Output MAI Distributions of Linear MMSE Multiuser Receivers in DS-CDMA Systems

Junshan Zhang, *Member, IEEE*, Edwin K. P. Chong, *Senior Member, IEEE*, and David N. C. Tse, *Member, IEEE*

***Abstract*—Multiple-access interference (MAI) in a code-division multiple-access (CDMA) system plays an important role in performance analysis and characterization of fundamental system limits. In this paper, we study the behavior of the output MAI of the minimum mean-square error (MMSE) receiver employed in the uplink of a direct-sequence (DS)-CDMA system. We focus on imperfect power-controlled systems with random spreading, and establish that in a synchronous system 1) the output MAI of the MMSE receiver is asymptotically Gaussian, and 2) for almost every realization of the signatures and received powers, the conditional distribution of the output MAI converges weakly to the same Gaussian distribution as in the unconditional case. We also extend our study to asynchronous systems and establish the Gaussian nature of the output interference. These results indicate that in a large system the output interference is approximately Gaussian, and the performance of the MMSE receiver is robust to the randomness of the signatures and received powers. The Gaussianity justifies the use of single-user Gaussian codes for CDMA systems with linear MMSE receivers, and implies that from the viewpoints of detection and channel capacity, signal-to-interference ratio (SIR) is the key parameter that governs the performance of the MMSE receiver in a CDMA system.**

***Index Terms*—Central limit theorem, martingale difference array, minimum mean-square error (MMSE) receiver, multiple-access interference (MAI), power control, random signature.**

I. Introduction

THE last 10 years have witnessed an explosion in the development of code-division multiple-access (CDMA) systems. It is well known that CDMA systems have the desirable features of dynamic channel sharing, wide range of operating environments, graceful degradation, and ease of cellular planning (see, e.g., [11], [31]). CDMA systems also offer the potential to support a heterogeneous mix of transmitting sources with a broad range of bursty traffic characteristics and quality-of-service (QoS) requirements. In this paper, we consider a model for the uplink of a single-cell direct-sequence CDMA (DS-CDMA) system, with the view that it will provide insight into analyzing multiple-cell systems. The system therein consists of numerous mobile subscribers communicating with one base station, which is typically interconnected to a backbone network via a wired infrastructure.

In a CDMA system, each user, say user i, is assigned a signature of length N, $s_i = \frac{1}{\sqrt{N}}(s_{i1}, \ldots, s_{iN})^t$, and its information symbols are spread onto a much larger bandwidth via its own signature. Since all the users "simultaneously" occupy the same spectrum, they create multiple-access interference (MAI) to one another because of the nonzero cross correlation of their signatures. This gives rise to significant challenges to system design and analysis.

It is well known that bit-error probability (BEP) is an important performance measure in wireless communications, and is determined by the overall interference consisting of the MAI and background noise. In a conventional CDMA system, the overall interference at the output of the matched filter is usually approximated as Gaussian, and the BEP can easily be obtained by using the first- and second-order statistics of the filter output (see, e.g., [32]). The above approximation is done via averaging over the signatures, which is valid in the scenario where users employ long pseudorandom spreading sequences with periods considerably larger than the number of chips per symbol interval (which is the length of the signatures). Sometimes, it is also of interest to study the case where the signatures of the users are repeated from symbol to symbol, but they are randomly and independently selected initially when the users are admitted to the system. In this case, the more interesting quantity is the conditional distribution of the filter output given the signatures. In an information-theoretical setting, Verdú and Shamai [31] assumed that all the users had *equal* received powers and obtained that for almost every choice of signatures, the output MAI of the matched-filter receiver converges (in the sense of divergence) to a Gaussian random variable.

In this paper, we study the distributions of the MAI at the output of the minimum mean-square error (MMSE) receiver. Recently, there has been a substantial literature devoted to the study of linear multiuser receivers because they are practically appealing (see, e.g., [14]–[16], [18], [22], [26], [27], [29], [31]). We focus on the case where the MMSE receiver is employed because the MMSE receiver is optimal in the class of linear multiuser receivers in the sense of minimizing the mean-square error [15]. Assuming the signatures are deterministic, Poor and Verdú [18] have established the Gaussian nature of the MAI-plus-noise at the output of the MMSE receiver in several asymptotic scenarios (the output MAI vanishes in these scenarios). In this

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J. Zhang is with the Department of Electrical Engineering, Arizona State University, Tempe, AZ 85287-7206 USA (e-mail: junshan.zhang@asu.edu).

E. K. P. Chong is with the School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN 47907-1285 USA (e-mail: echong@ecn.purdue.edu).

D. N. C. Tse is with the Department of Electrical Engineering and Computer Science, University of California, Berkeley, CA 94720 USA (e-mail: dtse@eecs.berkeley.edu).

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paper, we study systems where the signatures are randomly and independently chosen, and our results are for the cases where the received powers are *unequal* and not perfectly known to the receiver. More specifically, first we study the unconditional distribution of the output MAI. As in [32], we average over the signatures to calculate the statistics of the output MAI, which is valid in the scenarios where long pseudorandom spreading sequences are employed. Then we study the conditional distributions of the output MAI given the signatures and powers, which is particularly useful in the scenario where repetition of the same random signatures is adopted.

In a practical wireless system, fading is ubiquitous, making perfect power control impossible (see, e.g., [32, pp. 116–119]). Therefore, it is of considerable interest to study a more realistic scenario where the received powers are random. This is the focus of our study. Suppose there are K users in the system when the processing gain is N. We assume that each user is capable of decentralized power control, and that the received powers are independent across different users. We denote the received power of user i as P_i and its mean μ_i. In the development throughout this paper, we assume that the receiver has knowledge of the μ_i's, not the P_i's.

Our results are asymptotic in nature, with both K and N going to infinity. Throughout this paper, the ratio of K to N is denoted by $\alpha \triangleq \frac{K}{N}$ and taken to be fixed as $N \to \infty$, as is standard (see, e.g., [13], [26], [31], [34]). We consider user 1 without loss of generality. Roughly speaking, our main results for synchronous systems can be summarized as follows.

Unconditional Distribution: Assuming that the empirical distribution function[1] of $\{\mu_1, \ldots, \mu_K\}$ converges weakly to a distribution function F_μ as $N \to \infty$, we obtain that the output MAI of the MMSE receiver is asymptotically Gaussian.

Conditional Distribution: Assuming that for almost every realization of the received powers, the joint empirical distribution function of $\{(P_1, \mu_1), \ldots, (P_K, \mu_K)\}$ converges weakly to a distribution function $F_{P,\mu}$, we obtain that for almost every realization of the signatures and received powers, the conditional distribution of the output MAI converges weakly to the same Gaussian distribution as in the unconditional case.

Furthermore, it turns out that the variance of the limiting distribution of the output MAI, which we shall characterize exactly, is affected only by the imperfect power control of the desired user and the mean powers of the interferers, and that the power variations of the interferers do not come into play at all. A parallel result is that the signal-to-interference ratio (SIR) given P_1 converges with probability one to β_1, where β_1 is deterministic and is the unique positive solution to the following fixed-point equation:

$$\beta_1 = \frac{P_1}{\eta + \alpha \int_0^\infty I(\mu, P_1, \beta_1)\, dF_\mu(\mu)}$$

where $I(\mu, P_1, \beta_1) \triangleq \frac{P_1 \mu}{P_1 + \mu \beta_1}$, and η is the power spectral density of the background Gaussian noise. Clearly, the asymptotic SIR is affected only by the imperfect power control of the desired user and the distribution of the mean powers of the interferers, and the impact (on the SIR) by the power fluctuation of the interferers vanishes in a large system. We note that this result is a generalization of [26, Theorem 3.1], which proves convergence in probability of the SIR in the perfect power control case.

We then extend our study to asynchronous systems to establish the Gaussian nature of the output interference and characterize the SIR. Our results are useful for performance analysis such as the calculation of the BEP, and also useful for the characterization of fundamental system limits such as channel capacity. In particular, the Gaussianity justifies the use of single-user Gaussian codes for CDMA systems with linear MMSE receivers, and implies that from the viewpoints of detection and channel capacity, SIR is the key parameter that governs the performance of the MMSE receiver in a CDMA system.

The organization of the rest of this paper is as follows. In the next section, we introduce a discrete-time model for the uplink of a single-cell CDMA system and the structure of the MMSE receiver in this context. In Section III, we summarize the main results found in this paper on the distributions of the output MAI of the MMSE receiver. Sections IV and V contain the proofs of the main results. We present numerical examples and draw our conclusions in Section VI.

II. System Model

Fig. 1 depicts a simplified block diagram of the uplink of a CDMA system equipped with the MMSE receiver. We focus primarily on the following discrete-time baseband model for synchronous systems.

A. Symbol-Synchronous Model

We have the following discrete-time model for the uplink of a synchronous CDMA system. The baseband received signal before filtering in a symbol interval is

$$Y^{(N)} = \sum_{i=1}^{K} \sqrt{P_i} b_i s_i + V \tag{1}$$

where the b_i's are the transmitted information symbols, the P_i's are the received powers, the s_i's are the signatures, and V is $\mathcal{N}(0, \eta I)$ background noise that comes from the sampling of the ambient white Gaussian noise with power spectral density η. (We assume throughout that $\eta > 0$.) We assume that the P_i's and b_i's are independent.

We assume that users choose their signatures randomly and independently. In a DS-CDMA system, the signatures are binary-valued. The model for binary random signatures is as follows: $s_i = \frac{1}{\sqrt{N}} (s_{i1}, \ldots, s_{iN})^T$, where the s_{in}'s are independent and identically distributed (i.i.d.) with

$$P\{s_{\text{in}} = 1\} = P\{s_{\text{in}} = -1\} = \frac{1}{2}, \qquad n = 1, \ldots, N, \text{ and } i = 1, \ldots, K.$$

This model is applicable to several scenarios (see, e.g., [8], [16], [26]–[28], [31]). First, consider systems where users employ

[1]See [5, p. 279] and [2, p. 268] for the definitions of empirical distribution functions.

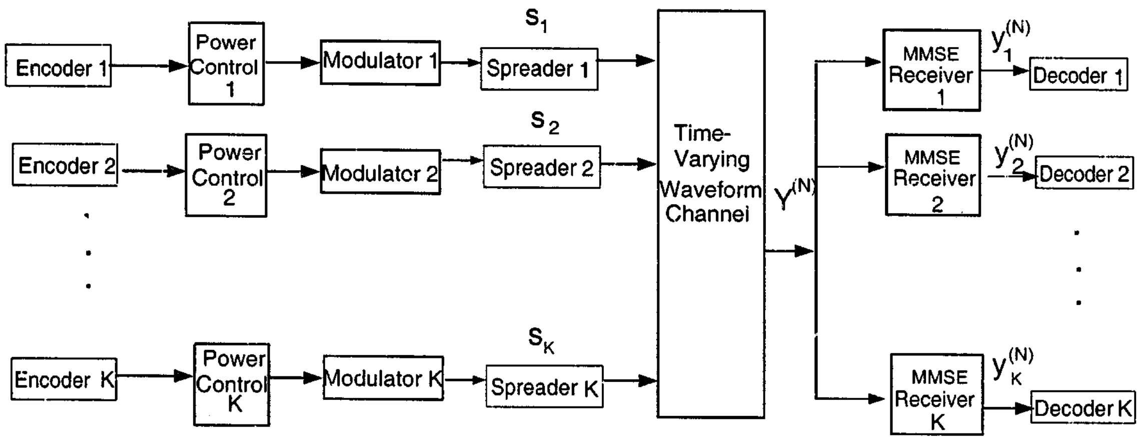

Fig. 1. A simplified block diagram of the uplink of a CDMA system with MMSE receivers as a front end.

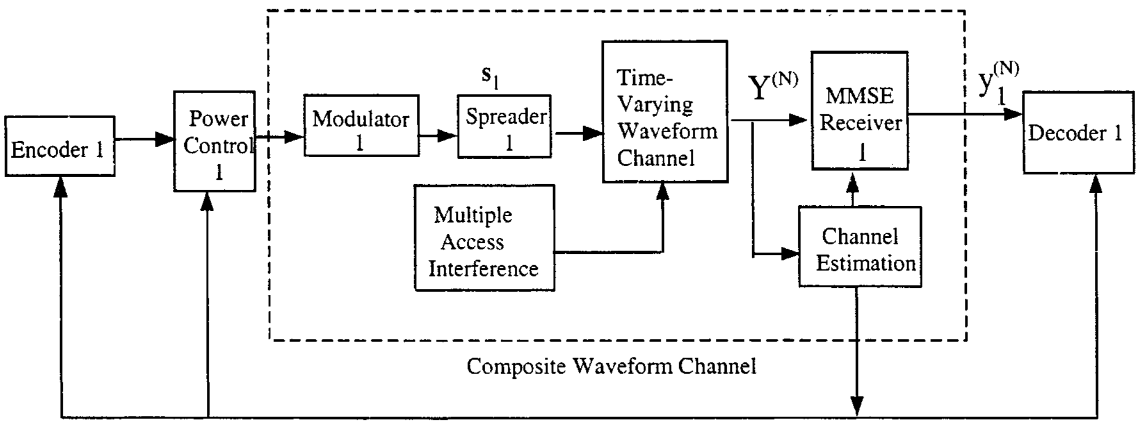

Fig. 2. The block diagram of a composite waveform channel for user 1.

long pseudorandom spreading sequences with periods considerably larger than the number of chips per symbol interval (such as in IS-95 systems), which is the length of the signatures. In this case, it is reasonable to adopt the model that the signatures are randomly and independently chosen and each user's signatures in different symbol intervals are also independent. A second scenario is the case where the signature of each user is repeated from symbol to symbol, but it is randomly and independently selected initially when the user is admitted into the system. Following the line of reasoning in [26], we assume that the signatures are known to the receivers once they are picked.

