Distributed Opportunistic Scheduling For Ad-Hoc Communications: An Optimal Stopping Approach*

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ABSTRACT

We consider distributed opportunistic scheduling (DOS) in wireless ad-hoc networks, where many links contend for the same channel using random access. In such networks, distributed opportunistic scheduling involves a process of joint channel probing and distributed scheduling. Due to channel fading, the link condition corresponding to a successful channel probing could be either good or poor. In the latter case, further channel probing, although at the cost of additional delay, may lead to better channel conditions and hence higher transmission rates. The desired tradeoff boils down to judiciously choosing the optimal stopping strategy for channel probing and the rate threshold. In this paper, we pursue a rigorous characterization of the optimal strategies from two perspectives, namely, a network-centric perspective and a user-centric perspective.

We first consider DOS from a network-centric point of view, where links cooperate to maximize the overall network throughput. Using optimal stopping theory, we show that the optimal strategy turns out to be a *pure threshold policy*, where the rate threshold can be obtained by solving a fixed point equation. We further devise an iterative algorithm for computing the threshold.

Next, we explore DOS from a user-centric perspective, where each links seeks to maximize its own throughput. We treat the problem of rate threshold selections for different links as a non-cooperative game. We explore the existence and uniqueness of the Nash equilibrium, and show that the Nash equilibrium can be approached by the best response strategy. We then develop an online stochastic iterative algorithm using local observations only, and establish its convergence. Finally, we observe that there is an efficiency loss in terms of the throughput at the Nash equilibrium, and introduce a pricing-based mechanism to mitigate the loss.

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Categories and Subject Descriptors

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Keywords

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1. INTRODUCTION

1.1 Motivation

Wireless ad hoc networks have emerged as a promising solution that can facilitate communications between wireless devices without a planned fixed infrastructure. Different from its wireline counterpart, the design of wireless ad hoc networks faces a number of unique challenges in wireless communications, including 1) co-channel interference among active links in a neighborhood; and 2) time varying channel conditions over fading channels. The traditional wisdom for wireless network design is to separate link losses caused by fading from those caused by interference. That is, the PHY layer addresses the issues of fading, and the MAC layer addresses the issue of contention. However, as shown in [3] [10], fading can often adversely affect the MAC layer in many realistic scenarios. The coupling between the timescales of fading and MAC calls for a unified PHY/MAC design for wireless ad-hoc networks, in order to achieve optimal throughput and latency.

Notably, there has recently been a surge of interest in channel-aware scheduling and channel-aware access control. Channel aware opportunistic scheduling was first developed for the downlink transmissions in multiuser wireless networks (see, e.g., [2], [5], [9], [16], [17], [25], [27]). Opportunistic scheduling originates from a holistic view: roughly speaking, in a multiuser wireless network, at each moment it is likely that there exists a user with good channel conditions; and by picking the instantaneous "on-peak" user for data transmission, opportunistic scheduling can utilize the wireless resource more efficiently. *A key assumption in these studies is that the scheduler has knowledge of the instantaneous channel conditions for all links, and therefore the scheduling is centralized.*

Channel aware random access has been investigated for the uplink transmissions in a many-to-one network, where channel probing can be realized by broadcasting pilot signals from the base station. Notably, [1, 20] study opportunistic Aloha under a collision model, with a basic idea being that in every slot each user transmits with a probability based on its own channel condition. While recent work [15] does not assume a base station in a wireless LAN, the transmitter node still needs to collect channel information from potential receivers, thereby serving as a tentative "virtual" base station. A key observation is that in the existing work on rate adaptation for ad hoc communications (see, e.g., [14, 19, 23]), a link continues transmission after a successful channel contention, no matter whether the channel condition is good or poor. Clearly, this leaves much room for improvement by devising channel-aware scheduling.

Unfortunately, little work has been done on developing channel-aware distributed scheduling to harvest rich diversity gains for enhancing ad hoc communications. This is perhaps due to the fact that channel-aware distributed scheduling is indeed challenging, since *the distributed nature of ad hoc communications dictates that each link has no knowledge of others' channel conditions* (in fact, even its own channel condition is unknown before channel probing). A principal goal of this study is to fill this void, and obtain a rigorous understanding of distributed opportunistic scheduling (DOS) for ad-hoc communications.

In this paper, we take some initial steps in this direction and consider a single-hop ad-hoc network where all links can hear others' transmissions. In such a network, links contend for the same channel using random access, and a collision model is assumed which indicates that at most one link can transmit successfully at each time. We assume that after a successful contention, the channel condition of the successful link is measured (e.g., by using some pilot signals embedded in the handshake packets). Due to channel fading, the link condition corresponding to this successful channel probing can be either good or poor. In the latter case, data packets have to be transmitted at low rates, leading to possible throughput degradation. A plausible alternative is to let this link give up this transmission opportunity, and allow all the links re-contend for the channel, in the hope that some link with a better channel condition can transmit after the re-contention. Intuitively speaking, because different links at different time slots experience different channel conditions, it is likely that after further probing, the channel can be taken by a link with a better channel condition, resulting in possible higher throughput. In this way, the multiuser diversity across links and the time diversity across slots can be exploited in an opportunistic manner. It is in this sense that we call this process of joint probing and scheduling "distributed opportunistic scheduling". We should caution that on the other hand, each channel probing comes with a cost in terms of the contention time, which could be used for data transmission.

Clearly, there is a *tradeoff* between the throughput gain from better channel conditions and the cost for further channel probings. The desired tradeoff boils down to judiciously choosing the optimal stopping time for channel probing, in order to maximize the overall network throughput. In this paper, we obtain a systematic characterization of this tradeoff by appealing to optimal stopping theory [11, 12, 26], and explore channel-aware distributed scheduling to exploit the multiuser diversity and time diversity for wireless ad-hoc networks in an opportunistic manner. We shall tackle this problem from the following two different approaches: 1) a network-centric perspective in which all links "cooperate" to maximize the overall network throughput; and 2) a user-centric view where each link seeks to maximize its own throughput selfishly. Accordingly, the results are organized into two thrusts.

1.2 Summary of Main Results

The common theme of the first thrust is distributed opportunistic scheduling from a network-centric perspective. We start with the basic case where all links have the same channel statistics. Recall that when a link discovers that its channel condition is relatively poor after a successful channel contention, it can skip the transmission opportunity so that some link with a better condition would have the chance to transmit in the next round channel probing. We should point out that there is no guarantee for this to happen due to the stochastic nature of random contention and time varying channel conditions. Nevertheless, as channel probing continues, the likelihood of reaching a better channel condition increases. In a nutshell, distributed opportunistic scheduling boils down to a process of joint channel probing and scheduling.

