networks*. Cambridge University Press, 2012.

[4] T. Govier, *Social trust and human communities*. McGill-Queen's Press-MQUP, 1997.

[5] N. Kayastha, D. Niyato, P. Wang, and E. Hossain, "Applications, architectures, and protocol design issues for mobile social networks: A survey," *Proceedings of the IEEE*, vol. 99, no. 12, pp. 2130–2158, 2011.

[6] FCC, *THIRD MEMORANDUM OPINION AND ORDER*, FCC Std., April 5, 2012.

[7] P.-A. Chen and D. Kempe, "Altruism, selfishness, and spite in traffic routing," in *the 9th ACM conference on Electronic commerce*, 2008.

[8] M. Hoefer and A. Skopalik, "Altruism in atomic congestion games," in *Algorithms-ESA 2009*. Springer, 2009, pp. 179–189.

[9] G. Kesidis, Y. Jin, A. P. Azad, and E. Altman, "Stable nash equilibria of aloha medium access games under symmetric, socially altruistic behavior," in *IEEE CDC*. IEEE, 2010, pp. 1071–1075.

[10] X. Chen, J. Huang, and H. Li, "Adaptive channel recommendation for opportunistic spectrum access," *IEEE Transactions on Mobile Computing*, pp. 1–1, 2012.

[11] P. Costa, C. Mascolo, M. Musolesi, and G. P. Picco, "Socially-aware routing for publish-subscribe in delay-tolerant mobile ad hoc networks," *IEEE JSAC*, vol. 26, no. 5, pp. 748–760, 2008.

[12] W. Gao, Q. Li, B. Zhao, and G. Cao, "Multicasting in delay tolerant networks: a social network perspective," in *ACM MOBIHOC*, 2009.

[13] X. Chen, B. Proulx, X. Gong, and J. Zhang, "Social trust and social reciprocity based cooperative d2d communications," in *ACM MOBIHOC*, 2013.

[14] L. Yang, H. Kim, J. Zhang, M. Chiang, and C. W. Tan, "Pricing-based decentralized spectrum access control in cognitive radio networks," *IEEE/ACM Transactions on Networking*, vol. 21, no. 2, pp. 522–535, 2013.

[15] X. Chen and J. Huang, "Database-assisted distributed spectrum sharing," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 11, pp. 2349 – 2361, 2013.

[16] K. N. Ramachandran, E. M. Belding-Royer, K. C. Almeroth, and M. M. Buddhikot, "Interference-aware channel assignment in multi-radio wireless mesh networks." in *INFOCOM*, vol. 6, 2006, pp. 1–12.

[17] D. Monderer and L. S. Shapley, "Potential games," *Games and economic behavior*, vol. 14, no. 1, pp. 124–143, 1996.

[18] M. Chen, S. C. Liew, Z. Shao, and C. Kai, "Markov approximation for combinatorial network optimization," in *IEEE INFOCOM*. IEEE, 2010, pp. 1–9.

[19] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.

[20] X. Chen, X. Gong, L. Yang, and J. Zhang, "Social group utility maximization framework with application in database assissted spectrum access," Arizona State University, Tech. Rep., 2013. [Online]. Available: http://www.public.asu.edu/~xchen179/SGAppendix.pdf

[21] M. E. Newman, D. J. Watts, and S. H. Strogatz, "Random graph models of social networks," *PANS*, vol. 99, no. Suppl 1, pp. 2566–2572, 2002.

[22] T. Roughgarden, *Selfish routing and the price of anarchy*. MIT press, 2005.

[23] E. Cho, S. A. Myers, and J. Leskovec, "Friendship and mobility: user movement in location-based social networks," in *ACM SIGKDD*, 2011, pp. 1082–1090.

ONLINE APPENDIX

B. Proof of Theorem 2

As mentioned, the system state of the spectrum access Markov chain is defined as the channel selection profile $\boldsymbol{a} \in \Theta$ of all users. Since it is possible to get from any state to any other state within finite steps of transition, the spectrum access Markov chain is hence irreducible and has a stationary distribution.

We then show that the Markov chain is time reversible by showing that the distribution in (10) satisfies the following detailed balance equations:

$$q_{\boldsymbol{a}}^* q_{\boldsymbol{a},\boldsymbol{a}'} = q_{\boldsymbol{a}'}^* q_{\boldsymbol{a}',\boldsymbol{a}}, \forall \boldsymbol{a}, \boldsymbol{a}' \in \Theta. \quad (22)$$

To see this, we consider the following two cases:

1) If $\boldsymbol{a}' \notin \Delta_{\boldsymbol{a}}$, we have $q_{\boldsymbol{a},\boldsymbol{a}'} = q_{\boldsymbol{a}',\boldsymbol{a}} = 0$ and the equation (22) holds.

2) If $\boldsymbol{a}' \in \Delta_{\boldsymbol{a}}$, according to (10) and (13), we have

$$q_{\boldsymbol{a}}^* q_{\boldsymbol{a},\boldsymbol{a}'}$$

$$= \frac{\tau_n}{|\mathcal{M}_n|} \frac{\exp(\theta\Phi(\boldsymbol{a}))}{\sum_{\hat{\boldsymbol{a}} \in \Theta} \exp(\theta\Phi(\hat{\boldsymbol{a}}))}$$

$$\times \frac{\exp\left(\theta S_n(a_n', a_{-n})\right)}{\max\{\exp\left(\theta S_n(a_n', a_{-n})\right), \exp\left(\theta S_n(a_n, a_{-n})\right)\}}$$

$$= \frac{\tau_n}{|\mathcal{M}_n|} \frac{\exp\left(\theta\left(\Phi(\boldsymbol{a}) + S_n(a_n', a_{-n})\right)\right)}{\sum_{\hat{\boldsymbol{a}} \in \Theta} \exp(\theta\Phi(\hat{\boldsymbol{a}}))}$$

$$\times \frac{1}{\max\{\exp\left(\theta S_n(a_n', a_{-n})\right), \exp\left(\theta S_n(a_n, a_{-n})\right)\}},$$

and similarly,

$$q_{\boldsymbol{a}'}^* q_{\boldsymbol{a}',\boldsymbol{a}}$$

$$= \frac{\tau_n}{|\mathcal{M}_n|} \frac{\exp\left(\theta\left(\Phi(\boldsymbol{a}') + S_n(a_n, a_{-n})\right)\right)}{\sum_{\hat{\boldsymbol{a}} \in \Theta} \exp(\theta\Phi(\hat{\boldsymbol{a}}))}$$

$$\times \frac{1}{\max\{\exp\left(\theta S_n(a_n', a_{-n})\right), \exp\left(\theta S_n(a_n, a_{-n})\right)\}}.$$

Thus, according to (5), we must have

$$q_{\boldsymbol{a}}^* q_{\boldsymbol{a},\boldsymbol{a}'} = q_{\boldsymbol{a}'}^* q_{\boldsymbol{a}',\boldsymbol{a}}.$$

The spectrum access Markov chain is hence time-reversible and has the unique stationary distribution as given in (10). $\square$