We consider user 1 without loss of generality. A discrete-time *composite waveform channel* (see, e.g., [19]) seen by user 1 is shown in Fig. 2, where all the interferers' signals contribute to the MAI. By definition, every linear receiver generates an output of the form $y_1^{(N)} \equiv c^t Y^{(N)}$ (see, e.g., [15], [18], [26]). Note that the outputs of the receiver depend on the processing gain N. To emphasize this dependence, we use $I_1^{(N)}$ to denote the output MAI, and $I_2^{(N)}$ the effect of background noise at the receiver output, that is,

$$I_1^{(N)} \triangleq \sum_{i=2}^{K} \sqrt{P_i} b_i c^t s_i$$
$$I_2^{(N)} \triangleq c^t V.$$

The MMSE receiver exploits the structure of the MAI provided by the signatures and received powers of the interferers. We assume that the MMSE receiver has knowledge of P_1 (the instantaneous received power of user 1), but has no knowledge of the instantaneous received powers of the interferers. We also assume that the MMSE receiver has knowledge of the μ_i's. Define

$$S_1 \triangleq [s_2, \ldots, s_K] \qquad S \triangleq [s_1, s_2, \ldots, s_K]$$
$$D_1 \triangleq \operatorname{diag}(P_2, \ldots, P_K) \qquad E_1 \triangleq \operatorname{diag}(\mu_2, \ldots, \mu_K)$$
$$M_I \triangleq S_1 E_1 S_1^H + \eta I \qquad M_I' \triangleq S_1 D_1 S_1^H + \eta I.$$

The MMSE receiver generates an output of the form of $c^t Y^{(N)}$, where c is chosen to minimize the mean-square error

$$J = \mathbb{E}\left[(c^t Y^{(N)} - b_1)^2 \middle| P_1, S \right].$$

The output at the MMSE receiver can easily be shown to be as follows (cf. [15], [26]):

$$\begin{aligned} y_1^{(N)} &= \frac{\sqrt{P_1}}{1 + P_1 s_1^t M_I^{-1} s_1} s_1^t M_I^{-1} Y^{(N)} \\ &= \frac{P_1 s_1^t M_I^{-1} s_1}{1 + P_1 s_1^t M_I^{-1} s_1} b_1 + I_1^{(N)} + I_2^{(N)} \end{aligned} \tag{2}$$

where

$$I_1^{(N)} = \sum_{i=2}^{K} \frac{\sqrt{P_1}}{1+P_1 s_1^t M_I^{-1} s_1} \, s_1^t M_I^{-1} \sqrt{P_i} b_i s_i$$
$$I_2^{(N)} = \frac{\sqrt{P_1}}{1+P_1 s_1^t M_I^{-1} s_1} \, s_1^t M_I^{-1} V.$$

The above construction of the MMSE receiver requires knowledge of the signature and instantaneous received power of user 1, and the signatures and mean received powers of the interferers. We note that the construction is applicable to systems of any size. If the processing gain N is large, the MMSE receiver would be difficult to implement if the signatures change from symbol to symbol. Therefore, from a practical viewpoint, repetition of the signatures may be more suitable for the implementation of MMSE receivers (see [8]). Since the received powers may vary from symbol to symbol, it is sensible to assume that the MMSE receiver has knowledge of the interferers' mean received powers instead of instantaneous received powers. Moreover, it turns out that in a large system, the knowledge of P_1 is in fact not crucial for the construction of the MMSE receiver. We will elaborate further on this in Section III. Worth noting is that the MMSE receiver does require knowledge of the timing of user 1 [9].

Since the MMSE receiver has no knowledge of the instantaneous powers of the interferers, the power of the overall interference is a function of D_1 and hence random, and can be shown to be as follows:

$$\mathbb{E}\left[(I_1^{(N)} + I_2^{(N)})^2 \middle| P_1, S, D_1 \right] = \frac{P_1}{(1+P_1 s_1^t M_I^{-1} s_1)^2} \, s_1^t M_I^{-1} M_I' M_I^{-1} s_1.$$

As is standard (see, e.g., [15]), the SIR is defined to be the ratio of the desired signal power to the sum of the power due to noise and MAI at the receiver output in a symbol interval. It then follows that the SIR of user 1 is

$$\text{SIR}_1^{(N)} = \frac{P_1 (s_1^t M_I^{-1} s_1)^2}{s_1^t M_I^{-1} M_I' M_I^{-1} s_1}. \tag{3}$$

Note that the SIR is random as well.

B. Symbol-Asynchronous Model

In the above symbol-synchronous model (1), symbol epochs are aligned at the receiver. This requires closed-loop timing control or providing the transmitters with access to a common clock. In CDMA systems, the design of the uplink is considerably simplified if the users need not be synchronized [30, Ch. 2]. In this section, we describe a symbol-asynchronous model. To facilitate the analysis, we assume that the system is chip-synchronous. More specifically, the offset (also called delay) of the interferer i relative to user 1, denoted as τ_i, is a nonnegative integer in terms of the number of chips, $i = 2, \ldots, K$. Following the line of reasoning in [12], we assume that the offsets are known to the receiver although they are *random*.

As pointed out in [12], [15], [30], a typical interferer has two different but consecutive symbols interfering with the symbol of user 1, and the interferer can be modeled as two *effective interferers*. Based on [12], we have the following discrete-time symbol-asynchronous baseband model:

$$Y^{(N)} = \sqrt{P_1} b_1 s_1 + \sum_{i=2}^{K} \sqrt{P_{i,1}} b_{i,1} s_{i,e1} + \sum_{i=2}^{K} \sqrt{P_{i,2}} b_{i,2} s_{i,e2} + V$$

where $b_{i,1}$ and $b_{i,2}$ are two consecutive symbols of user i interfering with the symbol of user 1, and $P_{i,1}$ and $P_{i,2}$ are the corresponding powers in the two symbol intervals. We assume that $b_{i,1}$ and $b_{i,2}$ are independent, and $P_{i,1} = P_{i,2}$. The *effective signatures* $s_{i,e1}$ and $s_{i,e2}$ are

$$(s_{i,e1})_n = \begin{cases} (s_i)_{(N-\tau_i+n)}, & 0 \le n \le \tau_i \\ 0, & \tau_i < n < N \end{cases}$$
$$(s_{i,e2})_n = \begin{cases} 0, & 0 \le n \le \tau_i \\ (s_i)_{(n-\tau_i)}, & \tau_i < n < N. \end{cases}$$

For asynchronous systems, we consider only the single-symbol asynchronous MMSE receiver, that is, we assume that the observation window of the MMSE receiver spans only the duration of the symbol of interest. Define

$$S_1' \triangleq [s_{2,e1}, \ldots, s_{K,e1}, s_{2,e2}, \ldots, s_{K,e2}]$$
$$D_1' \triangleq \text{diag}(P_2, \ldots, P_K, P_2, \ldots, P_K)$$
$$E_1' \triangleq \text{diag}(\mu_2, \ldots, \mu_K, \mu_2, \ldots, \mu_K)$$
$$M_{\text{II}} \triangleq S_1' E_1' (S_1')^H + \eta I$$
$$M_{\text{II}}' \triangleq S_1' D_1' (S_1')^H + \eta I.$$

Then the output MAI of the MMSE receiver in this setting is

$$I_1^{(N)} = \sum_{i=2}^{K} \frac{\sqrt{P_1}}{1+P_1 s_1^t M_{\text{II}}^{-1} s_1} \, s_1^t M_{\text{II}}^{-1} \cdot \left(\sqrt{P_{i,1}} b_{i,1} s_{i,e1} + \sqrt{P_{i,2}} b_{i,2} s_{i,e2} \right)$$

and the SIR at the output of the MMSE receiver is

$$\text{SIR}_{1,\text{II}}^{(N)} = \frac{P_1 (s_1^t M_{\text{II}}^{-1} s_1)^2}{s_1^t M_{\text{II}}^{-1} M_{\text{II}}' M_{\text{II}}^{-1} s_1}.$$

III. Main Results

In this section, we summarize the main conclusions found in this paper on the output MAI distributions. Because the proofs of our results are rather technical, we defer the details of the proofs to Sections IV and V.

A. The Symbol-Synchronous Case

First, we impose the following assumptions on the received powers.

- **(3.A1)** The empirical distribution function of $\{\mu_1, \ldots, \mu_K\}$ converges weakly to a distribution function F_μ as $N \to \infty$.
- **(3.A2)** The second moments of the received powers are bounded.

We note that the assumption (3.A1) is applicable to many practical scenarios. For example, in a practical system, typically users can be classified into a few classes according to their QoS requirements. One common approach for power control in practice is to drive the received powers for all the users having the same QoS requirements to be a fixed predetermined value, namely, *power balancing* (see, e.g., [32]). Correspondingly, the mean received powers are about the same for all the users having the same QoS requirements. Then it can be shown that the condition (3.A1) is satisfied.

We have the following result on the MAI unconditional distribution.

Theorem 3.1 (Symbol-Synchronous: Unconditional MAI): Suppose Conditions (3.A1) and (3.A2) hold. Then the output MAI of the MMSE receiver, $I_1^{(N)}$, has a limiting Gaussian distribution (as $N \to \infty$).

Theorem 3.1 establishes that the unconditional distribution of $I_1^{(N)}$ converges weakly to Gaussian. To show the almost sure convergence of the MAI conditional distribution, we need a stronger form of regularity on the received powers. The assumptions we impose on the received powers are as follows.

- **(3.C1)** The joint empirical distribution function of $\{(P_1, \mu_1), \ldots, (P_K, \mu_K)\}$ converges weakly to a deterministic distribution function $F_{P,\mu}$ with probability one.
- **(3.C2)** The P_i's are uniformly bounded above, and the μ_i's are bounded below by a positive number.

The assumption (3.C1) is general enough to cover many practical systems of interest. For example, Condition (3.C1) holds when there is a finite number of classes in the system, where users within each class have i.i.d. received powers. (This is a reasonable model because fading channel gains are typically assumed to be stationary and ergodic, and all the users in one class can be assumed to have i.i.d. channel gains [6], [7].)

For convenience, let $X^{(N)}$ denote the collection of the signatures and received powers when the processing gain is N, and $\boldsymbol{X}$ the sequence $\{X^{(N)}\}$. Let $P_N^{\boldsymbol{X}}$ denote the conditional distribution of $I_1^{(N)}$ given $\boldsymbol{X}$. Following [25], we say that $P_N^{\boldsymbol{X}}$ converges almost surely to a probability measure P_ν if for any bounded continuous function f, $\int f\, dP_N^{\boldsymbol{X}} \to \int f\, dP_\nu$ with probability one. We note that convergence almost surely of conditional distribution is an instance of convergence of a sequence of random measures. We have the following theorem on the MAI conditional distribution.

Theorem 3.2 (Symbol-Synchronous: Conditional MAI): Suppose Conditions (3.C1) and (3.C2) hold. Then the conditional distribution of the output MAI of the MMSE receiver, $I_1^{(N)}$, given the signatures and the receiver powers, converges almost surely (as $N \to \infty$) to the same Gaussian distribution as in the unconditional case.

Theorem 3.2 is somewhat surprising because we would expect that the asymptotic properties of the output MAI depend on the choice of signatures and hence are random. Indeed, there are some cases for which the output MAI of the MMSE receiver does not have an asymptotic Gaussian distribution (see [18]). But in fact under our assumptions, the limiting distribution of the output MAI conditioned on the signatures and received powers is the same as the unconditional one with probability one. The intuition behind the above result is as follows: In a large system, after the "whitening" of the linear MMSE receiver, averaging across the users acts as "good" as ensemble averaging over the signatures and received powers.

We have the following heuristic interpretation of Theorems 3.1 and 3.2. Theorem 3.1 reveals that in a fading environment, as long as the mean powers of the users satisfy Conditions (3.A1) and (3.A2), the output MAI of the MMSE receiver is approximately Gaussian in a large system. Theorem 3.2 establishes that for almost every realization of the signatures and received powers, the conditional distribution of the output MAI of the MMSE receiver is approximately Gaussian (the same Gaussian distribution as in the unconditional case) when the received powers satisfy Conditions (3.C1) and (3.C2). Therefore, from the viewpoints of detection and channel capacity, systems with the MMSE receiver achieve the same performance with probability one and are robust to the randomness of the signatures and received powers.