Mathematically speaking, we treat distributed opportunistic scheduling as a cooperative game. Building on optimal stopping theory [11, 12, 26], we cast the problem as a *maximal rate of return* problem, where the rate of return refers to the average throughput. As noted above, since the cost, in terms of the contention duration, is random, we use the Maximal Inequality to establish the existence of the optimal stopping rule. Then, we develop the optimal strategy for distributed opportunistic scheduling, by characterizing the optimal stopping rule to "control" the channel probing process and hence to maximize the overall throughput. In particular, we show that the optimal strategy is a *pure threshold policy*, [1] in the sense that the decision on further channel probing or data transmission is based on the local channel condition only, and the threshold is invariant in time. Therefore, it is amenable to distributed implementation. Furthermore, it turns out that the optimal threshold can be chosen to be the maximum network throughput, which can be obtained by solving a fixed point equation. We then generalize the above study to the case with heterogeneous links, where different links may have different channel statistics. Due to the channel heterogeneity, the channel conditions corresponding to consecutive successful channel probings may follow different distributions. Again, we show that the optimal strategy for joint channel probing and distributed scheduling is a pure threshold policy. Somewhat surprisingly, the optimal thresholds turn out to be the same across all the links regardless of the channel statistics and contention probabilities. We further devise an iterative algorithm to compute the optimal threshold.

In the second thrust, we focus on distributed opportunistic scheduling from the user-centric perspective, where each link seeks to maximize its own throughput in a selfish manner. We treat the rate threshold selections across different links as a non-cooperative game. We establish the existence of the Nash equilibrium, and show the uniqueness of the Nash equilibrium under some sufficient conditions. We then propose the best response strategy to approach the

[1] A threshold policy is called pure if the threshold is invariant in time.

Nash equilibrium in an iterative manner. Observing that the best response strategy requires global information, we develop an online stochastic iterative algorithm based on local observations only. In light of the asynchronous feature of the online algorithm, we appeal to recent results on asynchronous stochastic approximation algorithms [7] and establish its convergence under standard conditions. Finally, we examine the efficiency loss in terms of the throughput in the non-cooperative game, compared to the network-centric case, and introduce a pricing-based mechanism to reduce the loss.

In summary, the study in this paper on distributed opportunistic scheduling, for both the network-centric case and the user-centric case, reveals that rich PHY/MAC diversities are available for exploitation in ad-hoc communications. We believe that these initial steps open a new avenue for channel-aware distributed scheduling, and are useful for enhancing MAC protocol design for wireless LANs and wireless mesh networks.

1.3 Related Work and Organization

As noted above, there has been much work on centralized opportunistic scheduling (e.g., [2], [5], [9], [16], [17], [25], [27]), channel-aware Aloha (e.g., [1, 20]) and MAC design with rate adaptation (e.g., [14, 19, 23]). Most relevant to our study are perhaps (e.g., [1, 14, 20, 23]). The main differences between this study and the studies [1, 20] lie in the following two aspects: 1) We consider ad hoc communications assuming no centralized coordination, and the transmission scheduling is done distributively; and 2) the transmitter nodes have no knowledge of other links' channel conditions, and even their own channel conditions are not available before contention. These limitations, dictated by the distributed nature of ad hoc communications, pose great challenges for exploiting channel diversity in distributed scheduling. A major difference between our study and the studies in [14, 23] is that our scheme allows links to opportunistically utilize the channel whereas in the schemes in [14, 23] the transmission rate is adapted based on the current channel condition, regardless of whether the channel condition is poor or good.

Along a different avenue, opportunistic channel probing for single-user multichannel systems has been studied in [13, 22], where the basic idea is to opportunistically probe and select a transmission channel among multiple channels between the transmitter node and the receiver node. In contrast, in this study, we consider multiple links (each with its own transmitter and receiver) sharing one single channel and explore distributed scheduling, assuming that each link has no knowledge of other links' channel conditions.

We note that a game theoretic formulation on random access protocols has been investigated, and the most relevant to our study for the user-centric case are perhaps [4, 8], with one major difference being that none of these works exploit time-varying channel conditions.

The rest of the paper is organized as follows. Section 2 introduces the model for random-access based channel probing and scheduling. Section 3 and Section 4 present the problem formulation for joint channel probing and scheduling from the network-centric perspective and the user-centric perspective, respectively. Section 5 compares the efficiency of the cooperative game with that of the non-cooperative game, and proposes a pricing mechanism to mitigate the price of anarchy. In Section 6, we provide numerical examples to corroborate the theoretic results. Finally, Section 7 concludes the paper. Due to space limitation, most details of the proofs are omitted in this conference version.

2. SYSTEM MODEL

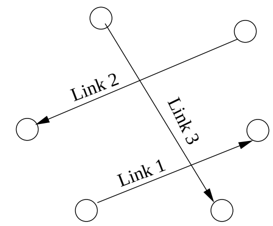

Figure 1: A single-hop ad-hoc network.

Random access is widely used for medium access control in wireless ad hoc networks. Consider a single-hop ad-hoc network with M links (see Fig. 1), where link m contends for the channel with probability p_m, $m = 1, \ldots, M$. A collision model is assumed for the random access, where a channel contention of a link is said to be successful if no other links transmit at the same time. We assume that the local channel condition can be obtained after a successful channel contention. Accordingly, the overall successful channel probing probability in each slot, p_s, is then given by $\sum_{m=1}^{M}(p_m \prod_{i \neq m}(1 - p_i))$ [20]. (To avoid triviality, we assume that $p_s > 0$.)

For convenience, we call the random duration of achieving a successful channel contention as one round of channel probing. It is clear that the number of slots (denoted by K) for a successful channel contention (probing) is a Geometric random variable, i.e., $K \sim Geometric(p_s)$. Let τ denote the duration of mini-slot for channel contention. It follows that the random duration corresponding to one round of channel probing is $K\tau$, with expectation τ/p_s.

Let $s(n)$ denote the successful link in the n-th round of channel probing, and $R_{n,s(n)}$ denote the corresponding transmission rate. In wireless communications, $R_{n,s(n)}$ depends on the time varying channel condition, and hence is random. Following the standard assumption on the block fading channel in wireless communications [14, 23], we assume that the rate $R_{n,s(n)}$ remains constant for a duration of T, where T is the data transmission duration and is no greater than the channel coherence time. [2]

To get a more concrete sense of joint channel probing and distributed scheduling, we depict in Fig. 2 an example with N rounds of channel probing and one single data transmission. Specifically, suppose after the first round of channel probing with a duration of $K_1\tau$, the rate of link $s(1)$, $R_{1,s(1)}$, is small (indicating a poor channel condition); and as a result, $s(1)$ gives up this transmission opportunity and let all the links re-contend. Then, after the second round of channel probing with a duration of $K_2\tau$, link $s(2)$ also gives up the transmission because $R_{2,s(2)}$ is small. This continues for N rounds until link $s(N)$ transmits because $R_{N,s(N)}$ is good.

In this paper, we provide a systematic study on distributed opportunistic scheduling by using optimal stopping

[2]Channel coherence time refers to the time during which the channel condition remains more or less unchanged.

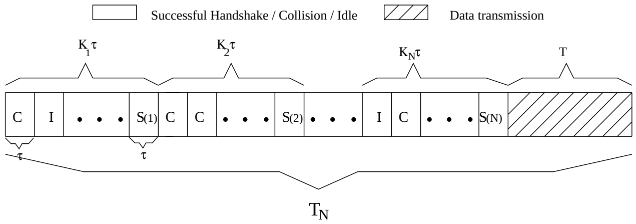

Figure 2: A sample realization of channel probing and data transmission

theory. We first impose the following assumption on the transmission rates across different rounds of channel probing.

A1) $\{R_{n,s(n)}, n = 1, 2, \ldots\}$ are independent.

We note that the above condition holds in many practical scenarios of interest, and the rational behind is as follows: 1) in a multi-user wireless network, the likelihood of one link (say link m) achieving two consecutive successful channel probing, $p_m^2 \prod_{i \neq m}(1-p_i)^2$, is fairly small, especially when the number of links is large; and 2), even if this happens, it is reasonable to assume that the channel conditions corresponding to two successful channel probings are independent since the channel probing duration in between is comparable to the channel coherence time.

3. DOS: A TEAM GAME VIEW

In this section, we treat distributed opportunistic scheduling, namely, joint channel probing and distributed scheduling, as a team game in which all links collaborate to maximize the overall network throughput. In particular, building on optimal stopping theory, we cast the problem as *maximizing the rate of return*, where the rate of return refers to the average throughput [12]. For convenience, let $R_{(n)}$ denote the rate corresponding to the n-th round successful channel probing, i.e., $R_{(n)} = R_{n,s(n)}$. Without loss of generality, we assume that the second moment of $R_{(n)}$ exists.

As illustrated in Fig. 2, after one round of channel probing, a stopping rule N decides whether the successful link carries out data transmission, or simply skips this opportunity and let all the links re-contend. Suppose that this game on joint channel probing and transmission is carried out L times, and let $\{N_1, N_2, \ldots, N_L\}$ denote the corresponding stopping times. let T_{N_l} denote the l-th realization of the duration for probing and data transmission. Then, appealing to the Renewal Theorem, we have that

$$x_L = \frac{\sum_{l=1}^{L} R_{(N_l)} T}{\sum_{l=1}^{L} T_{N_l}} \longrightarrow \frac{E[R_{(N)}T]}{E[T_N]} \quad a.s., \tag{1}$$

where $E[R_{(N)}T]/E[T_N]$ is the rate of return [12]. Clearly, $R_{(N)}$ and T_N are stopped random variables since N is a stopping time. Accordingly, the distribution of $R_{(N)}$ and T_N depend on that of the stopping time N. Define

$$Q \triangleq \{N : N \geq 1, E[T_N] < \infty\}. \tag{2}$$

It then follows that the problem of maximizing the long-term average throughput can be cast as a maximal-rate-of-return problem, in which a key step is to characterize the optimal stopping rule N^* and the optimal throughput x^*, as

$$N^* \triangleq \arg\max_{N \in Q} \frac{E[R_{(N)}T]}{E[T_N]}, \quad x^* \triangleq \sup_{N \in Q} \frac{E[R_{(N)}T]}{E[T_N]}. \tag{3}$$

3.1 Optimal Stopping Rule for Throughput Maximization

We now exploit optimal stopping theory [11, 12, 26] to solve the problem in (3).

3.1.1 The Case with Homogeneous Links

For ease of exposition, we first consider a network with homogenous links where all links have the same channel statistics with the same distribution $F_R(r)$. By **A1**, $\{R_{(n)}, n = 1, 2, \ldots\}$ is a sequence of i.i.d. random variables with distribution $F_R(r)$.

Observe that different from standard optimal stopping problems, the cost in terms of the probing duration is random due to the stochastic nature of channel probing. In light of this, we use the Maximal Inequality to establish the existence of the optimal stopping rule. We have the following proposition [28].

PROPOSITION 3.1. *a) The optimal stopping rule N^* exists, and is given by*

$$N^* = \min\{n \geq 1 : R_{(n)} \geq x^*\}. \tag{4}$$

b) The maximum throughput x^* is an optimal threshold, and is the unique solution to

$$E(R_{(n)} - x)^+ = \frac{x\tau}{p_s T}. \tag{5}$$

The proof is provided in Appendix A since it serves as the foundation for this study.

Remarks: Proposition 3.1 reveals that the optimal stopping rule N^* is a pure threshold policy, and the stopping decision can be made based on the current rate only. Accordingly, the optimal channel probing and scheduling strategy takes the following simple form: If the successful link discovers that the current rate $R_{(n)}$ is higher than the threshold x^*, it transmits the data with rate $R_{(n)}$; otherwise, it skips this transmission opportunity (e.g., by skipping CTS), and then the links re-contend.

We note that the maximum throughput x^* is unique, but the optimal threshold in (4) may not be unique in general. It is not difficult to show the uniqueness of the optimal threshold in the continuous rate case with $f(r) > 0$, $\forall\, r > 0$. In contrast, in the discrete rate case, changing the threshold in between two adjacent quantization levels would not affect its optimality since the new threshold policy achieves the same throughput. In what follows, for the discrete rate case, we treat the thresholds in between two adjacent quantization levels "effectively" the same.

3.1.2 The Case with Heterogeneous Links

In the above, it is assumed that all links have the same channel statistics. As a result, $R_{n,s(n)}$ follows the same distribution $F_R(r)$. In many practical scenarios, it is likely that different links may have different channel statistics. As a result, if $s(n+1) \neq s(n)$, $R_{n,s(n)}$ and $R_{n+1,s(n+1)}$ may follow different distributions. Nevertheless, we can treat $R_{n,s(n)}$ as a compound random variable. Accordingly, a key step is to characterize the distribution of $R_{n,s(n)}$ for the heterogeneous case.

To this end, let $F_m(\cdot)$ denote the distribution for each link $m \in \{1, 2, \ldots, M\}$. It can be shown that

$$P(R_{(n)} \le r) = E\left[P(R_{n,m} \le r) | s(n) = m\right] = \sum_{m=1}^{M} \frac{p_{s,m}}{p_s} F_m(r), \quad (6)$$

where $p_{s,m} \triangleq p_m \prod_{i \ne m}(1 - p_i)$ is the successful probing probability of user m. Based on (6), it is clear that $R_{(n)}$ is a compound random variable whose distribution is a "mixed" version of that across the links.

For convenience, let $\delta \triangleq \tau/T$. We have the following proposition regarding the optimal threshold policy.