Theorem 3.2 is particularly useful in the scenario where repetition of the same random signatures is adopted. In this case, the output MAIs are correlated across symbol intervals because they depend on the same random signatures. This lack of independence is usually difficult to deal with. For example, it complicates decoding and hinders us from simply analyzing the performance and characterizing system limits such as channel capacity. However, conditioned on the signatures and received powers, the output MAIs are independent across symbol intervals (under certain assumptions on the information symbols), and the asymptotic Gaussianity greatly simplifies the performance analysis and the characterization of system limits.

In the perfect power control case, Theorems 3.1 and 3.2 amount to the following result. (Note that $P_i = \mu_i$ in the perfect power control case.)

Corollary 3.1: Suppose Conditions (3.A1) and (3.C2) hold. Then as $N \to \infty$, the output MAI of the MMSE receiver has a limiting Gaussian distribution; moreover, the conditional distribution of the output MAI, given the signatures, converges almost surely to the same Gaussian distribution as in the unconditional case.

Theorems 3.1 and 3.2 allow us to incorporate easily the effect of the background noise and conclude that the overall interference is asymptotically Gaussian. Let $\Pi_e^{(N)}$ denote the BEP of user 1 when the processing gain is N. Then we have the following result on the asymptotic SIR and BEP.

Theorem 3.3 (Symbol-Synchronous: SIR and BEP): Suppose Conditions (3.C1) and (3.C2) hold. Then $\mathrm{SIR}_1^{(N)}$ (given P_1) converges almost surely to the unique positive solution β_1 of the following fixed-point equation:

$$\beta_1 = \frac{P_1}{\eta + \alpha \int_0^\infty I(\mu, P_1, \beta_1)\, dF_\mu(\mu)} \tag{4}$$

where

$$I(\mu, P_1, \beta_1) \triangleq \frac{P_1 \mu}{P_1 + \mu \beta_1}. \tag{5}$$

Moreover, if the modulation is antipodal, then $\Pi_e^{(N)}$ converges to $E[Q(\sqrt{\beta_1})]$, where the expectation is taken over β_1. Therefore, in a large system, we can approximate the BEP by $E[Q(\sqrt{\mathrm{SIR}})]$. This implies that the SIR is the key parameter that governs the performance of systems with the MMSE receiver in a fading environment. Moreover, we note that from the viewpoints of detection and channel capacity, the SIR is of fundamental interest. Since any (positive) scaled version of the MMSE receiver results in the same SIR, it suffices to use any filter of the form $c^t = ds_1^t M_I^{-1}$, $d \in \mathbb{R}$ [cf. (2)]. Therefore, there is no need for knowledge of the desired user's instantaneous received power for the construction of the MMSE receiver (strictly speaking, a (positive) scaled version of the MMSE receiver).

Heuristically, the SIR β_1 (given P_1) approximately satisfies the following fixed-point equation:

$$\beta_1 \approx \frac{P_1}{\eta + \frac{1}{N}\sum_{i=2}^{K} I(\mu_i, P_1, \beta_1)}.$$

That is, the total interference can be decoupled into the sum of the background noise and an interference term from each of the interferers. The quantity $I(\mu, P_1, \beta_1)$ is called the *effective interference function* [26]. Note that the effective interference function depends on the mean powers of the interferers (not the instantaneous powers), and the instantaneous power of the desired user, hence is random. This further indicates that the performance of the MMSE receiver is robust to the power variations of the interferers.

B. The Symbol-Asynchronous Case

As noted before, to facilitate the analysis, we assume that the system is chip-synchronous, that is, the offsets, denoted as τ_i, are nonnegative integers in terms of the number of chips. We impose the following assumption on the offsets $\{\tau_1, \ldots, \tau_K\}$.

(3.A3) The empirical distribution function of $\{\tau_1, \ldots, \tau_K\}$ converges weakly to a deterministic distribution function H_τ with probability one.

The above assumption is also very general to cover many practical systems of interest. For example, Condition (3.A3) holds when the offsets have (identical) uniform distributions, which is a typical model for the offsets in an asynchronous system (see, e.g., [12], [13], [15]).

Our main results for the symbol-asynchronous case make use of the notion of convergence in probability of conditional distribution. As in [25], we say that $P_N^{\boldsymbol{X}}$ converges in probability to P_ν if every subsequence $\{N'\}$ contains a further subsequence $\{N''\}$ for which $P_{N''}^{\boldsymbol{X}}$ converges almost surely to P_ν. We are now ready to present the results on the output MAI distributions and asymptotic SIR.

Theorem 3.4 (Symbol-Asynchronous: Unconditional and Conditional MAI): Suppose Conditions (3.A1), (3.C2), and (3.A3) hold. We have that as $N \to \infty$

a) the output MAI of the MMSE receiver has a limiting Gaussian distribution;

b) the conditional distribution of the output MAI of the MMSE receiver, given the signatures, received powers, and offsets, converges in probability to the same Gaussian distribution as in the unconditional case.

Theorem 3.5 (Symbol-Asynchronous: SIR): Suppose Conditions (3.A1), (3.C2), and (3.A3) hold. Then as $N \to \infty$, $\mathrm{SIR}_{1,\,\mathrm{II}}^{(N)}$ (given P_1) converges in probability to $\int_0^1 w(x)\,dx$, where $w(x)$ is the unique solution (in the class of functions $w(x) \geq 0$) to the following functional equation:

$$w(x) = \frac{P_1}{\eta + \alpha\int_0^\infty\int_0^\infty I_{\mathrm{eff}}(\mu, P_1, \tau, x, w(t))\,dF_\mu(\mu)\,dH_\tau(\tau)} \tag{6}$$

where

$$I_{\mathrm{eff}}(\mu, P_1, \tau, x, w(\cdot)) \triangleq I\left(\mu, P_1, \int_0^\tau w(t)\,dt\right) 1_{\{\tau \geq x\}} + I\left(\mu, P_1, \int_\tau^1 w(t)\,dt\right) 1_{\{\tau \leq x\}}.$$

[The quantity $I(\mu, P_1, \cdot)$ is defined in (5).]

We have the following heuristic interpretation of Theorem 3.4. Given the signatures, received powers, and offsets, the output MAI is approximately Gaussian with high probability in a large system. This further reveals that systems with the MMSE receiver are robust to the randomness of the offsets as well as the randomness of signatures and received powers. We note that Theorem 3.5 is a generalization of [12, Theorem 4.1].

It should be noted that Theorem 3.4 is "weaker" than Theorem 3.2 in the sense that the convergence mode of the conditional distribution is weaker. (The convergence almost surely of the conditional distribution appears considerably more difficult to obtain in the asynchronous case.) The proof of Theorem 3.4 involves a combination of the techniques used in proving Theorem 3.1 and those used in proving [12, Theorem 4.1]. Although more complicated, the proof of Theorem 3.4 essentially follows the same line as that of Theorem 3.1. We omit the details here. (See [33, Ch. 3] for more details.) The intuition behind Theorem 3.4 is as follows: In an asynchronous system, an asynchronous interferer can be regarded as two effective interferers with smaller powers [15]; and, as before, Gaussianity essentially comes from sums of many "small" terms.

In what follows, we prove Theorem 3.1 in Section IV. Section V contains the proofs of Theorems 3.2 and 3.3. For simplicity, we assume in the proofs that the modulation is antipodal, that is, $b_i \in \{-1, 1\}$. This assumption is not crucial, but simplifies the analysis. We will elaborate further on this assumption in Section V.

IV. Proof of Theorem 3.1

The proofs of our results make use of a theorem in random matrix theory. For convenience, we restate that theorem here. Denote the eigenvalues of the random matrix $S_1 E_1 S_1^t$ by $\lambda_1, \ldots, \lambda_N$ (they are random, depending on the realization of S_1), and the empirical distribution of the eigenvalues by G_N. The result of [24, Theorem 1.1] states that if the Condition (3.A1) holds, then G_N converges weakly (as $N \to \infty$) to a (nonrandom) distribution function G^* with probability one, and the Stieltjes transform $m(z)$ of G^* is the solution to the following functional equation:

$$m(z) = \frac{1}{-z + \alpha \int \frac{\mu}{1+\mu m(z)}\, dF_\mu(\mu)} \tag{7}$$

for all

$$z \in \mathbb{C}^+ \equiv \{z \in \mathbb{C},\, \mathrm{Im}\{z\} > 0\}.$$

It is further pointed out in [24] that the limiting distribution function G^* is unique.

In this section, we prove Theorem 3.1 via the dependent central limit theorem in [17], which is a central limit theorem for martingale difference arrays (cf. [13]). We begin with some technical lemmas.

A. Technical Lemmas

Define

$$a_g \triangleq \frac{\sqrt{P_1}}{1 + P_1 \int_0^\infty \frac{1}{\lambda+\eta}\, dG^*(\lambda)}$$

$$t_i^{(N)} \triangleq \sqrt{P_i} b_i s_1^t M_I^{-1} s_i, \qquad i = 2, \ldots, K.$$

We have the following lemmas, the proofs of which have been relegated to Appendixes A and B.

Lemma 4.1:

$$\frac{\sqrt{P_1}}{1 + P_1 s_1^t M_I^{-1} s_1} \xrightarrow{\text{a.s.}} a_g. \tag{8}$$

Lemma 4.2:

$$\lim_{N\to\infty} \mathbb{E}\left[\sum_{i=2}^K \left(t_i^{(N)}\right)^2\right] = \int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\, dG^*(\lambda). \tag{9}$$

Lemma 4.3: We have for $i = 2, \ldots, K$

$$\mathbb{E}\left[\left(t_i^{(N)}\right)^2\right] \le \frac{\mu_i}{N\eta^2} \tag{10}$$

$$\mathbb{E}\left[\left(t_i^{(N)}\right)^4\right] \le \frac{9E[P_i^2]}{N^2\eta^4} \tag{11}$$

$$\mathbb{E}\left[\left(t_i^{(N)}\right)^6\right] \le \frac{225E[P_i^3]}{N^3\eta^6}. \tag{12}$$

B. Proof of Theorem 3.1

Let $I_{1n}^{(N)} = \sum_{i=2}^K t_i^{(N)}$. First we show that $I_{1n}^{(N)}$ has a limiting Gaussian distribution. Observing that the $t_i^{(N)}$'s are dependent because every $t_i^{(N)}$ is a function of the random signatures S, we resort to the dependent central limit theorem in [17]. Define $\mathcal{F}_{N,i}$ as the σ-algebra generated by $\{t_2^{(N)}, \ldots, t_i^{(N)}\}$, that is

$$\mathcal{F}_{N,i} \triangleq \sigma\left(t_2^{(N)}, \ldots, t_i^{(N)}\right), \qquad i = 2, \ldots, K.$$

It is clear that

$$\mathbb{E}\left[t_i^{(N)} \,\middle|\, \mathcal{F}_{N,i-1}\right] = \mathbb{E}[b_i]\mathbb{E}\left[s_1^t M_I^{-1}\sqrt{P_i} s_i \,\middle|\, \mathcal{F}_{N,i-1}\right] = 0. \tag{13}$$

Therefore, the array $\{t_i^{(N)}\}$ is a martingale difference array with respect to $\{\mathcal{F}_{N,i}\}$. Thus, based on [17], it suffices to verify that the following three conditions are satisfied:

1) $\max_{2\le i\le K} |t_i^{(N)}|$ is bounded in L_2 norm;
2) $\max_{2\le i\le K} |t_i^{(N)}|$ converges to 0 in probability as $N \to \infty$;
3) $\sum_{i=2}^K (t_i^{(N)})^2$ converges to $\int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\, dG^*(\lambda)$ in probability as $N \to \infty$.

It is worth noting that these conditions are weaker than the standard Lindeberg conditions (see [17] for more detailed comparison).