PROPOSITION 3.2. *The maximum throughput x^* in the heterogeneous case is an optimal threshold, and is the unique solution to the following equation:*

$$x = \frac{\sum_{m=1}^{M} p_{s,m} \int_x^\infty r dF_m(r)}{\delta + \sum_{m=1}^{M} p_{s,m} \left(1 - F_m(x)\right)}. \quad (7)$$

Remarks: For the heterogeneous case, a priori, it is not clear that different links would have different thresholds or not since their channel statistics are different. However, Proposition 3.2 indicates that in the optimal strategy the threshold is the same for all the links (again, for the discrete rate case, we treat the thresholds in between two adjacent quantization levels "effectively" the same). Our intuition is as follows: when all the links have the same threshold, links with better channel conditions would have a higher likelihood to transmit accordingly.

3.2 Iterative Computation Algorithm for x^*

In the following, we devise an iterative algorithm to compute x^*. To this end, rewrite (7) as $x = \Phi(x)$, with

$$\Phi(x) \triangleq \frac{\sum_{m=1}^{M} p_{s,m} \int_x^\infty r dF_m(r)}{\delta + \sum_{m=1}^{M} p_{s,m} \left(1 - F_m(x)\right)}. \quad (8)$$

Accordingly, we propose the following iterative algorithm for computing x^*.

$$x_{k+1} = \Phi(x_k), \text{ for } k = 0, 1, 2, \ldots, \quad (9)$$

where x_0 is the initial value. We have the following proposition on the convergence of iterative algorithm (9).

PROPOSITION 3.3. *The iterates generated by algorithm (9) converge to x^* for any positive initial value x_0.*

A standard approach for establishing the convergence of iterative fixed point algorithms is via the Contraction (or Pseudo-Contraction) Mapping Theorem [6], which is unfortunately not applicable here since $\Phi(x)$ is not a pseudo-contraction mapping in some cases. For instance, suppose for any m, $f_m(r)$ is given by

$$f_m(r) = \begin{cases} 0, & x < 0 \\ 0.01, & 0 \le r < 96 \\ 0.005(r - 94), & 96 \le r < 98 \\ 0.02(r - 97)^{-3}, & r \ge 98 \end{cases} \quad (10)$$

Let $p_{s,m} = 0.99/\text{M}$ and $\delta = 0.05$. The corresponding optimal point $x^* = 72.82$. However,

$$|\Phi(95.5) - x^*| = |45.88 - 72.82| > |95.5 - 72.82|,$$

which violates the condition for pseudo-contraction mapping.

In what follows, we explore a different approach to examine the convergence of the iterative algorithm (9). We first need the following lemma.

LEMMA 3.1. *x^* is a global maximum point of $\Phi(x)$.*

It can be shown that $\Phi(x)$ is the average network throughput under the following stopping rule (cf. Lemma 4.1):

$$N = \min\{n \ge 1 : R_{(n)} \ge x\}.$$

The proof then follows from Proposition 3.1.

Proof of Proposition 3.3: From Lemma 3.1 and Proposition 3.2, it is clear that $y = \Phi(x)$ only intersects $y = x$ at the point x^*. This, together with the fact that $\Phi(0) > 0$, yields that

$$\Phi(x) \ge x, \forall\ x \le x^*; \Phi(x) \le x, \forall\ x > x^*. \quad (11)$$

Without loss of generality, we can assume that $x_0 \le x^*$ (we note that if $x_0 > x^*$, $x_1 = \Phi(x_0) \le \Phi(x^*) = x^*$ according to Lemma 3.1). Now suppose that $x_k \le x^*$. From (11), we obtain that $x_k \le \Phi(x_k) = x_{k+1} \le x^*$, where the last inequality is due to the fact that $\Phi(x_k) \le \Phi(x^*) = x^*$ from Lemma 3.1. Since $0 < x_0 \le x^*$, it follows that $\{x_k, k = 1, 2, \ldots\}$ is a monotonically increasing positive sequence with an upper-bound x^*. As a result, the sequence $\{x_k, k = 1, 2, \ldots\}$ converges to a limit, denoted as x_∞.

To show that $x_\infty = x^*$, we rewrite $x_{k+1} = \Phi(x_k)$ as

$$E[R_{(n)} - x_k]^+ - x_k \frac{\delta}{p_s} = (x_{k+1} - x_k) \left(\frac{\delta}{p_s} + \sum_{m=1}^{M} \frac{p_{s,m}}{p_s} \left(1 - F_m(x_k)\right) \right). \quad (12)$$

Observe that $E[R_{(n)} - x]^+$ is continuous in x (see the proof of Proposition 3.1), $x_{k+1} - x_k \to 0$ as $k \to \infty$, and $\frac{\delta}{p_s} + \sum_{m=1}^{M} \frac{p_{s,m}}{p_s} \left(1 - F_m(x_k)\right) \le \frac{\delta}{p_s} + 1 < \infty$. Therefore, taking limits on both sides of (12) yields that $E[R_{(n)} - x_\infty]^+ - \delta x_\infty / p_s = 0$.

Since from Proposition 3.2 $E[R_{(n)} - x]^+ = x\frac{\delta}{p_s}$ has a unique solution, we conclude that $x_\infty = x^*$. ∎

4. DOS: A NON-COOPERATIVE GAME PERSPECTIVE

4.1 Non-Cooperative Game Model for Threshold Selection

In the above sections, we formulate distributed opportunistic scheduling, namely joint channel probing and distributed scheduling, as a team game in which users collaborate together to optimize the network throughput. In this section, we treat joint channel probing and distributed scheduling as a non-cooperative game, where users seek to maximize their own throughput by choosing the thresholds $\{x_m, m = 1, 2, \ldots, M\}$ in a selfish manner. Towards this end, a first important step is to characterize the average throughput for each user for a given set of thresholds across users. Then, we cast the threshold selections of links as a non-cooperative game, in which each individual link chooses its threshold x_m to maximize its own throughput ϕ_m in a selfish manner. Specifically, let $G = [\{1, 2, \ldots, M\}, \times_{m \in \{1,2,\ldots,M\}} A_m, \{\phi_m, m \in \{1, 2, \ldots, M\}\}]$ denote the non-cooperative threshold selection game, where the links in $\{1, 2, \ldots, M\}$ are the players of the game, $A_m = \{x_m | 0 \le x_m < \infty\}$ is the action set of player m, and ϕ_m is treated as the utility function for player m. To examine the property of Game G, we first present the following lemma on $\phi_m(\mathbf{x})$.