By using (39) in Appendix A, we get that

$$\mathbb{E}\left[\max_{2\le i\le K}\left(t_i^{(N)}\right)^2\right] \le \mathbb{E}\left[\sum_{i=2}^K \left(t_i^{(N)}\right)^2\right] \le \frac{1}{\eta} \tag{14}$$

that is, Condition 1) is satisfied. Fix $\epsilon > 0$. By exploiting Lemma 4.2, we have that

$$\begin{aligned} P\left\{\max_{2\le i\le K}\left|t_i^{(N)}\right| > \epsilon\right\} &\le \sum_{i=2}^K P\left\{\left|t_i^{(N)}\right| > \epsilon\right\} \\ &\le \sum_{i=2}^K \frac{\mathbb{E}\left[\left(t_i^{(N)}\right)^4\right]}{\epsilon^4} \\ &\le \frac{9K}{N^2(\epsilon\eta)^4} \max_{2\le i\le K} \mathbb{E}[P_i^2] \\ &\to 0. \end{aligned} \tag{15}$$

Next, we verify that Condition 3) is satisfied. To this end, by Lemma 4.2, it suffices to show that

$$\mathrm{var}\left(\sum_{i=2}^K \left(t_i^{(N)}\right)^2\right) \longrightarrow 0.$$

Note that

$$\begin{aligned} \mathrm{var}\left(\sum_{i=2}^K \left(t_i^{(N)}\right)^2\right) &= \mathbb{E}\left[\mathrm{var}\left(\sum_{i=2}^K \left(t_i^{(N)}\right)^2 \,\middle|\, S\right)\right] \\ &\quad + \mathrm{var}\left(\mathbb{E}\left[\sum_{i=2}^K \left(t_i^{(N)}\right)^2 \,\middle|\, S\right]\right) \end{aligned} \tag{16}$$

where we used the following well-known conditional variance formula [20, p. 51]:

$$\mathrm{var}(X) = \mathbb{E}[\mathrm{var}(X|Y)] + \mathrm{var}(\mathbb{E}[X|Y]).$$

For the first term on the right side of (16), we have that

$$\begin{aligned} \mathbb{E}\left[\mathrm{var}\left(\sum_{i=2}^K \left(t_i^{(N)}\right)^2 \,\middle|\, S\right)\right] &\overset{(a)}{=} \mathbb{E}\left[\sum_{i=2}^K \mathrm{var}\left(\left(t_i^{(N)}\right)^2 \,\middle|\, S\right)\right] \\ &\overset{(b)}{\le} \mathbb{E}\left[\sum_{i=2}^K \mathbb{E}\left[\left(t_i^{(N)}\right)^4 \,\middle|\, S\right]\right] \\ &= \sum_{i=2}^K \mathbb{E}\left[\left(t_i^{(N)}\right)^4\right] \\ &\overset{(c)}{\le} \sum_{i=2}^K \mathbb{E}[P_i^2]\frac{9}{N^2\eta^4} \\ &\xrightarrow{(d)} 0 \end{aligned} \tag{17}$$

where (a) follows from the fact that the $t_i^{(N)}$'s are independent conditioned on S, (b) from the fact that $\mathrm{var}(X) \le \mathbb{E}[X^2]$ for any random variable X, (c) from Lemma 4.3, and (d) from the assumption that the second moments of the received powers are bounded.

We proceed to show that the second term on the right side of (16) goes to zero as $N \to \infty$. Combining Lemma 4.2 with the fact that x^2 is continuous in x, we obtain that

$$\lim_{N\to\infty} \left(\mathbb{E}\left[\mathbb{E}\left[\sum_{i=2}^{K} \left(t_i^{(N)}\right)^2 \middle| S \right]\right]\right)^2 = \left(\int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\, dG^*(\lambda) \right)^2. \quad (18)$$

Furthermore, based on (38) in Appendix A, we have that

$$\begin{aligned}\mathbb{E}\left[\sum_{i=2}^{K} \left(t_i^{(N)}\right)^2 \middle| S \right] &= \sum_{i=2}^{K} \mathrm{trace}(\mu_i s_i s_i^t M_I^{-1} s_1 s_1^t M_I^{-1}) \\ &= \mathrm{trace}((M_I - \eta I) M_I^{-1} s_1 s_1^t M_I^{-1}) \\ &= s_1^t (M_I^{-1} - \eta M_I^{-2}) s_1\end{aligned}$$

where we denote $(M_I^{-1})^2$ by M_I^{-2} (we use this notation throughout). For any realization of S_1, the corresponding M_I is symmetric and positive definite, and hence can be written in the form of $U^t \Lambda U$, where $\Lambda \triangleq \mathrm{diag}(\lambda_1+\eta, \ldots, \lambda_N+\eta)$. It follows that

$$\begin{aligned}\mathbb{E}&\left[\left(\mathbb{E}\left[\sum_{i=2}^{K} \left(t_i^{(N)}\right)^2 \middle| S \right] \right)^2 \right] \\ &= \mathbb{E}[(s_1^t(M_I^{-1} - \eta M_I^{-2}) s_1)^2] \\ &= \mathbb{E}[\mathbb{E}[((Us_1)^t (\Lambda^{-1} - \eta \Lambda^{-2}) U s_1)^2 | S_1]]. \quad (19)\end{aligned}$$

Observe the equation at the bottom of this page. Clearly, $\mathbb{E}[s_{1k_1} s_{1k_2} s_{1k_3} s_{1k_4}]$ is nonzero (equal to 1) only in the following three cases: 1) $k_1 = k_2$ and $k_3 = k_4$; 2) $k_1 = k_3$ and $k_2 = k_4$; and 3) $k_1 = k_4$ and $k_2 = k_3$. We let Z_1 denote the sum of all the terms for which $k_1 = k_2$ and $k_3 = k_4$, Z_2 the sum of all the terms for which $k_1 = k_3$ and $k_2 = k_4$ excluding $k_1 = k_2 = k_3 = k_4$, and Z_3 the sum of all the terms for which $k_1 = k_4$ and $k_3 = k_2$ excluding $k_1 = k_2 = k_3 = k_4$. Then it follows that

$$\begin{aligned}Z_1 &= \left(\frac{1}{N} \sum_{j=1}^{N} \frac{\sum_{k_1=1}^{N} U_{jk_1}^2 \lambda_j}{(\lambda_j+\eta)^2} \right)^2 \\ &= \left(\int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\, dG_N(\lambda) \right)^2\end{aligned}$$

and

$$\begin{aligned}Z_2 &\le \frac{1}{N^2} \sum_{j_1} \sum_{j_2} \frac{\sum_{k_1}\sum_{k_2} U_{j_1k_1} U_{j_1k_2} U_{j_2k_1} U_{j_2k_2} \lambda_{j_1} \lambda_{j_2}}{(\lambda_{j_1}+\eta)^2 (\lambda_{j_2}+\eta)^2} \\ &= \frac{1}{N^2} \sum_{j_1} \sum_{j_2} \frac{\left(\sum_k U_{j_1k} U_{j_2k}\right)^2 \lambda_{j_1} \lambda_{j_2}}{(\lambda_{j_1}+\eta)^2 (\lambda_{j_2}+\eta)^2} \\ &= \frac{1}{N} \int_0^\infty \left(\frac{\lambda}{(\lambda+\eta)^2} \right)^2 dG_N(\lambda) \\ &\le \frac{1}{N\eta^2}.\end{aligned}$$

Similarly, it can be shown that

$$Z_3 \le \frac{1}{N\eta^2}.$$

Observe that

$$\lim_{N\to\infty} \mathbb{E}[Z_1] = \lim_{N\to\infty} \mathbb{E}\left[\left(\int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\, dG_N(\lambda) \right)^2 \right] \quad (20)$$

$$\overset{(a)}{=} \mathbb{E}\left[\lim_{N\to\infty} \left(\int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\, dG_N(\lambda) \right)^2 \right]$$

$$= \left(\int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\, dG^*(\lambda) \right)^2 \quad (21)$$

where (a) follows from the Lebesgue Dominated Convergence Theorem [21, p. 91] because $\int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\, dG_N(\lambda)$ is positive and upper-bounded by $\frac{1}{\eta}$.

In summary, we have that

$$\begin{aligned}\lim_{N\to\infty} &\mathrm{var}\left(\mathbb{E}\left[\sum_{i=2}^{K} \left(t_i^{(N)}\right)^2 \middle| S \right] \right) \\ &= \lim_{N\to\infty} \mathbb{E}[Z_1 + Z_2 + Z_3] \\ &\quad - \lim_{N\to\infty} \left(\mathbb{E}\left[\mathbb{E}\left[\sum_{i=2}^{K} \left(t_i^{(N)}\right)^2 \middle| S \right]\right]\right)^2 \\ &= \lim_{N\to\infty} \mathbb{E}[Z_2 + Z_3] \\ &\le \lim_{N\to\infty} \frac{2}{N\eta^2} = 0. \quad (22)\end{aligned}$$

It then follows that

$$\sum_{i=2}^{K} \left(t_i^{(N)}\right)^2 \xrightarrow{P} \int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\, dG^*(\lambda). \quad (23)$$

$$\begin{aligned}\mathbb{E}[((Us_1)^t(\Lambda^{-1} - \eta\Lambda^{-2}) U s_1)^2 | S_1] &= \mathbb{E}\left[\left(\frac{1}{N} \left[\sum_{j=1}^{N} \frac{\sum_{k_1=1}^{N} \sum_{k_2=1}^{N} U_{jk_1} U_{jk_2} s_{1k_1} s_{1k_2} \lambda_j}{(\lambda_j+\eta)^2} \right] \right)^2 \middle| S_1 \right] \\ &= \frac{1}{N^2} \sum_{j_1=1}^{N} \sum_{j_2=1}^{N} \frac{\lambda_{j_1} \lambda_{j_2} \sum_{k_1=1}^{N} \sum_{k_2=1}^{N} \sum_{k_3=1}^{N} \sum_{k_4=1}^{N} U_{j_1k_1} U_{j_1k_2} U_{j_2k_3} U_{j_2k_4} \mathbb{E}[s_{1k_1} s_{1k_2} s_{1k_3} s_{1k_4}]}{(\lambda_{j_1}+\eta)(\lambda_{j_2}+\eta)}.\end{aligned}$$

Combining (14), (15), and (23), we conclude, by using the dependent central limit theorem in [17], that $I_{1n}^{(N)}$ has a limiting Gaussian distribution with mean 0 and variance

$$\int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\, dG^*(\lambda).$$

Furthermore, combining the above result with Lemma 4.1, we apply Slutsky's theorem [3, Theorem 5.3.5] and conclude that $I_1^{(N)}$ has a limiting Gaussian distribution with mean 0 and variance

$$a_g^2 \int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\, dG^*(\lambda)$$

thereby completing the proof.

V. Proofs of Theorems 3.2 and 3.3

In this section, we prove the almost sure convergence of the conditional distribution, and derive an expression for the SIR. Throughout this section, we assume that Conditions (3.C1) and (3.C2) hold. More specifically, for Condition (3.C2), we assume that the P_i's are uniformly bounded above by d_1 and the μ_i's are bounded below by $d_2 > 0$.

A. Technical Lemmas

First we define for $i = 2, \ldots, K$

$$\begin{aligned} S_i &\triangleq [s_2, \ldots, s_{i-1}, s_{i+1}, \ldots, s_K] \\ E_i &\triangleq \operatorname{diag}(\mu_2, \ldots, \mu_{i-1}, \mu_{i+1}, \ldots, \mu_K) \\ M_i &\triangleq S_i E_i S_i^t + \eta I. \end{aligned}$$

Lemma 5.1:

$$\max_{2\le i\le K} t_i^{(N)} \xrightarrow{\text{a.s.}} 0.$$

Lemma 5.2: Fix $\epsilon > 0$. Given N and any $i \in \{2, \ldots, K\}$,

$$P\left\{\left|\frac{s_i^t M_i^{-2} s_i}{(1+\mu_i s_i^t M_i^{-1} s_i)^2} - \frac{\frac{1}{N}\operatorname{trace}(M_i^{-2})}{\left(1+\frac{\mu_i}{N}\operatorname{trace}(M_i^{-1})\right)^2}\right| > \epsilon\right\} \le \frac{c_1(\epsilon)}{N^3}$$

where the constant $c_1(\epsilon)$ does not depend upon N.

The proofs of the above two lemmas are in Appendix B.

B. Proof of Theorem 3.2

We now complete the proof of Theorem 3.2.

Observe that conditioned on the signatures and received powers, the array $\{t_i^{(N)}\}$ still forms a martingale difference array with respect to $\{\mathcal{F}_{N,i}\}$. In what follows, we show that for almost every realization of the signatures and received powers, $I_{1n}^{(N)}$ has a limiting Gaussian distribution. To this end, first we show that $\max_{2\le i\le K} |t_i^{(N)}|$ is uniformly bounded in L_2 norm for any given signatures and received powers. Appealing to [10, p. 470], we have[2]

$$\begin{aligned} M_I^{-1}\left(\sum_{i=2}^K P_i s_i s_i^t\right) M_I^{-1} &\preceq M_I^{-1}\left(\sum_{i=2}^K d_1 s_i s_i^t\right) M_I^{-1} \\ &\preceq \frac{d_1}{d_2} M_I^{-1}\left(\sum_{i=2}^K \mu_i s_i s_i^t\right) M_I^{-1}. \end{aligned} \tag{24}$$

It follows that for given signatures and received powers

$$\begin{aligned} &\mathbb{E}\left[\left(\max_{2\le i\le K}\left|t_i^{(N)}\right|\right)^2 \middle| S, D_1\right] \\ &\le \mathbb{E}\left[s_1^t M_I^{-1}\left(\sum_{i=2}^K P_i s_i s_i^t\right) M_I^{-1} s_1 \middle| S, D_1\right] \\ &\le \frac{d_1}{d_2} s_1^t M_I^{-1} s_1, \qquad \text{since } \eta s_1^t M_I^{-2} s_1 \ge 0 \\ &\le \frac{d_1}{d_2 \eta} \end{aligned} \tag{25}$$

where the last step follows from the fact that $0 \prec M_I^{-1} \preceq \frac{1}{\eta} I$ because $M_I \succeq \eta I \succ 0$ (see [10, p. 471]).