LEMMA 4.1. *Assume that the threshold for user m is x_m, $m = 1, 2, \ldots, M$. Then, the average throughput of user m is given by*

$$\phi_m(\mathbf{x}) = \frac{p_{s,m} \int_{x_m}^{\infty} r dF_m(r)}{\delta + \sum_{i=1}^{M} p_{s,i} \left(1 - F_i(x_i)\right)}. \quad (13)$$

To get a more concrete understanding of $\phi_m(\mathbf{x})$, we rewrite (13) as follows:

$$\phi_m(\mathbf{x}) = \frac{\frac{\int_{x_m}^{\infty} r dF_m(r)}{1-F_m(x_m)} T}{\frac{\tau + \sum_{i \neq m} p_{s,i}(1-F_i(x_i))T}{p_{s,m}(1-F_m(x_m))} + T}. \quad (14)$$

It can be seen that the numerator in (14) is the expected throughput of user m, whereas the denominator can be decomposed into two parts: 1) the expected channel probing time $\frac{\tau + \sum_{i \neq m} p_{s,i}(1-F_i(x_i))T}{p_{s,m}(1-F_m(x_m))}$, and 2) the data transmission time T. Furthermore, in the expected channel probing time, $p_{s,m}(1 - F_m(x_m))$ is the successful probing and transmission probability of user m, while $\tau + \sum_{i \neq m} p_{s,i} \left(1 - F_i(x_i)\right) T$ can be viewed as the *effective channel probing time* for user m, consisting of the constant probing time τ and the average transmission time of other users $\sum_{i \neq m} p_{s,i} \left(1 - F_i(x_i)\right) T$.

4.2 Nash Equilibrium for Non-Cooperative Game

Treating the rate threshold selection as a non-cooperative game, we first examine the existence of Nash equilibrium [18]. By definition, a threshold vector $\mathbf{x}^*$ is said to be a Nash equilibrium of Game G, if

$$\phi_m(x_m^*, \mathbf{x}_{-m}^*) \geq \phi_m(x_m, \mathbf{x}_{-m}^*), \ 0 \leq x_m < \infty, \ \forall\ m, \quad (15)$$

where $\mathbf{x}_{-m} \triangleq [x_1, \cdots, x_{m-1}, x_{m+1}, \cdots, x_M]^T$. In other words, at the Nash equilibrium, no link can increase its throughput by unilaterally deviating its threshold from the equilibrium, given the thresholds of other links.

Based on [18, Proposition 20.3], by showing that $\phi_m(\mathbf{x})$ is a quasi-concave function on x_m, we have the following proposition on the existence of the Nash equilibrium for the threshold selection game.

PROPOSITION 4.1. *There exists a Nash equilibrium in the threshold selection game G, which satisfies the following set of equations: for $m = 1, 2, \ldots, M$,*

$$x_m^* = \phi_m(x_m^*, \mathbf{x}_{-m}^*) = \frac{p_{s,m} \int_{x_m^*}^{\infty} r dF_m(r)}{\delta + \sum_{i=1}^{M} p_{s,i} \left(1 - F_i(x_i^*)\right)}. \quad (16)$$

We further have the following proposition on the property of the Nash equilibria that satisfy (16).

PROPOSITION 4.2. **[Componentwise monotonicity]:** *Suppose $\mathbf{x}^*$ and $\mathbf{y}^*$ are two Nash equilibrium points satisfying (16). If there exists $k \in \{1, 2, \ldots, M\}$ such that $x_k^* < y_k^*$, then $x_m^* < y_m^*$, $\forall\ m \in \{1, 2, \ldots, M\}$, i.e., $\mathbf{x}^* < \mathbf{y}^*$.*

4.3 Uniqueness of Nash Equilibrium

Needless to say, the uniqueness of Nash equilibrium is an important issue for the non-cooperative game. Unfortunately, in general, the Nash equilibrium that satisfies (16) is not necessarily unique, as illustrated by the following simple example. Suppose that there are two links in the network, with the same rate distribution as

$$R(r) = \begin{cases} \text{2Mbps}, & \text{w.p. } 0.5, \\ \text{12Mbps}, & \text{w.p. } 0.5. \end{cases} \quad (17)$$

Let $p_{s,1} = p_{s,2} = 0.2$ and $\delta = 0.35$. Then, there exist two Nash equilibria at $\mathbf{x} = (1.867, 1.867)$ and $\mathbf{x} = (2.18, 2.18)$ that satisfy (16).

In what follows, we provide some sufficient conditions for establishing the uniqueness of Nash equilibrium. Consider a network with homogeneous links, where all links have the same channel statistics $F(r)$ and the same contention probability. Then, (16) boils down to

$$x_m^* = \phi_m(x_m^*, \mathbf{x}_{-m}^*) = \frac{\frac{p_s}{M} \int_{x_m^*}^{\infty} r dF(r)}{\delta + \frac{p_s}{M} \sum_{i=1}^{M} (1 - F(x_i^*))}. \quad (18)$$

4.3.1 Continuous Rate over Rayleigh Fading

Consider a special case, where the transmission rate is given by the Shannon channel capacity:

$$R(h) = \log(1 + \rho h) \text{ nats/s/Hz}, \quad (19)$$

where ρ is the normalized average SNR, and h is the random channel gain corresponding to Rayleigh fading.

PROPOSITION 4.3. *The Nash equilibrium of the threshold selection game G is unique under the rate model in (19).*

4.3.2 General Continuous Rate Case

Consider a homogenous network where the transmission rate follows a general continuous distribution with pdf $f(r) \geq 0$, $\forall\ r > 0$. We have the following sufficient condition regarding the uniqueness of Nash equilibrium.

PROPOSITION 4.4. *The Nash equilibrium of the threshold selection game G is unique if $rf(r) < \frac{\delta}{p_s(M-1)}$, $\forall\ r > 0$.*

4.4 Best Response Strategy

Based on the structure of game G, we can use the following *best response strategy* to iteratively compute the Nash equilibrium: $\forall\ m \in \{1, 2, \cdots, M\}$,

$$x_m(k+1) = x_m^*(k), \text{ for } k = 0, 1, 2, \cdots \quad (20)$$

where $x_m^*(k)$ is the unique solution to the equation

$$x_m = \phi_m(x_m, \mathbf{x}_{-m}(k)).$$

Remarks: The algorithm in (20) is a two time-scale iterative algorithm: on the smaller time-scale, each link can use an iterative algorithm to compute $x_m^*(k)$; and on the larger time-scale, each link updates its threshold based on (20).

PROPOSITION 4.5. *Suppose that the Nash equilibrium is unique. Then, for any non-negative initial value $\mathbf{x}(0)$, the sequence $\{\mathbf{x}(k)\}$ generated by algorithm (20) converge to the Nash equilibrium $\mathbf{x}^*$, as $k \to \infty$.*

4.5 Online Algorithm for Computing Nash Equilibrium

Observe that in (20), computing x_m^* requires the knowledge of $\sum_{i=1}^{M} p_{s,i}(1 - F_i(x_i^*))$, which involves the channel information of all links. In this section, a distributed asynchronous iterative algorithm is proposed in which each link independently computes the optimal threshold x_m^*, $\forall\ m \in \{1, 2, \cdots, M\}$, based on local observations only.

Rewrite (16) as

$$x_m^* = \frac{p_{s,m} \int_{x_m^*}^{\infty} r dF_m(r) - x_m^* \delta}{\sum_{i=1}^{M} p_{s,i}(1 - F_i(x_i^*))}, \quad \forall\ m.$$

Define

$$g_m(\mathbf{x}) \triangleq \frac{p_{s,m}\int_{x_m}^{\infty} r dF_m(r) - x_m\delta}{\sum_{i=1}^{M} p_{s,i}(1-F_i(x_i))} - x_m.$$

If the Nash equilibrium is unique, then $\mathbf{x}^*$ is the unique root to the equation $g(\mathbf{x}) = 0$.

Recall that the collision model is assumed for channel contention, indicating that at most one link can successfully occupy the channel each time. As a result, only the successful link can update its threshold. Clearly, the updating is *asynchronous* across the links.

Let $v(k)$ denote the duration of channel probing between the $(k-1)$th transmission and the kth transmission, which can be observed locally. Later, we show that $v(k)$ is a local "unbiased estimation" of $1/\sum_{i=1}^{M} p_{s,i}(1-F_i(x_i(k)))$. Define

$$\widetilde{g_m}(k) \triangleq v(k)\left[p_{s,m}\int_{x_m(k)}^{\infty} r dF_m(r) - x_m(k)\delta\right] - x_m(k). \quad (21)$$

It is clear that $\widetilde{g_m}(k)$ involves local information only. Let N^m be an infinite subset of $\mathcal{N}$ indicating the set of times at which an update of x_m is performed. Based on stochastic approximation theory, the distributed iterative algorithm can be written as

$$x_m(k+1) = [x_m(k) + a_m(k)\,[\widetilde{g_m}(\mathbf{x}(k))]\, I\{k \in N^m\}]_0^b\,, \quad (22)$$

where $I\{\cdot\}$ is the indicating function, $[\cdot]_0^b$ is the projection between 0 and b, and $a_m(k)$ is the stepsize defined as

$$a_i(k) = a(i, \sum_{l=1}^{k} I\{k \in N^i\}).$$

Based on [7], we impose the following conditions.

B1) The sequence $\{a(i,k)\}$ satisfy

$$\sum_{k=1}^{\infty} a(i,k) = \infty, \quad \text{and} \quad \sum_{k=1}^{\infty} a(i,k)^2 < \infty.$$

and for $\beta \in (0,1)$, $\forall\, i, j$

$$\lim_{k\to\infty} \frac{\sum_{l=1}^{\lfloor \beta k \rfloor} a(i,l)}{\sum_{l=1}^{k} a(i,l)} = 1, \quad \text{and} \quad \lim_{k\to\infty} \frac{\sum_{l=1}^{k} a(i,l)}{\sum_{l=1}^{k} a(j,l)} > 0.$$

B2) The Nash equilibrium defined in (16) is unique.

PROPOSITION 4.6. *Under Conditions* **B1** *and* **B2***, for any non-negative initial value* $\mathbf{x}(0)$*, the sequence* $\{\mathbf{x}(k)\}$ *generated by (22) converge to the Nash equilibrium* $\mathbf{x}^*$ *almost surely, as* $k \to \infty$*.*

Sketch of the proof: We first examine the asynchronism term and the unbiased estimation term in (22), and show that the effect of those terms would diminish as $k \to \infty$. Then, the convergence can be shown by using the Arzela-Ascoli Theorem.

5. THE PRICE OF ANARCHY

In this section, we compare the efficiency of the cooperative game with that of the non-cooperative game. Towards this end, let x_{co}^* denote the optimal network throughput in the cooperative case. Recall $x_{co}^* = x^*$ where x^* is the root to (7). Let x_{nco}^* denote the network throughput at the Nash equilibrium point $\mathbf{x}^*$ for the non-cooperative case, and that $x_{nco}^* = \sum_{m=1}^{M} \phi_m(\mathbf{x}^*)$. Clearly, the optimal network throughput in the cooperative game is no less than the network throughput at the Nash equilibrium in the non-cooperative game, i.e., $x_{co}^* \geq x_{nco}^*$. We have the following result regarding the efficiency of the two different games.

PROPOSITION 5.1. *If* $m \geq 2$ *and* $f_m(r) > 0$, $\forall\, m, r$*, then the optimal network throughput in the cooperative game is always larger than that at the Nash equilibrium in the non-cooperative game, i.e.,* $x_{co}^* > x_{nco}^*$*.*

The Nash equilibrium offers a solution to the non-cooperative game, where no link can improve its throughput any further through individual effort. However, the non-cooperative game approach is inefficient due to the selfish decisions made by individual links, and this is the so-called *price of anarchy* [21].

The price of anarchy can be reduced by introducing a pricing-based mechanism, in which users are "encouraged" to adopt a social behavior. In the above study, each link aims to maximize its own throughput $\phi_m(x)$ by adjusting its threshold x_m, but the overhead it imposes on other links is ignored. In order to mitigate the overhead, a plausible pricing function is given by $c_m(\mathbf{x}) = c\alpha_m(\mathbf{x})$, where c is a preset-parameter for all links and $\alpha_m(\cdot)$ is defined as

$$\alpha_m(\mathbf{x}) \triangleq \frac{p_{s,m}(1-F_m(x_m))}{\delta + \sum_{i=1}^{M} p_{s,i}(1-F_i(x_i))}, \quad (23)$$

which points to the portion of time link m transmits. It is a usage-based pricing policy, where the cost(charge) is proportional to the amount of services consumed by the link [24]. Accordingly, define the utility function as $u_m(\mathbf{x}) \triangleq \phi_m(\mathbf{x}) - c_m(\mathbf{x})$. Then, the "new" non-cooperative game is as follows:

$$(\tilde{\mathbf{G}}) \quad \max_{x_m}\; u_m(\mathbf{x}),\; m = 1, 2, \ldots, M. \quad (24)$$

PROPOSITION 5.2. *There exists a Nash equilibrium* $\tilde{\mathbf{x}}^*$ *in the new game* $\tilde{\mathbf{G}}$*, which outperforms the one without pricing mechanism, i.e.,* $\tilde{x}_{nco}^* \triangleq \sum_{m=1}^{M} \phi_m(\tilde{\mathbf{x}}^*) \geq x_{nco}^*$*.*

In Section 6.3, we will compare the results in games with and without pricing, and show the price of anarchy could be reduced by the pricing-based mechanism.

6. NUMERICAL RESULTS

6.1 Numerical Examples for the Cooperative Game

Needless to say, a key performance metric is the throughput gain of distributed opportunistic scheduling over the approaches without using optimal stopping. Consider the case that the transmission rate is given by the Shannon channel capacity:

$$R(h) = \log(1+\rho h) \text{ nats/s/Hz},$$

where ρ is the normalized average SNR, and h is the random channel gain corresponding to Rayleigh fading. For convenience, define the throughput gain as

$$g \triangleq \frac{x^* - x^L}{x^L}. \quad (25)$$

where x^L is the average throughput of the OAR scheme [23] without using optimal stopping, and $x^L = \Phi(0)$.