Lemma 5.1 indicates that for almost every realization of the signatures and received powers, $\max_{2\le i\le K} t_i^{(N)}$ converges almost surely to 0. Therefore, based on the central limit theorem in [17], it suffices to show that for almost every realization of the signatures and received powers, $\sum_{i=2}^K (t_i^{(N)})^2$ converges almost surely to

$$\int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\, dG^*(\lambda).$$

To that end, note that

$$\sum_{i=2}^K \left(t_i^{(N)}\right)^2 = s_1^t \left[M_I^{-1}\left(\sum_{i=2}^K P_i s_i s_i^t\right) M_I^{-1}\right] s_1.$$

First we verify that the spectral radius of

$$M_I^{-1}\left(\sum_{i=2}^K P_i s_i s_i^t\right) M_I^{-1}$$

is bounded above. Because

$$M_I^{-1}\left(\sum_{i=2}^K \mu_i s_i s_i^t\right) M_I^{-1} = M_I^{-1} - \eta M_I^{-2}$$

it follow that the spectral radius of $M_I^{-1}(\sum_{i=2}^K \mu_i s_i s_i^t) M_I^{-1}$ is bounded above by $\frac{1}{\eta}$. Using (24), we conclude that the spectral radius of $M_I^{-1}(\sum_{i=2}^K P_i s_i s_i^t) M_I^{-1}$ is bounded above by $\frac{d_1}{d_2 \eta}$.

Fix a realization of $\{(P_1, \mu_1), \ldots, (P_K, \mu_K)\}$ for which the empirical distribution function converges weakly to $F_{P,\mu}$. By appealing to Lemma B.1 and using the same techniques as in the proof of Lemma 4.1, it can be shown that

$$s_1^t\left[M_I^{-1}\left(\sum_{i=2}^K P_i s_i s_i^t\right) M_I^{-1}\right] s_1 - \frac{1}{N}\operatorname{trace}\left(M_I^{-1}\left(\sum_{i=2}^K P_i s_i s_i^t\right) M_I^{-1}\right) \xrightarrow{\text{a.s.}} 0. \tag{26}$$

[2]By matrix inequality $A \succeq B$, we mean that $A - B$ is positive semidefinite.

Note that $M_I = M_i + \mu_i s_i s_i^t$. Using the matrix inverse lemma (see, e.g., [4, p. 175]), we have that

$$M_I^{-1} = M_i^{-1}\left[I - \frac{\mu_i s_i s_i^t M_i^{-1}}{1+\mu_i s_i^t M_i^{-1} s_i}\right] \tag{27}$$

which implies that

$$\begin{aligned}\frac{1}{N}\operatorname{trace}\left(M_I^{-1}\left(\sum_{i=2}^K P_i s_i s_i^t\right)M_I^{-1}\right) &= \frac{1}{N}\sum_{i=2}^K P_i s_i^t M_I^{-2} s_i\\ &= \frac{1}{N}\sum_{i=2}^K P_i \frac{s_i^t M_i^{-2} s_i}{(1+\mu_i s_i^t M_i^{-1} s_i)^2}.\end{aligned}$$

Using (27) again, we obtain that for $i = 2, \ldots, K$

$$\begin{aligned}\left|\frac{1}{N}\operatorname{trace}(M_i^{-1}) - \frac{1}{N}\operatorname{trace}(M_I^{-1})\right| &= \frac{1}{N}\left|\frac{\mu_i s_i^t M_i^{-2} s_i}{1+\mu_i s_i^t M_i^{-1} s_i}\right|\\ &\le \frac{1}{N}\frac{\mu_i}{\eta^2}\\ &\le \frac{c_5}{N}\end{aligned} \tag{28}$$

and

$$\begin{aligned}\left|\frac{1}{N}\operatorname{trace}(M_i^{-2}) - \frac{1}{N}\operatorname{trace}(M_I^{-2})\right| &\le \frac{1}{N}\left|\frac{2\mu_i s_i^t M_i^{-3} s_i}{1+\mu_i s_i^t M_i^{-1} s_i} - \frac{(\mu_i s_i^t M_i^{-2} s_i)^2}{1+\mu_i s_i^t M_i^{-1} s_i}\right|\\ &\le \frac{1}{N}\left(\frac{2\mu_i}{\eta^3} + \frac{\mu_i^2}{\eta^4}\right)\\ &\le \frac{c_6}{N}.\end{aligned} \tag{29}$$

We note that both c_5 and c_6 are constants independent of N. Their existence comes from the boundedness of the μ_i's.

Fix $\epsilon > 0$. Based on (28) and (29), it follows that there exists $N_0(\epsilon)$ such that for all $N \ge N_0(\epsilon)$

$$\begin{aligned}&P\left\{\left|\frac{s_i^t M_i^{-2} s_i}{(1+\mu_i s_i^t M_i^{-1} s_i)^2} - \frac{\frac{1}{N}\operatorname{trace}(M_I^{-2})}{\left(1+\frac{\mu_i}{N}\operatorname{trace}(M_I^{-1})\right)^2}\right| > \epsilon\right\}\\ &= P\left\{\left|\frac{s_i^t M_i^{-2} s_i}{(1+\mu_i s_i^t M_i^{-1} s_i)^2} - \frac{\frac{1}{N}\operatorname{trace}(M_i^{-2})}{\left(1+\frac{\mu_i}{N}\operatorname{trace}(M_i^{-1})\right)^2}\right| > \epsilon\right\}\\ &\le \frac{c_1(\epsilon)}{N^3}\end{aligned} \tag{30}$$

where the last step follows from Lemma 5.2. We use the union bound and get that for $N \ge N_0(\epsilon)$

$$P\left\{\max_{2\le i\le K}\left|\frac{\alpha P_i s_i^t M_i^{-2} s_i}{(1+\mu_i s_i^t M_i^{-1} s_i)^2} - \frac{\alpha P_i \frac{1}{N}\operatorname{trace}(M_I^{-2})}{\left(1+\frac{\mu_i}{N}\operatorname{trace}(M_I^{-1})\right)^2}\right| > \epsilon\right\} \le \frac{c_7(\epsilon)}{N^2}$$

which implies that

$$P\left\{\left|\frac{1}{N}\sum_{i=2}^K P_i \frac{s_i^t M_i^{-2} s_i}{(1+\mu_i s_i^t M_i^{-1} s_i)^2} - \frac{1}{N}\sum_{i=2}^K P_i \frac{\frac{1}{N}\operatorname{trace}(M_I^{-2})}{\left(1+\frac{\mu_i}{N}\operatorname{trace}(M_I^{-1})\right)^2}\right| > \epsilon\right\} \le \frac{c_8(\epsilon)}{N^2}$$

where $c_7(\epsilon)$ and $c_8(\epsilon)$ are positive constants. Since ϵ is arbitrary, we use the Borel–Cantelli lemma [23, p. 253] and conclude that

$$\frac{1}{N}\operatorname{trace}\left(M_I^{-1}\left(\sum_{i=2}^K P_i s_i s_i^t\right)M_I^{-1}\right) - \frac{1}{N}\sum_{i=2}^K P_i \frac{\frac{1}{N}\operatorname{trace}(M_I^{-2})}{\left(1+\frac{\mu_i}{N}\operatorname{trace}(M_I^{-1})\right)^2} \xrightarrow{\text{a.s.}} 0. \tag{31}$$

Observe that

$$\begin{aligned}\frac{1}{N}\sum_{i=2}^K P_i \frac{\frac{1}{N}\operatorname{trace}(M_I^{-2})}{\left(1+\frac{\mu_i}{N}\operatorname{trace}(M_I^{-1})\right)^2} &= \frac{1}{N}\sum_{i=2}^K P_i \frac{\frac{1}{N}\sum_{j=1}^N \frac{1}{(\lambda_j+\eta)^2}}{\left(1+\frac{\mu_i}{N}\sum_{j=1}^N \frac{1}{\lambda_j+\eta}\right)^2}\\ &= \int \frac{P\int_0^\infty \frac{1}{(\lambda+\eta)^2}\,dG_N(\lambda)}{\left(1+\mu\int_0^\infty \frac{1}{\lambda+\eta}\,dG_N(\lambda)\right)^2}\,dF_{P,\mu}^{(N)}\end{aligned} \tag{32}$$

where the outer integral is with respect to $F_{P,\mu}^{(N)}$, which is the joint empirical distribution of $\{(P_1, \mu_1), \ldots, (P_K, \mu_K)\}$ when the processing gain is N. Moreover, it is straightforward to see that

$$\begin{aligned}\int_0^\infty \frac{1}{\lambda+\eta}\,dG_N(\lambda) &\xrightarrow{\text{a.s.}} \int_0^\infty \frac{1}{\lambda+\eta}\,dG^*(\lambda)\\ \int_0^\infty \frac{1}{(\lambda+\eta)^2}\,dG_N(\lambda) &\xrightarrow{\text{a.s.}} \int_0^\infty \frac{1}{(\lambda+\eta)^2}\,dG^*(\lambda)\end{aligned} \tag{33}$$

where G^* depends on F_μ, the limiting empirical distribution of $\{\mu_1, \ldots, \mu_K\}$ [namely, $F_\mu(\cdot) = \lim_{p\to\infty} F_{P,\mu}(p, \cdot)$]. Using a telescoping argument, after some algebraic manipulation, we get that

$$\frac{1}{N}\sum_{i=2}^K P_i \frac{\frac{1}{N}\operatorname{trace}(M_I^{-2})}{\left(1+\frac{\mu_i}{N}\operatorname{trace}(M_I^{-1})\right)^2} \xrightarrow{\text{a.s.}} \alpha\mathbb{E}\left[\frac{P\int_0^\infty \frac{1}{(\lambda+\eta)^2}\,dG^*(\lambda)}{\left(1+\mu\int_0^\infty \frac{1}{\lambda+\eta}\,dG^*(\lambda)\right)^2}\right] \tag{34}$$

where the expectation is taken over $F_{P,\mu}$.

Recall that the joint empirical distribution function of $\{(P_1, \mu_1), \ldots, (P_K, \mu_K)\}$ converges weakly to $F_{P,\mu}$ with probability one, and the signatures and received powers are independent. Therefore, we combine (26), (31), and (34) to conclude that for almost every realization of the signatures and received powers

$$\sum_{i=2}^K \left(t_i^{(N)}\right)^2 \xrightarrow{\text{a.s.}} \alpha\mathbb{E}\left[\frac{P\int_0^\infty \frac{1}{(\lambda+\eta)^2}\,dG^*(\lambda)}{\left(1+\mu\int_0^\infty \frac{1}{\lambda+\eta}\,dG^*(\lambda)\right)^2}\right]. \tag{35}$$

The right-hand side of (35) can be further simplified as follows:

$$\begin{aligned}
&\alpha\mathbb{E}\left[\frac{P\int_0^\infty \frac{1}{(\lambda+\eta)^2}\,dG^*(\lambda)}{\left(1+\mu\int_0^\infty \frac{1}{\lambda+\eta}\,dG^*(\lambda)\right)^2}\right]\\
&=\alpha\mathbb{E}\left[\frac{\mu\int_0^\infty \frac{1}{(\lambda+\eta)^2}\,dG^*(\lambda)}{\left(1+\mu\int_0^\infty \frac{1}{\lambda+\eta}\,dG^*(\lambda)\right)^2}\right]\\
&=\alpha\mathbb{E}\left[\frac{d}{d\eta}\left(\frac{1}{1+\mu\int_0^\infty \frac{1}{\lambda+\eta}\,dG^*(\lambda)}\right)\right]\\
&\overset{(a)}{=}\alpha\frac{d}{d\eta}\left(\mathbb{E}\left[1-\frac{\mu\int_0^\infty \frac{1}{\lambda+\eta}\,dG^*(\lambda)}{1+\mu\int_0^\infty \frac{1}{\lambda+\eta}\,dG^*(\lambda)}\right]\right)\\
&=\frac{d}{d\eta}\left(\frac{-\alpha\mathbb{E}[\mu]}{1+\mu\int_0^\infty \frac{1}{\lambda+\eta}\,dG^*(\lambda)}\int_0^\infty \frac{1}{\lambda+\eta}\,dG^*(\lambda)\right)\\
&\overset{(b)}{=}\frac{d}{d\eta}\left(\eta\int_0^\infty \frac{1}{\lambda+\eta}\,dG^*(\lambda)\right)\\
&=\int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\,dG^*(\lambda)
\end{aligned}$$

where (a) follows from the Lebesgue Dominated Convergence Theorem, and (b) is obtained by using (7). Therefore, the conditional distribution of $I_{1n}^{(N)}$, given the signatures and received powers, converges almost surely to the Gaussian distribution with mean 0 and variance

$$\int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\,dG^*(\lambda).$$

Combining the above result with Lemma 4.1, we conclude that for almost every realization of the signatures and received powers, $I_1^{(N)}$ has a limiting Gaussian distribution with mean 0 and variance

$$a_g^2\int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\,dG^*(\lambda)$$

completing the proof.