In the following, we illustrate the gain via numerical examples for homogeneous networks and heterogeneous networks.

6.1.1 Homogeneous Networks

It follows from (5) that

$$x^* = \Phi(x^*) = \frac{x^* \exp\left(-\frac{\exp(x^*)}{\rho}\right) + E_1\left(\exp(x^*)/\rho\right)}{\frac{\exp(-1/\rho)\delta}{p_s} + \exp\left(-\frac{\exp(x^*)}{\rho}\right)}, \quad (26)$$

where $E_1(x)$ is the *exponential integral function*.

We have the following results on the optimal throughput x^* and the throughput gain $g(\rho)$.

COROLLARY 6.1. *The optimal throughput x^* is an increasing function of the average SNR ρ. Furthermore, $x^*(\rho) \longrightarrow 0$ as $\rho \longrightarrow 0$, and $x^*(\rho) \longrightarrow \infty$ as $\rho \longrightarrow \infty$. The throughput gain $g(\rho)$ is a monotonically decreasing function of ρ, and approaches the maximum when $\rho \longrightarrow 0$, i.e.,*

$$g(\rho) \longrightarrow \left(1 + \frac{\delta}{p_s}\right) \frac{dx^*(\rho)}{d\rho}\bigg|_{\rho=0} - 1, \quad as\ \rho \longrightarrow 0, \quad (27)$$

where $\frac{dx^*(\rho)}{d\rho}\Big|_{\rho=0}$ is the root of $x\exp(x) = \frac{p_s}{\delta}$.

We provide numerical examples to illustrate the above results. Unless otherwise specified, we assume that τ, T, p, and m are chosen such that $\delta = 0.1, p_s = \exp(-1)$.

Table 1 illustrates that $g(\rho)$ is more significant in the low SNR region, and is a decreasing function of ρ. In Table 2, we present the maximum throughput gain $g(0)$ as a function of δ/p_s. It can be observed that $g(0)$ increases as the value of δ/p_s decreases. Intuitively speaking, a smaller value of δ indicates that the channel probing incurs less overhead; and a larger value of p_s implies that the random access scheme yields higher throughput.

Table 1: Throughput gain

ρ	0.5	1	2	5	10
x^*	0.40	0.60	0.90	1.40	1.80
x^L	0.28	0.47	0.73	1.17	1.58
$g(\rho)$	42.8%	27.7%	23.3%	19.7%	13.9 %

6.1.2 Heterogeneous Networks

Based on (7), it can be shown that the optimal threshold for the heterogeneous case, x^*, satisfies the following equation:

$$x^* = \frac{1}{\delta} \sum_{m=1}^{M} p_{s,m} \exp\left(\frac{1}{\rho_m}\right) E_1\left(\frac{\exp(x^*)}{\rho_m}\right). \quad (28)$$

Note that the average throughput without using optimal stopping rule is given by

$$x^L = \frac{\sum_{m=1}^{M} p_{s,m} \exp(1/\rho_m) E_1\left(1/\rho_m\right)}{\delta + p_s}. \quad (29)$$

In the following example, we consider a heterogeneous network model with 5 users, each with different transmission probabilities and channel statistics. The performance of the iterative algorithm (9) is examined in Table 3. Clearly, the iterative algorithm (9) exhibits fast convergence rate.

Table 2: Maximum throughput gain

δ/p_s	0.136	0.271	0.544	1.359	2.718
g (numerical)	76.4%	47.0%	25.7%	9.2%	3.5%
g (by (27))	76.6%	47.2%	25.7%	9.2%	3.5%

Table 3: Convergence behavior of the iterative algorithm (9)

ρ (dB)	x_0	x_1	x_2	x_3	x^*
[0 10 10 8.5 6]	0.684	1.259	1.382	1.385	1.385
[10 10 10 8.5 6]	0.026	1.620	1.877	1.892	1.892
[20 10 10 8.5 6]	0.777	2.695	3.054	3.073	3.073

As is clear in (28), the optimal threshold x^* (namely the maximum throughput) depends on all SNR parameters $\{\rho_m, \forall\ m\}$ across links, and is monotonically increasing in each ρ_m. However, different from the homogeneous case, the gain g is no longer monotonically decreasing in each individual SNR. To get a more concrete sense, we plot in Fig. 3 the relationship between g and ρ_1, with other SNR parameters fixed. As illustrated in the figure, g decreases as ρ_1 increases from -10dB to 10dB. This is because when ρ_1 is small, the optimal throughput x^* is determined mainly by other SNR parameters and remains almost constant, whereas the throughput without using optimal stopping strategy (x^L) always increases. Furthermore, g increases when ρ_1 exceeds 10dB. Our intuition is that in this SNR regime user 1 becomes the dominating user in the system, and therefore x^* increases much faster than x^L.

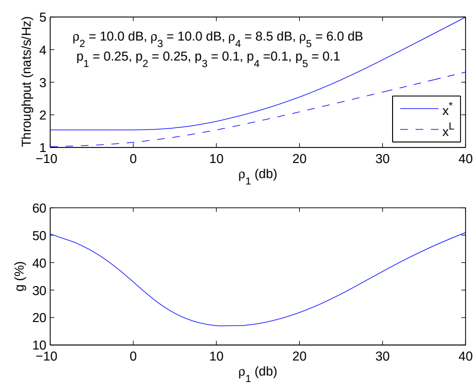

Figure 3: Throughput gain $g(\rho)$ as a function of average SNR ρ_1

6.2 Numerical Examples for the Non-Cooperative Game

Table 4 illustrates the convergence behavior of the best response strategy defined in (20), for 2 links randomly picked from the 5 links. It can be seen that with the knowledge of global information, the threshold converges to the optimal point within a few iterations. Fig. 4 depicts the convergence behavior of the online algorithm for computing Nash Equilibrium. As expected, it takes hundreds of iterations for the proposed asynchronous distributed stochastic algorithm to converge. Moreover, both those two algorithms converge to the same equilibrium point.