Theorem 3.2 shows that for almost every realization of the signatures and received powers, the conditional distribution of the output MAI of the MMSE receiver is asymptotically Gaussian (the same Gaussian distribution as in the unconditional case). This strong result tells us that the MMSE receiver performs well in a large system in the sense that the MMSE receiver is robust to the time-varying channel conditions and the randomness of signatures. In particular, this result is useful in the scenarios where the received powers change relatively slowly compared to the symbol rate, and correspondingly the received powers are correlated across symbol intervals.

Since the proofs are rather technical, for simplicity, we have confined ourselves to antipodal modulation. For general modulation, we can use similar techniques to establish the Gaussianity under the following assumption on the information symbols: $\mathbb{E}[b_i]=0, \mathbb{E}[b_i^2]=1$, and the $|b_i|$'s are uniformly bounded below and above. Note that this covers many practical modulation methods of interest.

C. Proof of Theorem 3.3

Recall

$$\mathrm{SIR}_1^{(N)}=\frac{P_1(s_1^tM_I^{-1}s_1)^2}{s_1^tM_I^{-1}M_I'M_I^{-1}s_1}.$$

Since

$$s_1^tM_I^{-1}M_I'M_I^{-1}s_1=\sum_{i=2}^K\left(t_i^{(N)}\right)^2+\eta s_1^tM_I^{-2}s_1$$

combining (35) and (33) with the above yields that

$$\mathrm{SIR}_1^{(N)}\xrightarrow{\text{a.s.}}P_1\int_0^\infty \frac{1}{\lambda+\eta}\,dG^*(\lambda). \tag{36}$$

Next we show that $\lim_{N\to\infty}\Pi_e^{(N)}=\mathbb{E}[Q(\sqrt{\beta_1})]$. Since the modulation is antipodal, by symmetry, it is straightforward to see that

$$\begin{aligned}
\Pi_e^{(N)}&=P\left\{y_1^{(N)}\geq 0\middle|\, b_1=-1\right\}\\
&=\mathbb{E}\left[P\left\{y_1^{(N)}\geq 0\middle|\, b_1=-1,\,P_1\right\}\right]
\end{aligned}$$

where the expectation is taken over P_1. We combine Lemma 4.1 with Theorem 3.1 to conclude that $y_1^{(N)}$ (given b_1 and P_1) has a limiting Gaussian distribution. Appealing to the Lebesgue Dominated Convergence Theorem and [1, Theorem 5.2], we have that

$$\begin{aligned}
\lim_{N\to\infty}\Pi_e^{(N)}&=\mathbb{E}\left[\lim_{N\to\infty}P\left\{y_1^{(N)}\geq 0\middle|\, b_1=-1,\,P_1\right\}\right]\\
&=\mathbb{E}\left[P\left\{\lim_{N\to\infty}y_1^{(N)}\geq 0\middle|\, b_1=-1,\,P_1\right\}\right]\\
&=\mathbb{E}\left[Q\left(\sqrt{\beta_1}\right)\right]
\end{aligned} \tag{37}$$

which completes the proof.

Theorem 3.3 suggests that the SIR is the key parameter that governs the performance of a large system with the MMSE receiver. Based on (36), we have that the SIR corresponding to a particular P_1 is simply $P_1\beta'$, where

$$\beta'\triangleq\int_0^\infty \frac{1}{\lambda+\eta}\,dG^*(\lambda).$$

The quantity β' can be interpretated as the SIR achieved by unit received power [26]. The calculation of the SIR and hence the BEP then boils down to that of β', which is a constant. Using (7), β' can be shown to be the unique positive solution to the following fixed-point equation:

$$\beta'=\frac{1}{\eta+\alpha\int_0^\infty \frac{\mu}{1+\mu\beta'}\,dF_\mu(\mu)}.$$

Heuristically, we can say that in a large system, β' approximately satisfies

$$\beta'\approx\frac{1}{\eta+\frac{1}{N}\sum_{i=2}^K I(\mu_i,\,\beta')}.$$

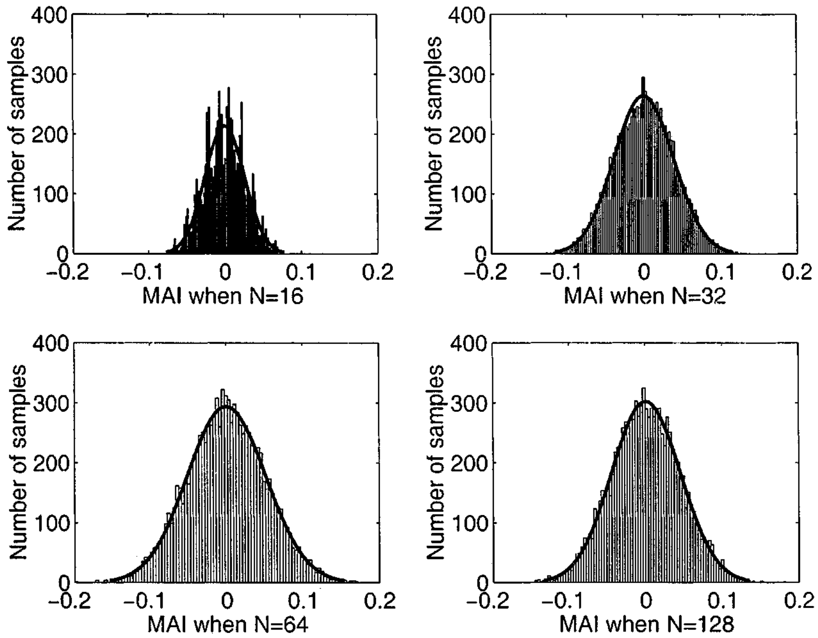

Fig. 3. The output MAI for a fixed set of randomly generated signatures, for $N = 16, 32, 64, 128$. Here $P_i = \mu_i$ and $\frac{P_i}{\eta} = 20$ dB, $i = 1, \ldots, K$.

where $I(\mu, \beta') \triangleq \frac{\mu}{1+\mu\beta'}$. In general, there is no closed-form solution for β'. However, when the users in the system can be classified into a small number of classes (according to their received powers), β' corresponds to the unique positive root of a polynomial.

VI. Numerical Examples and Conclusions

In this paper, we consider a model for the uplink of a single-cell DS-CDMA system with the MMSE receiver, assuming the received powers of users are random because of time-varying channel conditions. We have studied primarily the asymptotic distributions of the output MAI in CDMA systems with random spreading. Roughly speaking, we have found that if the empirical distribution function of $\{\mu_1, \ldots, \mu_K\}$ converges weakly, then the output MAI of the MMSE receiver is asymptotically Gaussian; if the joint empirical distribution function of $\{(P_1, \mu_1), \ldots, (P_K, \mu_K)\}$ converges weakly with probability one, then for almost every realization of the signatures and received powers, the conditional distribution of the output MAI converges weakly to the same Gaussian distribution as in the unconditional case. These results are quite general and applicable to many practical systems of interest. For example, we can obtain almost sure convergence of the conditional distribution when there are finite number of classes in the system (the received powers of the users within each class are i.i.d.). We have also extended our study to asynchronous systems and established the Gaussian nature of the output interference.

To illustrate our asymptotic results, we provide two numerical examples. In each example, we simulated for a fixed set of randomly chosen signatures corresponding to the processing gains $N = 16, 32, 64$, and 128, respectively. The entries of the signatures are either $\frac{1}{\sqrt{N}}$ or $\frac{-1}{\sqrt{N}}$, and the information symbols are random, either 1 or -1. We fixed $\alpha = 0.75$ in both examples. In Example 1, we assume that all the users have equal received powers with signal-to-noise ratio (SNR) $\frac{P}{\eta} = 20$ dB. In Example 2, we assume that all the users' received powers have log-normal distributions, $\frac{K}{4}$ interferers having mean SNR 20 dB and variance 13.3 dB, another $\frac{K}{4}$ interferers having mean SNR 16 dB and variance 11.4 dB, and the remaining $\frac{K}{2} - 1$ interferers having mean SNR 30 dB and variance 23.3 dB. In Example 2, we randomly generate the powers once according to the distributions and then fix them, and the corresponding plots are taken as the distributions of the output MAI conditioned on both the signatures and powers. We obtained the MAI values of 10 000 samples for each plot. Examples 1 and 2 are shown in Figs. 3 and 4. We observe that the distributions of the output MAI are close to Gaussian when N is large, corroborating our theoretical results.

Our results are useful for design of channel codes and performance analysis such as the calculation of the BEP, and also useful for the characterization of fundamental system limits such as channel capacity. For example, the BEP (assuming the modulation is antipodal) can be expressed as $\mathbb{E}[Q(\sqrt{\beta})]$, where β is the SIR and is random (due to the imperfect power control). If the MMSE receiver is followed by a single-user decoder (as has been advocated in [31] and the references therein), then the achievable information-theoretic rate (channel capacity) for each user is $\frac{1}{2}\mathbb{E}[\log(1+\beta)]$ bits per symbol time [6].

Although the results in this paper are for single-cell systems with MMSE receivers, we believe that these results can be extended to multiple-cell systems. Indeed, in a multiple-cell setting, interferers from other cells have smaller powers. Thus, the Gaussian approximation may be even more appropriate in a multiple-cell asynchronous setting because essentially Gaussianity comes from sums of many "small" terms.

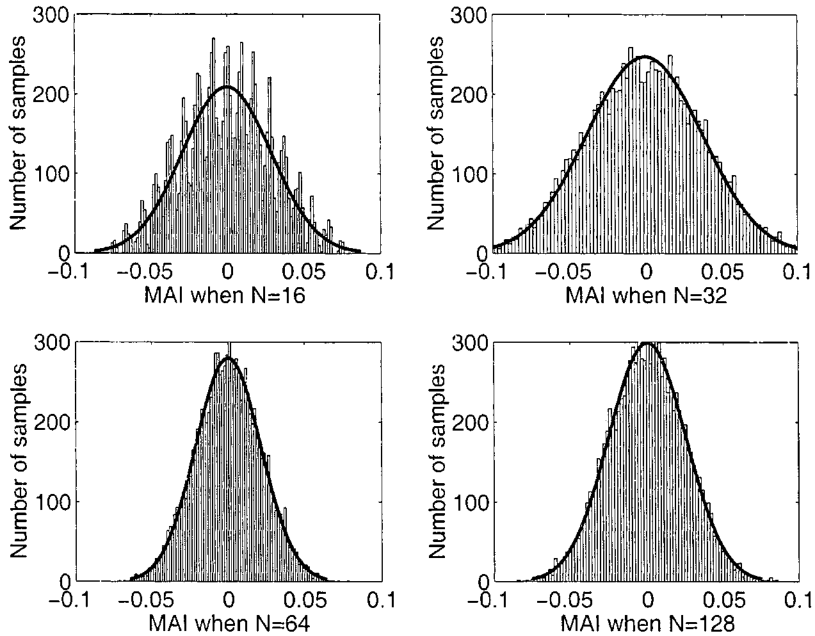

Fig. 4. The output MAI for a fixed set of randomly generated signatures, for $N = 16, 32, 64, 128$. Each user's received power is randomly generated according to a log-normal distribution and then fixed. Here $\frac{P_1}{\eta}$ = 20 dB, $\frac{\mu_i}{\eta}$ = 20 dB, $i = 2, \ldots, \frac{K}{4}+1$; $\frac{\mu_i}{\eta}$ = 16 dB, $i = \frac{K}{4}+2, \ldots, \frac{K}{2}+1$; $\frac{\mu_i}{\eta}$ = 30 dB, $i = \frac{K}{2}+2, \ldots, K$.

Appendix A
Proofs of Lemmas 4.2 and 4.2

A. Proof of Lemma 4.2

Observe that for $i = 2, \ldots, K$

$$\begin{aligned}\left(t_i^{(N)}\right)^2 &= (s_1^t M_I^{-1} s_i)^2 P_i b_i^2 \\ &= \text{trace}(s_i^t M_I^{-1} s_1 s_1^t M_I^{-1} s_i) P_i \\ &= \text{trace}(P_i s_i s_i^t M_I^{-1} s_1 s_1^t M_I^{-1})\end{aligned} \tag{38}$$

which implies that[3]

$$\begin{aligned}&\mathbb{E}\left[\sum_{i=2}^{K}\left(t_i^{(N)}\right)^2\right] \\ &= \mathbb{E}\left[\text{trace}\left(\sum_{i=2}^{K} P_i s_i s_i^t M_I^{-1} s_1 s_1^t M_I^{-1}\right)\right] \\ &= \mathbb{E}\left[\text{trace}\left(\sum_{i=2}^{K} \mathbb{E}[P_i] s_i s_i^t M_I^{-1} \mathbb{E}(s_1 s_1^t) M_I^{-1}\right)\right] \\ &= \frac{1}{N}\mathbb{E}\left[\text{trace}\left(\sum_{i=2}^{K} \mu_i s_i s_i^t M_I^{-2}\right)\right] \\ &\qquad \left(\text{since } \mathbb{E}[s_1 s_1^t] = \frac{1}{N} I\right) \\ &= \frac{1}{N}\mathbb{E}[\text{trace}(S_1 E_1 S_1^t M_I^{-2})] \\ &= \mathbb{E}\left[\frac{1}{N}\sum_{j=1}^{N}\frac{\lambda_j}{(\lambda_j+\eta)^2}\right] \\ &= \mathbb{E}\left[\int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\, dG_N(\lambda)\right].\end{aligned} \tag{39}$$

[3]The expectation in different lines may be taken over different random elements.

Therefore, we have

$$\begin{aligned}\lim_{N\to\infty}\mathbb{E}\left[\sum_{i=2}^{K}\left(t_i^{(N)}\right)^2\right] &= \lim_{N\to\infty}\mathbb{E}\left[\int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\, dG_N(\lambda)\right] \\ &\overset{(a)}{=} \mathbb{E}\left[\lim_{N\to\infty}\int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\, dG_N(\lambda)\right] \\ &\overset{(b)}{=} \int_0^\infty \frac{\lambda}{(\lambda+\eta)^2}\, dG^*(\lambda)\end{aligned} \tag{40}$$

where (a) follows from the Lebesgue Dominated Convergence Theorem, and (b) follows from [1, Theorem 5.2] because the quantity $\frac{\lambda}{(\lambda+\eta)^2}$ is a continuous function of λ and bounded by $\frac{1}{\eta}$.

B. Proof of Lemma 4.3

Recall that

$$\begin{aligned}S_i &\triangleq [s_2, \ldots, s_{i-1}, s_{i+1}, \ldots, s_K] \\ E_i &\triangleq \text{diag}(\mu_2, \ldots, \mu_{i-1}, \mu_{i+1}, \ldots, \mu_K) \\ M_i &\triangleq S_i E_i S_i^t + \eta I.\end{aligned}$$

We denote the eigenvalues of $S_i E_i S_i^t$ by $v_1, \ldots, v_N$ and the empirical distribution of the eigenvalues G'_N. Note that the μ_i's are bounded because $\mathbb{E}[P_i^2]$ is bounded for $i = 1, \ldots, K$. Therefore, $\{\mu_2, \ldots, \mu_{i-1}, \mu_{i+1}, \ldots, \mu_K\}$ also converges weakly to F_μ as $N \to \infty$. By appealing to [24, Theorem 1.1] again, G'_N converges weakly to G^* almost surely as $N \to \infty$.

Using the matrix inverse lemma, we have that

$$\begin{aligned}t_i^{(N)} &= \sqrt{P_i} b_i s_1^t M_I^{-1} s_i \\ &= s_1^t b_i M_i^{-1}\left[I - \frac{\mu_i s_i s_i^t M_i^{-1}}{1+\mu_i s_i^t M_i^{-1} s_i}\right]\sqrt{P_i} s_i \\ &= \frac{b_i s_1^t M_i^{-1}\sqrt{P_i} s_i}{1+\mu_i s_i^t M_i^{-1} s_i}.\end{aligned}$$

For any realization of S_i, the corresponding realization of M_i is symmetric and positive definite, and hence can be written in the form of $Q^t\Upsilon Q$, where $\Upsilon = \text{diag}(\upsilon_1+\eta, \ldots, \upsilon_N+\eta)$. By our assumption, P_i, b_i, and S_i are independent. Moreover, $\mu_i s_i^t M_i^{-1} s_i > 0$ for every realization of S_i, which implies that $1+\mu_i s_i^t M_i^{-1} s_i > 1$. Hence, we have

$$\mathbb{E}\left[\left(t_i^{(N)}\right)^2 \middle| S_i\right] \le \mathbb{E}[P_i]\mathbb{E}\left\{\frac{1}{N^2}\left[\sum_{j=1}^N \frac{\sum_{k_1=1}^N\sum_{k_2=1}^N Q_{jk_1}Q_{jk_2}s_{1k_1}s_{ik_2}}{\upsilon_j+\eta}\right]^2 \middle| S_i\right\}$$

$$\mathbb{E}\left[\left(t_i^{(N)}\right)^4 \middle| S_i\right] \le \mathbb{E}[P_i^2]\mathbb{E}\left\{\frac{1}{N^4}\left[\sum_{j=1}^N \frac{\sum_{k_1=1}^N\sum_{k_2=1}^N Q_{jk_1}Q_{jk_2}s_{1k_1}s_{ik_2}}{\upsilon_j+\eta}\right]^4 \middle| S_i\right\}.$$

Since s_1, s_i, and S_i are independent, we have the equation shown at the bottom of the page, which implies that

$$\begin{aligned}\mathbb{E}\left[\left(t_i^{(N)}\right)^2\right] &= \mathbb{E}\left[\mathbb{E}\left[\left(t_i^{(N)}\right)^2 \middle| S_i\right]\right] \\ &\le \mathbb{E}[P_i]\mathbb{E}[b_i^2]\mathbb{E}\left[\frac{1}{N^2}\sum_{j=1}^N \frac{1}{(\upsilon_j+\eta)^2}\right] \\ &= \frac{\mu_i}{N}\mathbb{E}\left[\int_o^\infty \frac{1}{(\upsilon+\eta)^2}\,dG_N(\lambda)\right] \\ &\le \frac{\mu_i}{N\eta^2}.\end{aligned} \tag{41}$$

Next we use techniques similar to those in the above argument to show that

$$\mathbb{E}[(t_i^{(N)})^4] \le \frac{9\mathbb{E}[P_i^2]}{N^2\eta^4}$$

for $i = 2, \ldots, M$. Observe that

$$\begin{aligned}&\left[\sum_{j=1}^N \frac{\sum_{k_1=1}^N\sum_{k_2=1}^N Q_{jk_1}Q_{jk_2}s_{1k_1}s_{ik_2}}{\upsilon_j+\eta}\right]^4 \\ &= \left[\sum_{j_1=1}^N \frac{\sum_{k_{11}=1}^N\sum_{k_{21}=1}^N Q_{j_1k_{11}}Q_{j_1k_{21}}s_{1k_{11}}s_{ik_{21}}}{\upsilon_{j_1}+\eta}\right] \cdot \left[\sum_{j_2=1}^N \frac{\sum_{k_{12}=1}^N\sum_{k_{22}=1}^N Q_{j_2k_{12}}Q_{j_2k_{22}}s_{1k_{12}}s_{ik_{22}}}{\upsilon_{j_2}+\eta}\right] \\ &\quad\cdot \left[\sum_{j_3=1}^N \frac{\sum_{k_{13}=1}^N\sum_{k_{23}=1}^N Q_{j_3k_{13}}Q_{j_3k_{23}}s_{1k_{13}}s_{ik_{23}}}{\upsilon_{j_3}+\eta}\right] \cdot \left[\sum_{j_4=1}^N \frac{\sum_{k_{14}=1}^N\sum_{k_{24}=1}^N Q_{j_4k_{14}}Q_{j_4k_{24}}s_{1k_{14}}s_{ik_{24}}}{\upsilon_{j_4}+\eta}\right].\end{aligned}$$

Note that $\mathbb{E}[s_{1k_{11}}s_{1k_{12}}s_{1k_{13}}s_{1k_{14}}]$ is nonzero (equal to 1) only in the following three cases: 1) $k_{11}=k_{12}$ and $k_{13}=k_{14}$; 2) $k_{11}=k_{13}$ and $k_{12}=k_{14}$; and 3) $k_{11}=k_{14}$ and $k_{12}=k_{13}$. Moreover, $\mathbb{E}[s_{ik_{21}}s_{ik_{22}}s_{ik_{23}}s_{ik_{24}}]$ is nonzero (equal to 1) only in the following three cases: 1) $k_{21}=k_{22}$ and $k_{23}=k_{24}$; 2) $k_{21}=k_{23}$ and $k_{22}=k_{24}$; and 3) $k_{21}=k_{24}$ and $k_{22}=k_{23}$. Let Z_4 denote the sum of all the terms corresponding to the case $k_{11}=k_{12}=k_{13}=k_{14}$, $k_{21}=k_{22}$, and $k_{23}=k_{24}$; and Z_5 the sum of all the terms corresponding to the case $k_{11}=k_{12}=k_{13}=k_{14}$ and $k_{21}=k_{22}=k_{23}=k_{24}$. Using the properties of orthogonal matrices, it follows that $\mathbb{E}[Z_4|S_i]$ equals the expression shown

$$\begin{aligned}&\mathbb{E}\left[\left(\sum_{j=1}^N \frac{\sum_{k_1=1}^N\sum_{k_2=1}^N Q_{jk_1}Q_{jk_2}s_{1k_1}s_{ik_2}}{\upsilon_j+\eta}\right)^2 \middle| S_i\right] \\ &= \mathbb{E}\left[\sum_{j_1=1}^N\sum_{j_2=1}^N \frac{\sum_{k_{11}=1}^N\sum_{k_{21}=1}^N\sum_{k_{12}=1}^N\sum_{k_{22}=1}^N Q_{j_1k_{11}}Q_{j_1k_{21}}Q_{j_2k_{12}}Q_{j_2k_{22}}s_{1k_{11}}s_{ik_{21}}s_{1k_{12}}s_{ik_{22}}}{(\upsilon_{j_1}+\eta)(\upsilon_{j_2}+\eta)} \middle| S_i\right] \\ &= \sum_{j_1=1}^N\sum_{j_2=1}^N \frac{\sum_{k_1=1}^N\sum_{k_2=1}^N Q_{j_1k_1}Q_{j_1k_2}Q_{j_2k_1}Q_{j_2k_2}}{(\upsilon_{j_1}+\eta)(\upsilon_{j_2}+\eta)} = \sum_{j_1=1}^N\sum_{j_2=1}^N \frac{\left(\sum_{k=1}^N Q_{j_1k}Q_{j_2k}\right)^2}{(\upsilon_{j_1}+\eta)(\upsilon_{j_2}+\eta)} = \sum_{j=1}^N \frac{1}{(\upsilon_j+\eta)^2}\end{aligned}$$

at the bottom of this page. Similarly, we have that $\mathbb{E}[Z_5|S_i] \le \frac{1}{N^3\eta^4}$. Then (42) follows, as shown at the bottom of the page. This implies that

$$\mathbb{E}\left[\left(t_i^{(N)}\right)^4\right] \le \frac{9\mathbb{E}[P_i^2]}{N^2\eta^4}.$$

Similarly, we have for $i = 2, \ldots, K$

$$\mathbb{E}\left[\left(t_i^{(N)}\right)^6\right] \le \frac{225\mathbb{E}[P_i^3]}{N^3\eta^6}.$$

The proof is completed.

Appendix B
Proofs of Lemmas 4.1, 5.1, and 5.2

We use the following lemma repeatedly, which follows directly from [24, Lemma 3.1].

Lemma B.1: Let $A = (a_{ij})$, $a_{ij} \in \mathbb{C}$, be an $N \times N$ matrix with spectral radius bounded in N, and

$$Y = \frac{1}{\sqrt{N}}(y_1, \ldots, y_N)^t$$

where the y_n's are i.i.d. with $P\{y_n = 1\} = P\{y_n = -1\} = \frac{1}{2}$. Then

$$\mathbb{E}\left[\left|Y^tAY - \frac{1}{N}\operatorname{trace}(A)\right|^6\right] \le \frac{c}{N^3}$$

where the constant c does not depend on N and A.