Table 4: Convergence behavior of the best response strategy

Link index	x_0	x_1	x_2	x_3	x^*
Link 1 ($\rho_1 = 3$dB)	1.00	0.267	0.298	0.300	0.30
Link 2 ($\rho_2 = 5$dB)	1.00	0.175	0.389	0.390	0.39

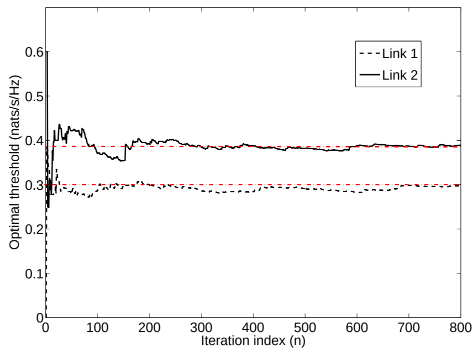

Figure 4: Convergence behavior of the online algorithm for computing Nash Equilibrium

6.3 Numerical Examples for Price of Anarchy

Table 5: The Price of Anarchy

Number of links	1	2	3	4	5
x^*_{co}	0.586	0.664	1.085	1.217	1.364
x^*_{nco}	0.586	0.624	0.994	1.043	1.127
η	100%	94.0%	91.6%	85.7%	82.6%
$x^*_{pricing}$	0.586	0.650	1.055	1.170	1.293
η'	100%	97.9%	97.2%	96.1%	94.8%

In Table 5, we examine the efficiency loss due to the selfish behavior of individual links, i.e., the price of anarchy. The efficiency is defined as $\eta \triangleq x^*_{nco}/x^*_{co}$ [21].

It can be seen from Table 5 that the efficiency is strictly less than 1 when two or more links exist in the network, which corroborates the conclusion of Proposition 5.1.

In Table 5, we also present the efficiency improvement by using the pricing mechanism. Let $x^*_{pricing}$ denote the network throughput at the Nash equilibrium for the non-cooperative game with pricing $\mathbf{G}'$ defined in (24). The efficiency η' is defined as $\eta' = x^*_{pricing}/x^*_{co}$. It can be seen from Table 5 that by carefully choosing the parameter c, the efficiency loss can be significantly reduced. However, it is still unable to achieve the optimal throughput in the cooperative case (the social optimum).

7. CONCLUSIONS

In this paper, we considered ad-hoc communications based on random access, and studied distributed opportunistic scheduling to resolve collisions therein while exploiting channel variation. In ad-hoc communications, distributed opportunistic scheduling boils down to a process of joint channel probing and distributed scheduling, and we investigated distributed opportunistic scheduling from two different perspectives, namely, the network-centric perspective and the user-centric perspective.

We first considered distributed opportunistic scheduling from a network-centric point of view, where links cooperate to maximize the overall network throughput. Specifically, we treated the channel probing and scheduling as a maximal-rate-of-return problem, and characterized the optimal strategy that yields the maximum overall throughput, for both homogenous networks and heterogeneous networks. We showed that the optimal strategy is a pure threshold policy, where the threshold is the maximum throughput, and is the solution to a fixed point equation. Furthermore, we devised an iterative algorithm to compute it.

Next, we studied distributed opportunistic scheduling from a user-centric perspective, where links seek to maximize their own throughput. We treated the problem of threshold selections for different links as a non-cooperative game. Accordingly, we explored the existence and uniqueness of the Nash equilibrium, and showed that the Nash equilibrium can be approached by the best response strategy. We then developed an online stochastic iterative algorithm based on local observations only, and we established its convergence under standard conditions, using recent results on asynchronous stochastic approximation algorithms. As expected, we observed an efficiency loss at the Nash equilibrium, and we proposed a pricing-based mechanism to mitigate the loss.

Due to space limitation, we omit the generalization to cases with general utility functions, and skip most details of the proofs.

8. ACKNOWLEDGEMENT

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APPENDIX

A. PROOF OF PROPOSITION 3.1

The proof of Proposition 3.1 hinges heavily on the tools in optimal stopping theory [12]. More specifically, based on Theorem 1 in [12, Chapter 6], in order to maximize the average throughput $\frac{E[R_{(N)}T]}{E[T_N]}$, a key step is to find an optimal stopping algorithm $N(x)$ such that

$$V^*(x) = E[R_{(N(x))}T - xT_{N(x)}] = \sup_{N \in Q} E[R_{(N)}T - xT_N].$$

It then follows from Theorem 1 in [12, Chapter 3] that $N(x)$ exists if the following conditions are satisfied:

$$E\sup_n Z_n < \infty, \quad \text{and} \quad \limsup_{n \longrightarrow \infty} Z_n = -\infty \ a.s., \tag{30}$$

where $Z_n \triangleq R_{(n)}T - xT_n$, $T_n \triangleq \sum_{j=1}^n K_j\tau + T$, and $K_j, j = 1, 2, \ldots, n$, denote the number of contentions during the jth channel probing.

The rest of the proof has two main steps. Step 1: we establish the existence of the optimal stopping rule $N(x)$; Step 2: we characterize the optimal strategy N^*.

Step 1: It is clear that $\limsup_{n\to\infty} Z_n \longrightarrow -\infty$.

Observe that $E[\sup_n Z_n]$ is upper-bounded by

$$\begin{aligned} E[\sup_n Z_n] \quad &\leq \quad E\left[\sup_n \left\{R_{(n)}T - nx\tau\left(\frac{1}{p_s} - \epsilon\right)\right\}\right] - Tx \\ &\quad + E\left[\sup_n \sum_{j=1}^n x\tau\left(\frac{1}{p_s} - \epsilon - K_j\right)\right], \end{aligned} \tag{31}$$

where ϵ is chosen such that $0 < \epsilon < 1/p_s$. It then follows from the Maximal Inequalities in Theorem 1 and Theorem 2 in [12, Chapter 4] that the first term and the last term of the right hand side of (31) are both finite, and hence $E[\sup_n Z_n] < \infty$.

Step 2: Next, we characterize $N(x)$ and N^*. It can be shown that the optimal stopping rule $N(x)$ is given by

$$N(x) = \min\{n \geq 1 : R_{(n)}T \geq V^*(x) + xT\}, \tag{32}$$

and $V^*(x)$ satisfies the following *optimality equation*:

$$E[\max(R_{(n)}T - xT - Kx\tau, V^*(x) - Kx\tau)] = V^*(x). \tag{33}$$

Note that $V^*(x^*) = 0$ from Theorem 1 in [12, Chapter 6] and (33) becomes $E[R_{(n)} - x^*]^+ = \frac{x^*\tau}{p_sT}$ since $E[K] = 1/p_s$. The optimal stopping rule (32) now becomes $N^* = \min\{n \geq 1 : R_{(n)} \geq x^*\}$.

Next we show that (5) has a unique solution. We first note that $f(x) \triangleq E[R_{(n)} - x]^+$ is continuous in x. To see this, let $\{x_m, m = 1, 2, \ldots\}$ be a sequence of real positive numbers, and $\lim_{m\to\infty} x_m = x$, then $R_{(n)} - x_m \to R_{(n)} - x$ almost surely. Since $|R_{(n)} - x_m| \leq R_{(n)}$, we have that $f(x_m) \to f(x)$ using Dominated Convergence Theorem [11]. Since $f(x)$ decreases from $E[R_{(n)}]$ to 0 and the right hand side of (5) strictly increases from 0 to ∞ as x grows, it follows that (5) has a unique finite solution, thereby concluding the proof.