A. Proof of Lemma 4.1

For any given S_1, we have that $M_I \succeq \eta I \succ 0$, which implies that $0 \prec M_I^{-1} \preceq \frac{1}{\eta}I$. That is, the spectral radius of M_I^{-1} is upper-bounded by $\frac{1}{\eta}$ for all N. Since s_1 and S_1 are independent, by appealing to Lemma B.1, we obtain

$$\begin{aligned}\mathbb{E}&\left[\left|s_1^tM_I^{-1}s_1 - \frac{\operatorname{trace}(M_I^{-1})}{N}\right|^6\right] \\ &= \mathbb{E}\left[\mathbb{E}\left(\left|s_1^tM_I^{-1}s_1 - \frac{\operatorname{trace}(M_I^{-1})}{N}\right|^6 \middle| S_1\right)\right] \le \frac{c_1}{N^3}\end{aligned}$$

where c_1 does not depend on M_I, N, nor on s_1. Fixed $\epsilon > 0$. Using Markov's inequality [2, p. 283], it follows that

$$\begin{aligned}P&\left\{\left|s_1^tM_I^{-1}s_1 - \frac{\operatorname{trace}(M_I^{-1})}{N}\right| \ge \epsilon\right\} \\ &\le \frac{\mathbb{E}\left[\left|s_1^tM_I^{-1}s_1 - \frac{\operatorname{trace}(M_I^{-1})}{N}\right|^6\right]}{\epsilon^6}.\end{aligned}$$

$$\begin{aligned}\mathbb{E}[Z_4|S_i] &= \frac{1}{N^4}\left\{\sum_{j_1}\sum_{j_2}\sum_{j_3}\sum_{j_4}\frac{\sum_{k_{11}}\sum_{k_{21}}\sum_{k_{23}} Q_{j_1k_{11}}Q_{j_2k_{11}}Q_{j_3k_{11}}Q_{j_4k_{11}}Q_{j_1k_{21}}Q_{j_2k_{21}}Q_{j_3k_{23}}Q_{j_4k_{23}}}{(\upsilon_{j_1}+\eta)(\upsilon_{j_2}+\eta)(\upsilon_{j_3}+\eta)(\upsilon_{j_4}+\eta)}\right\} \\ &= \frac{1}{N^4}\sum_{k_{11}}\sum_{k_{21}}\sum_{k_{23}}\left(\sum_{j_1}\sum_{j_3}\frac{Q_{j_1k_{11}}Q_{j_3k_{11}}Q_{j_1k_{21}}Q_{j_3k_{23}}}{(\upsilon_{j_1}+\eta)(\upsilon_{j_3}+\eta)}\right)^2 \\ &\le \frac{1}{N^4\eta^4}\sum_{k_{11}}\sum_{k_{21}}\sum_{k_{23}}\left(\sum_{j_1}Q_{j_1k_{11}}Q_{j_1k_{21}}\sum_{j_3}Q_{j_3k_{11}}Q_{j_3k_{23}}\right)^2 = \frac{1}{N^3\eta^4}.\end{aligned}$$

$$\begin{aligned}\frac{\mathbb{E}\left[\left(t_i^{(N)}\right)^4\middle|S_i\right]}{\mathbb{E}[P_i^2]} &\le \frac{9}{N^4}\left\{\sum_{j_1}\sum_{j_2}\sum_{j_3}\sum_{j_4}\frac{\sum_{k_{11}}\sum_{k_{13}}\sum_{k_{21}}\sum_{k_{23}} Q_{j_1k_{11}}Q_{j_2k_{11}}Q_{j_3k_{13}}Q_{j_4k_{13}}Q_{j_1k_{21}}Q_{j_2k_{21}}Q_{j_3k_{23}}Q_{j_4k_{23}}}{(\upsilon_{j_1}+\eta)(\upsilon_{j_2}+\eta)(\upsilon_{j_3}+\eta)(\upsilon_{j_4}+\eta)}\right\} \\ &= \frac{9}{N^4}\left(\sum_{j_1}\sum_{j_2}\frac{\sum_{k_{11}}\sum_{k_{21}} Q_{j_1k_{11}}Q_{j_1k_{21}}Q_{j_2k_{11}}Q_{j_2k_{21}}}{(\upsilon_{j_1}+\eta)(\upsilon_{j_2}+\eta)}\right)^2 = \frac{9}{N^4}\left(\sum_{j_1}\sum_{j_2}\frac{\left(\sum_k Q_{j_1k}Q_{j_2k}\right)^2}{(\upsilon_{j_1}+\eta)(\upsilon_{j_2}+\eta)}\right)^2 \\ &= \frac{9}{N^2}\left(\int\frac{1}{(\upsilon+\eta)^2}\,dG_N(\lambda)\right)^2 \le \frac{9}{N^2\eta^4}. \qquad (42)\end{aligned}$$

Therefore, we have that

$$\sum_{N=1}^{\infty} P\left\{\left|s_1^t M_I^{-1} s_1 - \frac{\mathrm{trace}(M_I^{-1})}{N}\right| \geq \epsilon\right\} \leq \sum_{N=1}^{\infty} \frac{c_1}{N^3\epsilon^6} < \infty.$$

Using the Borel–Cantelli lemma, we conclude that

$$s_1^t M_I^{-1} s_1 - \frac{\mathrm{trace}(M_I^{-1})}{N} \xrightarrow{\text{a.s.}} 0. \tag{43}$$

Furthermore, it is straightforward to show that

$$\frac{\mathrm{trace}(M_I^{-1})}{N} \xrightarrow{\text{a.s.}} \int_0^{\infty} \frac{1}{\lambda+\eta}\, dG^*(\lambda). \tag{44}$$

Combining (43) and (44) leads to

$$s_1^t M_I^{-1} s_1 \xrightarrow{a.s.} \int_0^{\infty} \frac{1}{\lambda+\eta}\, dG^*(\lambda)$$

which implies that

$$\frac{\sqrt{P_1}}{1+P_1 s_1^t M_I^{-1} s_1} \xrightarrow{\text{a.s.}} a_g.$$

B. Proof of Lemma 5.1

Fix $\epsilon > 0$. It follows that

$$\begin{aligned} P\left\{\max_{2\leq i\leq K}\left|t_i^{(N)}\right| > \epsilon\right\} &\leq \sum_{i=2}^{K} P\left\{\left|t_i^{(N)}\right| > \epsilon\right\} \\ &\overset{(a)}{\leq} \sum_{i=2}^{K} \frac{\mathbb{E}\left[\left(t_i^{(N)}\right)^6\right]}{\epsilon^6} \\ &\overset{(b)}{\leq} \sum_{i=2}^{K} \frac{225 d_1^3}{N^3(\epsilon\eta)^6} \end{aligned}$$

where (a) follows from Markov's inequality [2, p. 283], and (b) from Lemma 4.3. Therefore, we have

$$\sum_{N=1}^{\infty} P\left\{\max_{2\leq i\leq K}\left|t_i^{(N)}\right| > \epsilon\right\} \leq \frac{225\alpha d_1^3}{(\epsilon\eta)^6} \sum_{N=1}^{\infty} \frac{1}{N^2} < \infty.$$

Using the Borel–Cantelli lemma, we conclude that

$$\max_{2\leq i\leq K}\left|t_i^{(N)}\right| \xrightarrow{\text{a.s.}} 0. \tag{45}$$

C. Proof of Lemma 5.2

Observe that

$$\begin{aligned} &P\left\{\left|\frac{s_i^t M_i^{-2} s_i}{(1+\mu_i s_i^t M_i^{-1} s_i)^2} - \frac{\frac{1}{N}\mathrm{trace}(M_i^{-2})}{\left(1+\frac{\mu_i}{N}\mathrm{trace}(M_i^{-1})\right)^2}\right| > \epsilon\right\} \\ &= P\left\{\left|\frac{s_i^t M_i^{-2} s_i - \frac{1}{N}\mathrm{trace}(M_i^{-2})}{(1+\mu_i s_i^t M_i^{-1} s_i)^2} + \frac{\frac{1}{N}\mathrm{trace}(M_i^{-2})}{(1+\mu_i s_i^t M_i^{-1} s_i)^2} \right.\right. \\ &\qquad \left.\left. - \frac{\frac{1}{N}\mathrm{trace}(M_i^{-2})}{\left(1+\frac{\mu_i}{N}\mathrm{trace}(M_i^{-1})\right)^2}\right| > \epsilon\right\} \\ &\leq P\left\{\left|\frac{s_i^t M_i^{-2} s_i - \frac{1}{N}\mathrm{trace}(M_i^{-2})}{(1+\mu_i s_i^t M_i^{-1} s_i)^2}\right| > \frac{\epsilon}{2}\right\} \\ &\quad + P\left\{\frac{1}{N}\mathrm{trace}(M_i^{-2})\left|\frac{1}{(1+\mu_i s_i^t M_i^{-1} s_i)^2} \right.\right. \\ &\qquad \left.\left. - \frac{1}{\left(1+\frac{\mu_i}{N}\mathrm{trace}(M_i^{-1})\right)^2}\right| > \frac{\epsilon}{2}\right\}. \end{aligned} \tag{46}$$

Since $M_i \succeq \eta I \succ 0$, we have $0 \prec M_i^{-1} \preceq \frac{1}{\eta} I$. Hence we have the following inequalities:

$$\begin{aligned} s_i^t M_i^{-1} s_i &\leq \frac{1}{\eta} \\ \frac{1}{N}\mathrm{trace}(M_i^{-1}) &\leq \frac{1}{\eta} \\ \frac{1}{N}\mathrm{trace}(M_i^{-2}) &\leq \frac{1}{\eta^2}. \end{aligned}$$

Using the above inequalities, we obtain (47) shown at the bottom of the page. Furthermore, since the spectral radii of M_i^{-1} and M_i^{-2} are bounded by $\frac{1}{\eta}$ and $\frac{1}{\eta^2}$ respectively, we appeal to Lemma B.1 and get

$$\begin{aligned} \mathbb{E}\left[\left(s_i^t M_i^{-1} s_i - \frac{1}{N}\mathrm{trace}(M_i^{-1})\right)^6\right] &\leq \frac{c_2}{N^3} \\ \mathbb{E}\left[\left(s_i^t M_i^{-2} s_i - \frac{1}{N}\mathrm{trace}(M_i^{-2})\right)^6\right] &\leq \frac{c_3}{N^3} \end{aligned}$$

$$\begin{aligned} &P\left\{\frac{1}{N}\mathrm{trace}(M_i^{-2})\left|\frac{1}{(1+\mu_i s_i^t M_i^{-1} s_i)^2} - \frac{1}{\left(1+\frac{\mu_i}{N}\mathrm{trace}(M_i^{-1})\right)^2}\right| > \frac{\epsilon}{2}\right\} \\ &= P\left\{\frac{1}{N}\mathrm{trace}(M_i^{-2})\frac{\left[2\mu_i+\mu_i^2\left(\frac{1}{N}\mathrm{trace}(M_i^{-1})+s_i^t M_i^{-1} s_i\right)\right]\left|\frac{1}{N}\mathrm{trace}(M_i^{-1}) - s_i^t M_i^{-1} s_i\right|}{(1+\mu_i s_i^t M_i^{-1} s_i)^2\left(1+\frac{\mu_i}{N}\mathrm{trace}(M_i^{-1})\right)^2} > \frac{\epsilon}{2}\right\} \\ &\leq P\left\{\frac{2}{\eta^2}\left[\mu_i + \frac{\mu_i^2}{\eta}\right]\left|\frac{1}{N}\mathrm{trace}(M_i^{-1}) - s_i^t M_i^{-1} s_i\right| > \frac{\epsilon}{2}\right\}. \end{aligned} \tag{47}$$

where c_2 and c_3 are some positive constants. Using Markov's inequality [2, p. 283], we have

$$\begin{aligned} &P\left\{\left|\frac{s_i^t M_i^{-2} s_i - \frac{1}{N}\,\mathrm{trace}(M_i^{-2})}{(1+\mu_i s_i^t M_i^{-1} s_i)^2}\right| > \frac{\epsilon}{2}\right\} \\ &\quad\le P\left\{\left|s_i^t M_i^{-2} s_i - \frac{1}{N}\,\mathrm{trace}(M_i^{-2})\right| > \frac{\epsilon}{2}\right\} \\ &\quad\le \frac{\mathbb{E}\left[\left(s_i^t M_i^{-2} s_i - \frac{1}{N}\,\mathrm{trace}(M_i^{-2})\right)^6\right]}{\left(\frac{\epsilon}{2}\right)^6} \\ &\quad\le \frac{64 c_3}{N^3 \epsilon^6} \end{aligned} \tag{48}$$

and

$$\begin{aligned} &P\left\{\frac{1}{N}\,\mathrm{trace}(M_i^{-2})\left|\frac{1}{(1+\mu_i s_i^t M_i^{-1} s_i)^2}\right.\right. \\ &\quad\left.\left. - \frac{1}{\left(1+\frac{\mu_i}{N}\,\mathrm{trace}(M_i^{-1})\right)^2}\right| > \frac{\epsilon}{2}\right\} \le \frac{c_4}{N^3\epsilon^6} \end{aligned} \tag{49}$$

where c_4 is some positive constant. Combining (48) and (49), we conclude that for $i = 2, \ldots, K$

$$\begin{aligned} &P\left\{\left|\frac{s_i^t M_i^{-2} s_i}{(1+\mu_i s_i^t M_i^{-1} s_i)^2} - \frac{\frac{1}{N}\,\mathrm{trace}(M_i^{-2})}{\left(1+\frac{\mu_i}{N}\,\mathrm{trace}(M_i^{-1})\right)^2}\right| > \epsilon\right\} \\ &\qquad\qquad \le \frac{c_1(\epsilon)}{N^3} \end{aligned}$$

where $c_1(\epsilon)$ is some positive constant that does not depend on the index i. The existence of $c_1(\epsilon)$ follows from the assumption that the μ_i's are bounded. This completes the proof.

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